Control Systems

28-07-2014

BASICS OF CONTROL SYSTEMS

* **Open Loop System :-**
  - In case of open loop system, its final point is undefined. We give some input and corresponding to that input, output is coming which is not pre-defined, It is behaviour of open loop system.

* **Closed Loop system :-**
  - Here, final point is defined and system will try to achieve that final point.

* In case of open loop systems, its location of Pole is independent of any parameter while in case of Closed loop systems, its location of pole is Parameter-dependent and under specific condition, There is possibility that pole can lie on Right-Half of Imaginary-axis.

  So, Open Loop system is more stable than Closed Loop System.

* Zero in any system only changes the output response. Zero won't affect the stability of the system.

* Due to Pole, exponential terms exist in the output response and if after settling time exponential terms are 0, then that system will be stable system otherwise Unstable System.
  - If exponential terms do not exist in Output Response, only Sinusoidal terms exist, then system is Marginally Stable System.

* If pole locates on Left-half of s-plane, then exponential terms will settle after t_s and that system will be Stable.

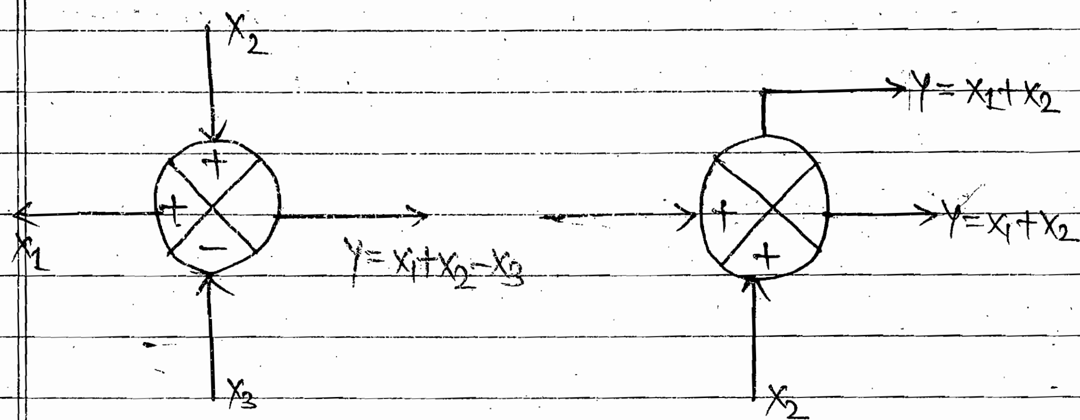
* If pole locate itself in Right-half side, exponential terms will not settle after t_s and so, system will be Unstable.

Chapter :- 1

BLOCK DIAGRAM AND SIGNAL FLOW GRAPH

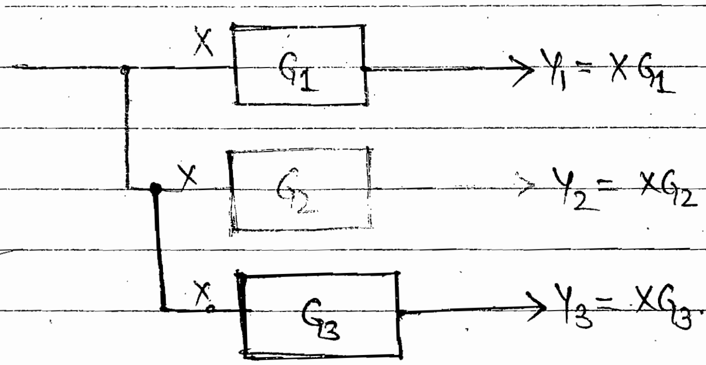
* → BLOCK DIAGRAM :-

* → Summation Point :-



* → Take-off Point :-

- When two different systems require common input, in that case, we will use Take-off point and from Equipotential line, we can take infinite number of Take-off points.



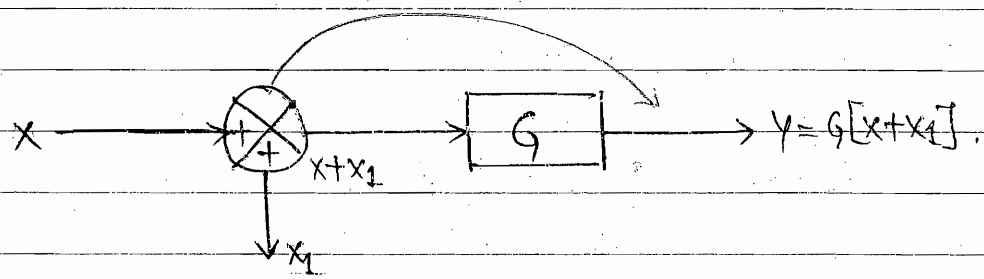
* → Forward Gain :-

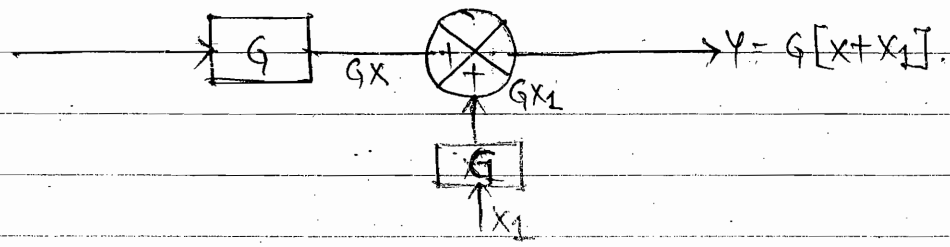
- In case of Forward Gain, direction is flow is from Input towards Output and Transfer function is Ratio of output with respect to input.

Under specific condition, a Closed Loop System can act as Forward Gain but it will never act like Feedback Gain.

Case:- I

**• To move summation on other side of Take-off point in direction of Flow. :-

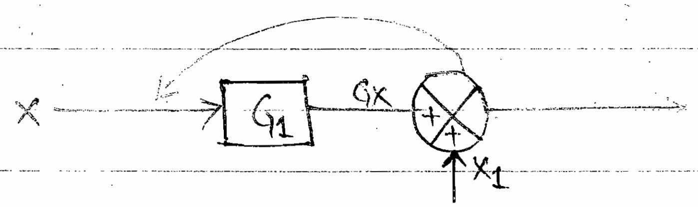


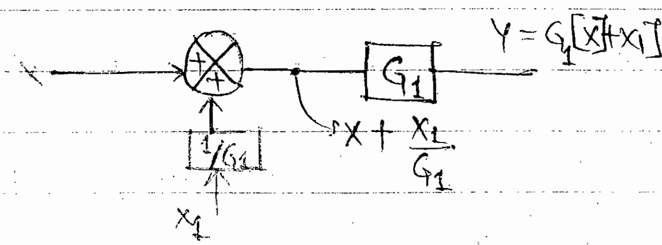


• To do this, Gain term (G) will be introduced at its second input.

Case:- II

•• To move summation on other side of Take-off point in direction opposite to that of flow :-

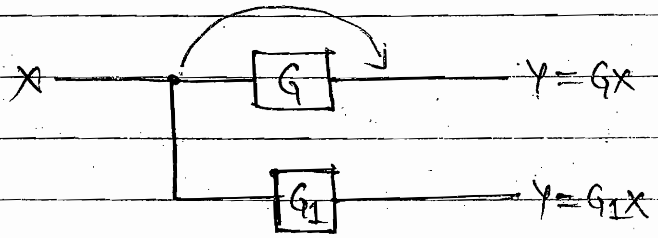


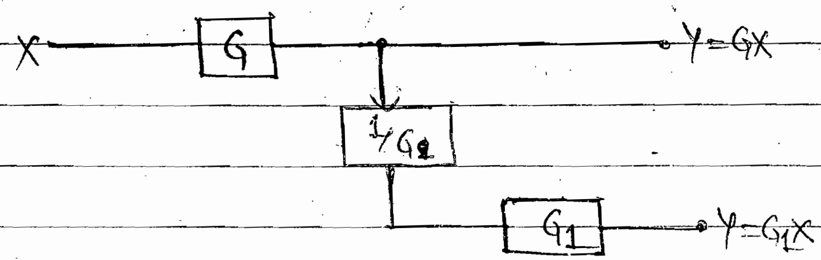


• Inverse of Gain term (1/G) will be introduced at its second input.

Case:- III

** • To move take-off point on other side of Gain in the direction of Flow :-

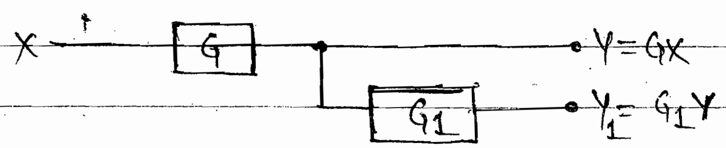


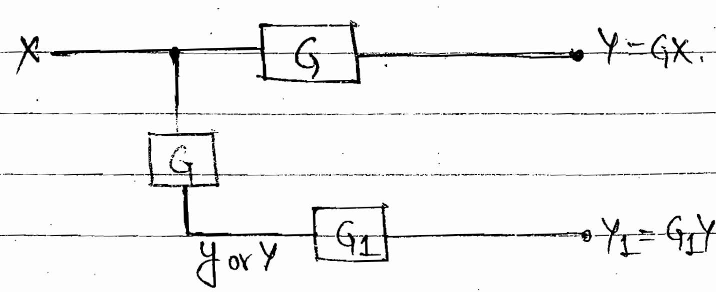


• Inverse of Gain term will be introduced at take-off point.

Case:- IV

** To move take-off point on other side of Gain in the direction opposite to that of flow :-

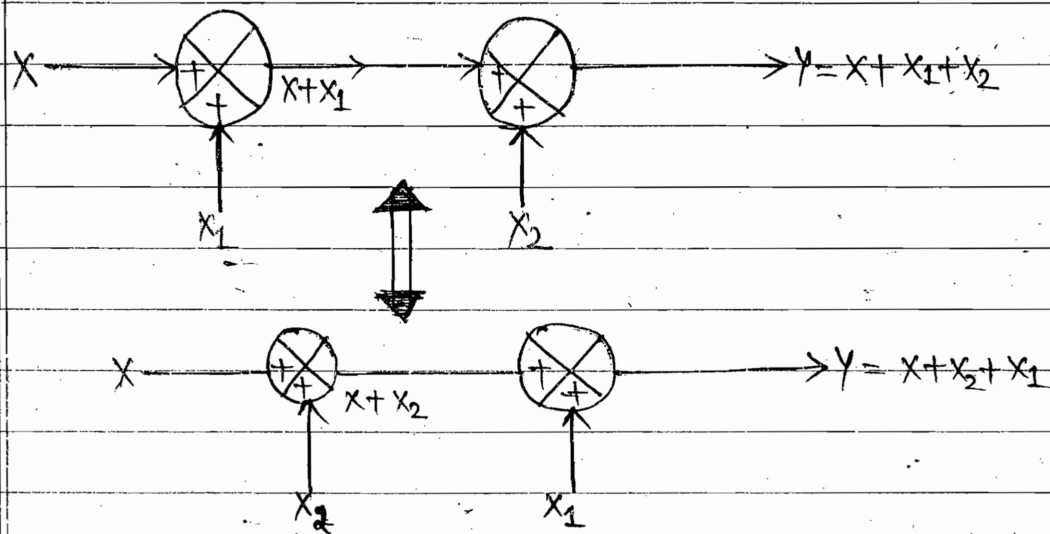




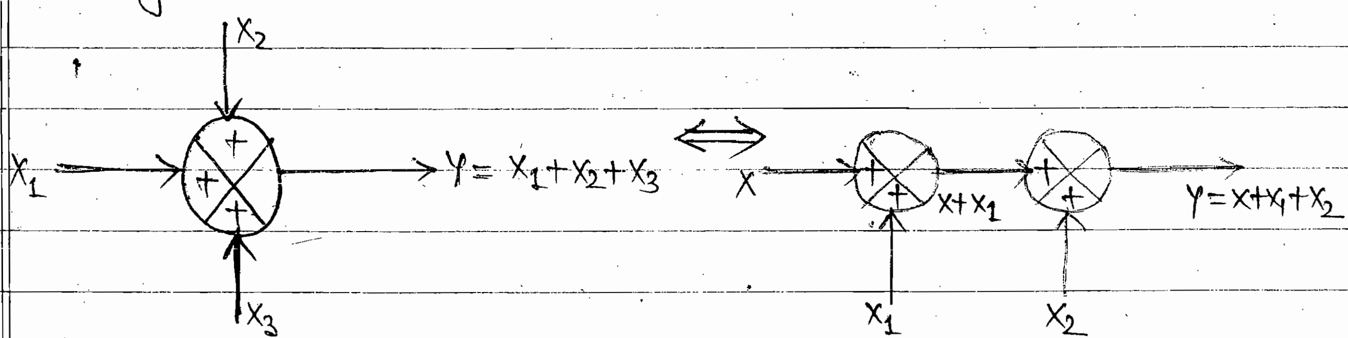
• Gain term (G) will be introduced at Take-off point.

* → Key Factors in Solving Block Diagrams :-

(i) Always try to bring two summation side by side and if required, interchange their respective inputs.

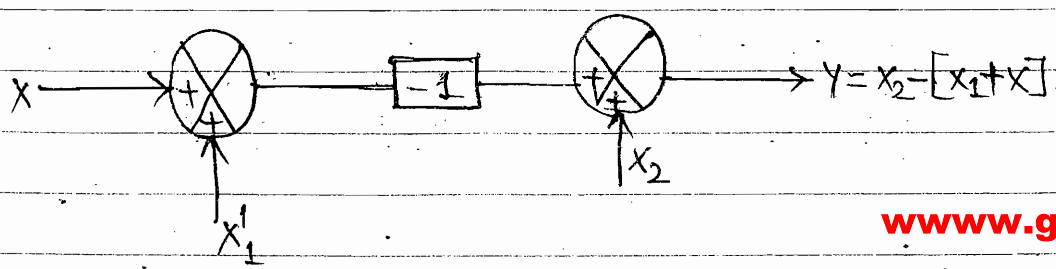


(ii) Always convert single three-input summation into two 2-input summation side by side.

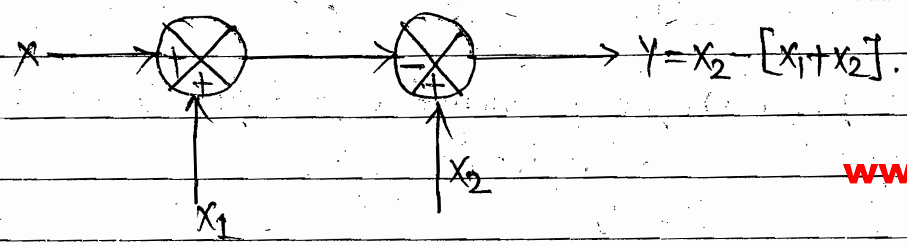


(3-Input summation) (2-input summation).

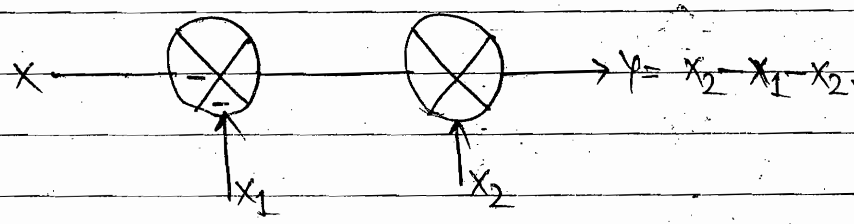
- Further :→



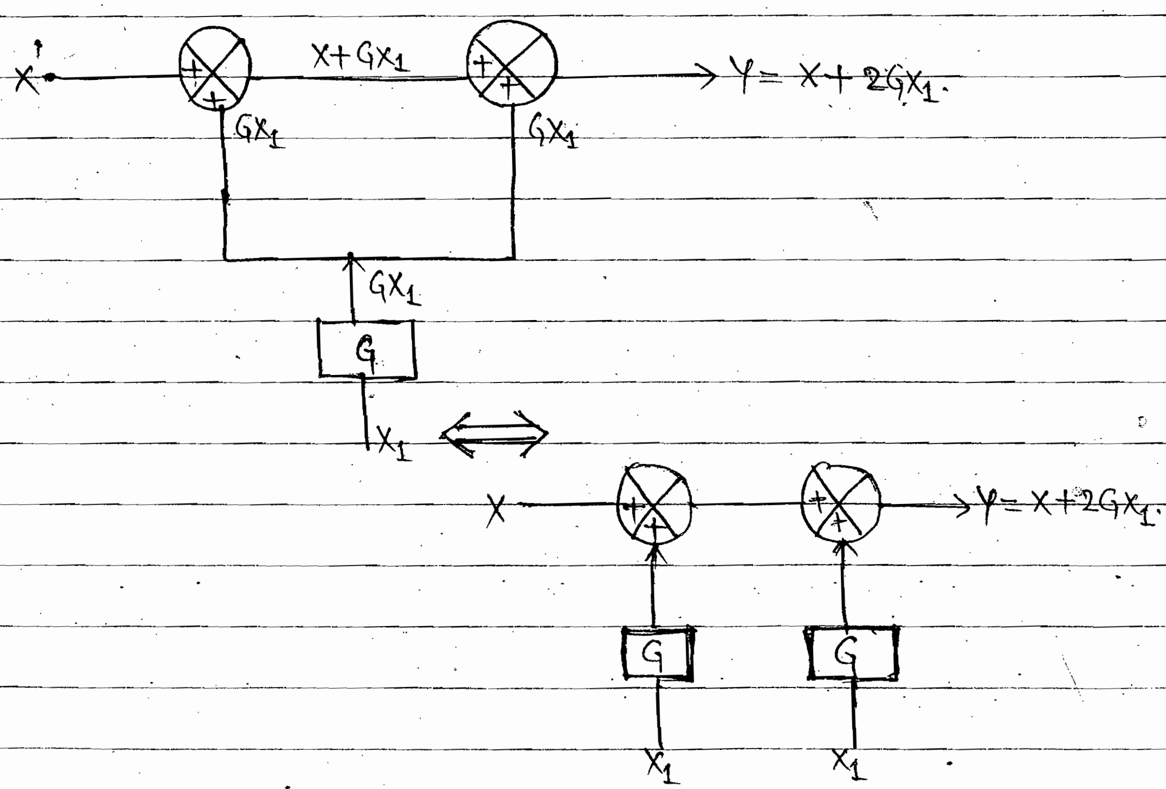
So, it is equivalent to :-



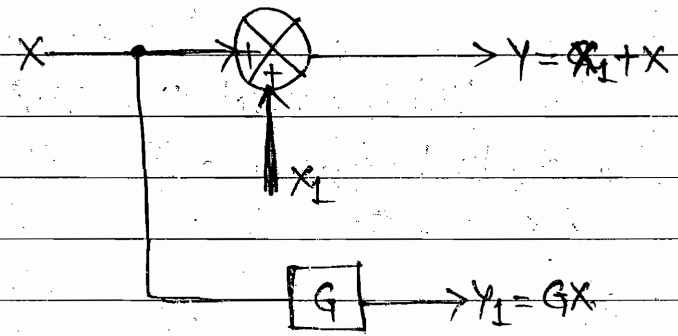
Similarly,

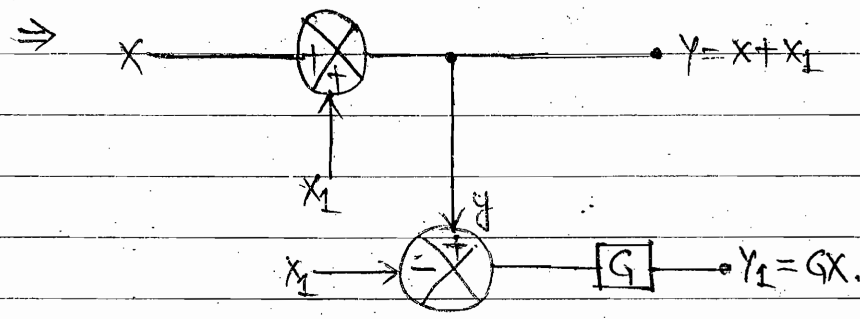


(iii) If any path is common between two summation, we can break that path into two parallel paths.



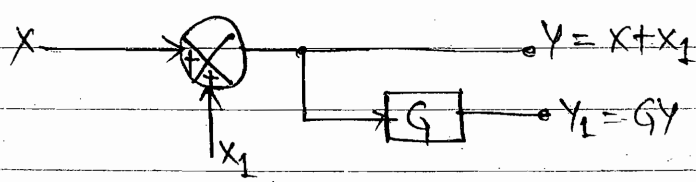
**** To move Take-off point on other side of summation in the direction of Flow :-**

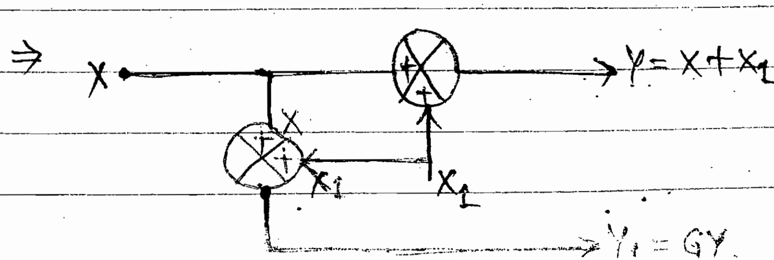




- Second summation will be introduced at take-off point and second input of that summation will be same as that of First summation but in Reverse Polarity.

**** To move Take-off point on other side of summation in opposite direction to flow :-**

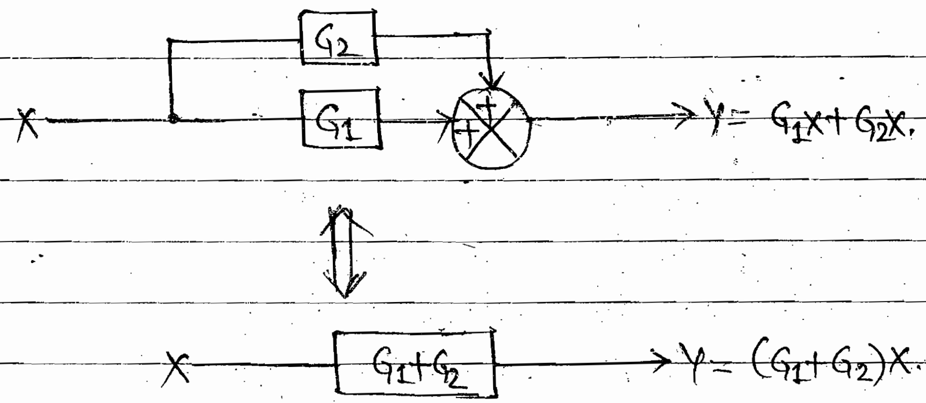




- Hence, a second summation will be introduced at take-off point whose second input will be same as second input of First summation with same polarity.

** Gain connected in Parallel :-

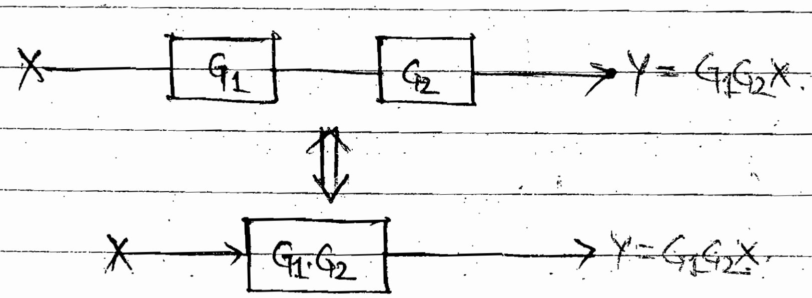
- For two Gains to be connection in parallel, there input should be same, summation should be common and direction of Flow should be same, then Overall Gain will be : Sum of two Gains.



** Gain connected in Cascade :-

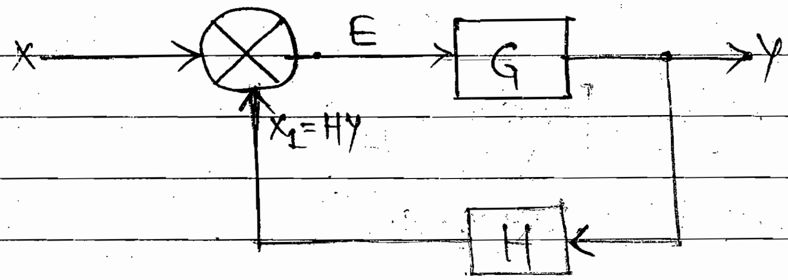
- For two Gains to be connected in Cascade, output of one should be input to other Gain. So, overall Gain will be : Product of two gains.

**



** Closed Loop Systems :-

(i) Negative Feedback Closed Loop System :-



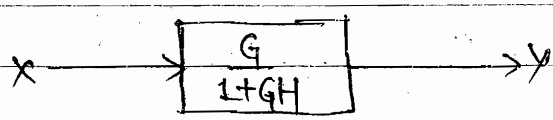
Here, $E = X - X_1 = X - HY$.

$\therefore \quad Y = GE = G(X - HY)$.

$\Rightarrow \quad Y = GX - GHY \Rightarrow Y(1 + GH) = GX$.

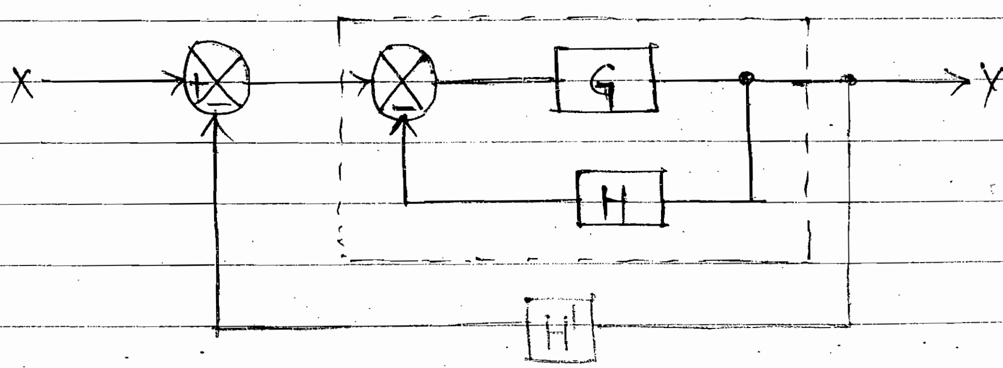
$$\therefore \boxed{\frac{Y}{X} = \frac{G}{1+GH}} \; *$$

Or,



$Y = \frac{G \cdot X}{(1+GH)}$

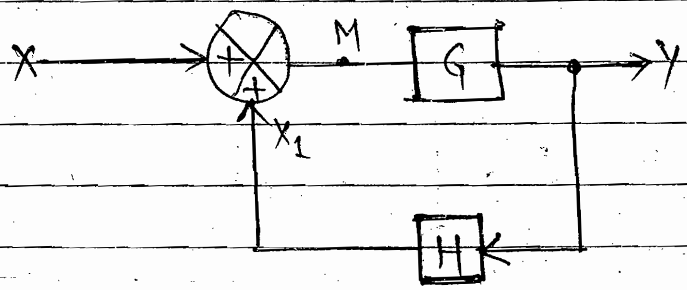
Also,



$$\therefore \boxed{\frac{Y}{X} = \frac{G}{1 + G(H + H')}} \; *$$

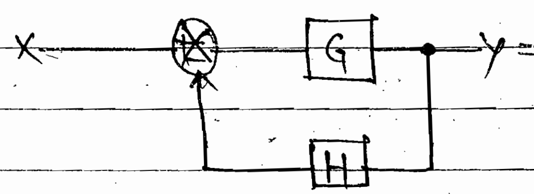
(ii) Positive Feedback Closed Loop System :-

- It acts as Mixer.

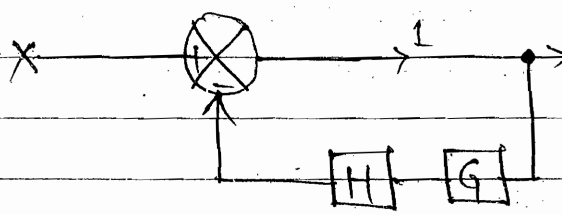


$X_1 = HY \Rightarrow X + X_1 = M.$

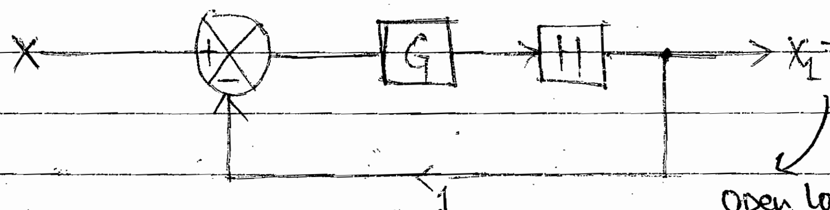
$$\therefore \boxed{\frac{Y}{X} = \frac{G}{1-GH}} *$$

*Example :- 

$Y = \frac{G}{1+GH}$



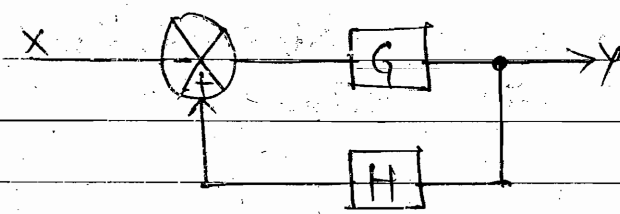
$E = \frac{1}{1+GH}$



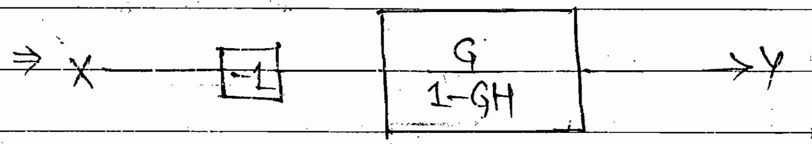
$X_1 = \frac{GH}{1+GH}$

Open Loop Transfer Function.

* Example :-

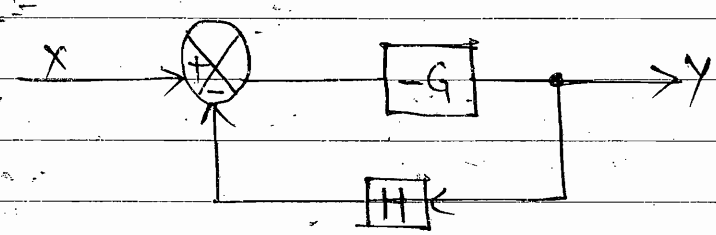


$\Rightarrow$



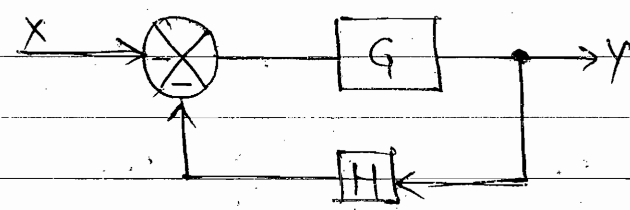
$\therefore \boxed{Y/X = \dfrac{-G}{1-GH}}$ * So, Given system uses Positive Feedback.

* Example :-



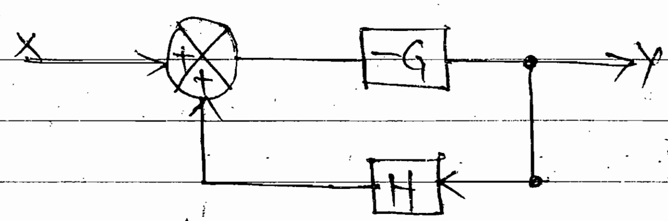
$\Rightarrow$ Here, $\dfrac{Y}{X} = \dfrac{-G}{1+(-GH)} = \dfrac{-G}{1-GH}$, So given system uses Positive Feedback.

* Example :-



$Y = \dfrac{-G}{1+GH}$, Hence System uses Negative Feedback.

* Example :-

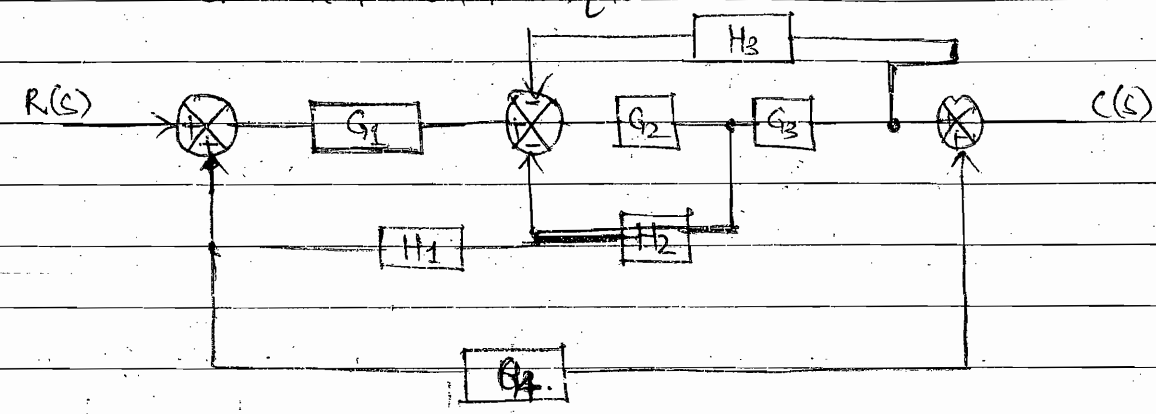


$Y = \dfrac{G}{1+GH}$, Hence system uses Negative Feedback.

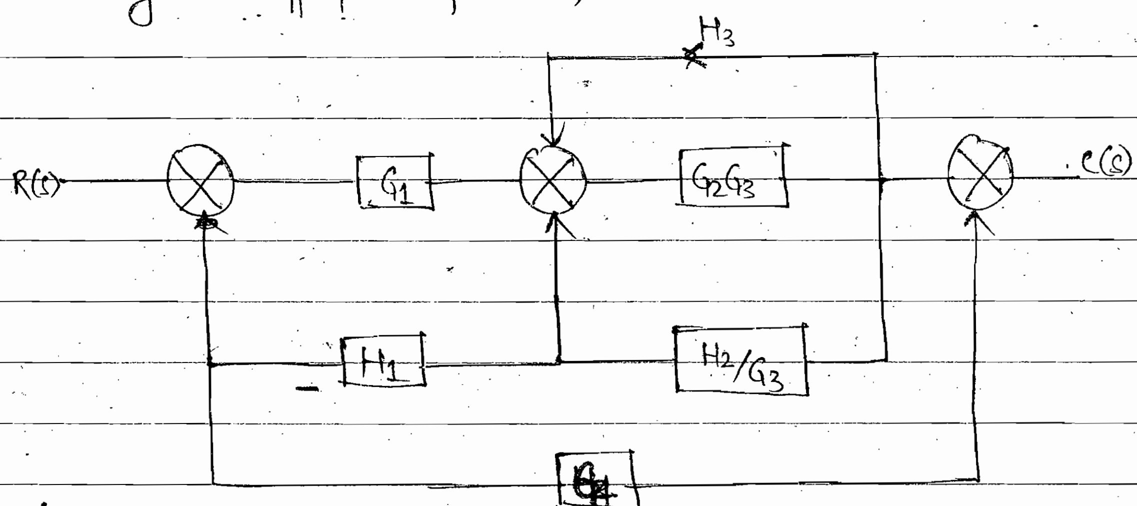
-07-20

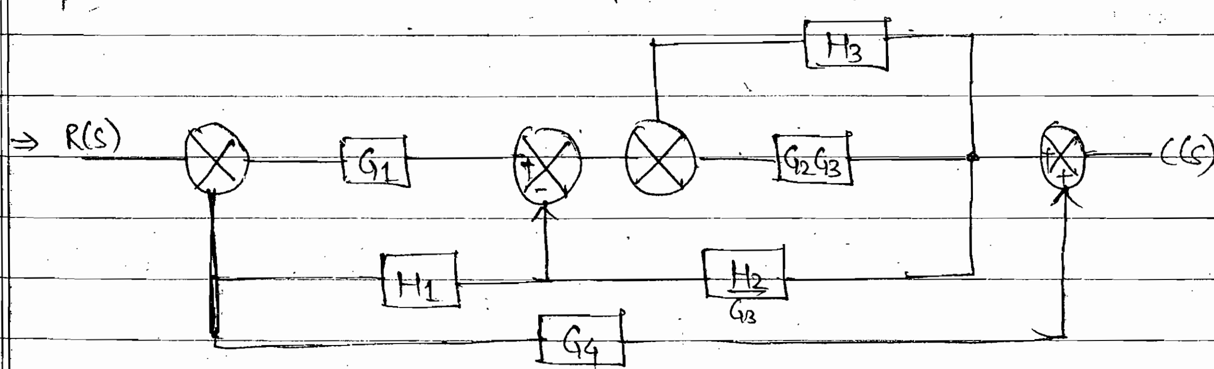
Numericals

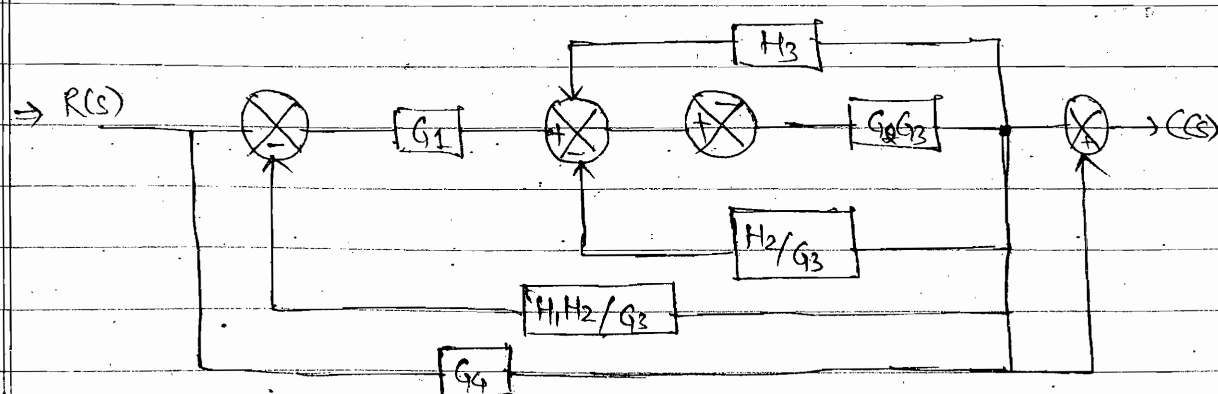
Ques:- (1) For the given block diagram, calculate transfer function using Block Reduction Technique.

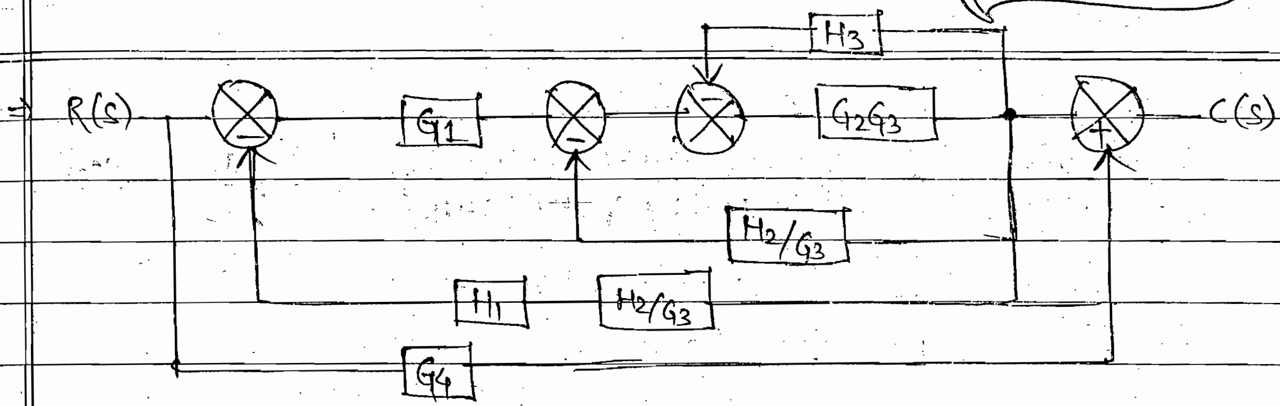


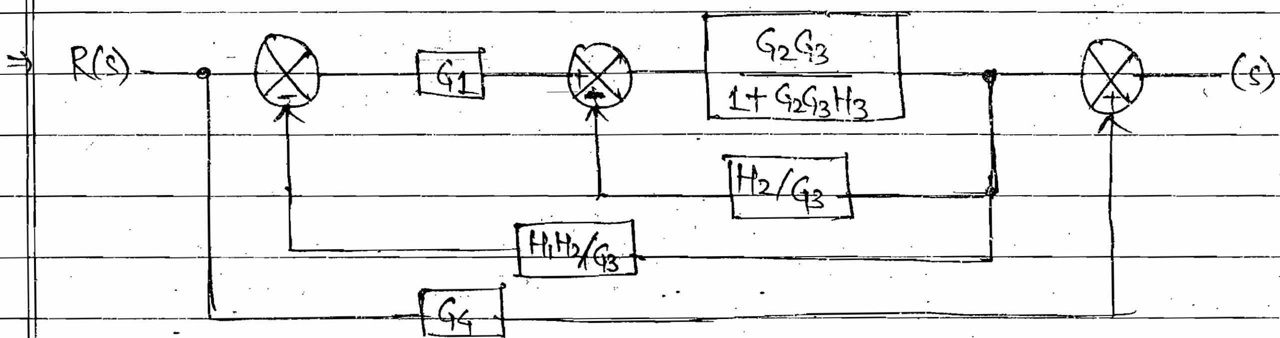
Ans:- (1) Taking take-off point forward,

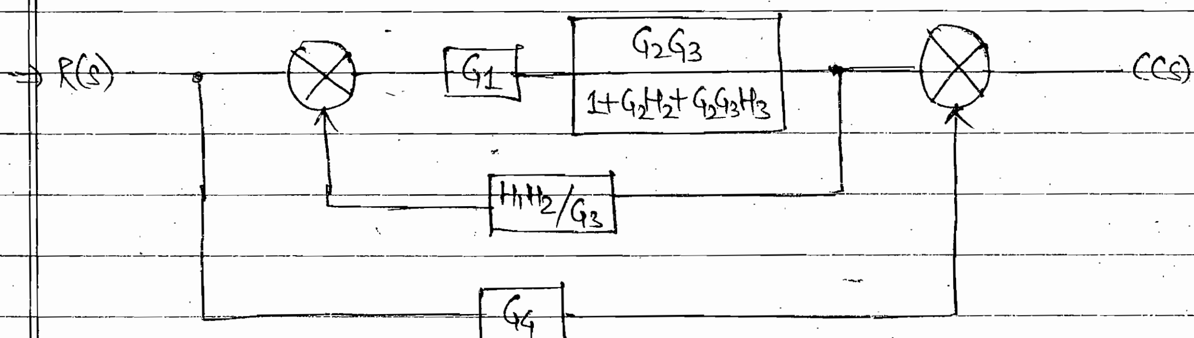




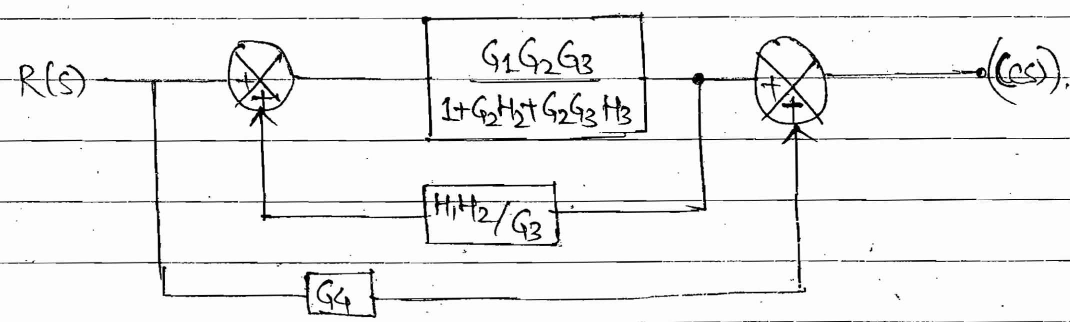




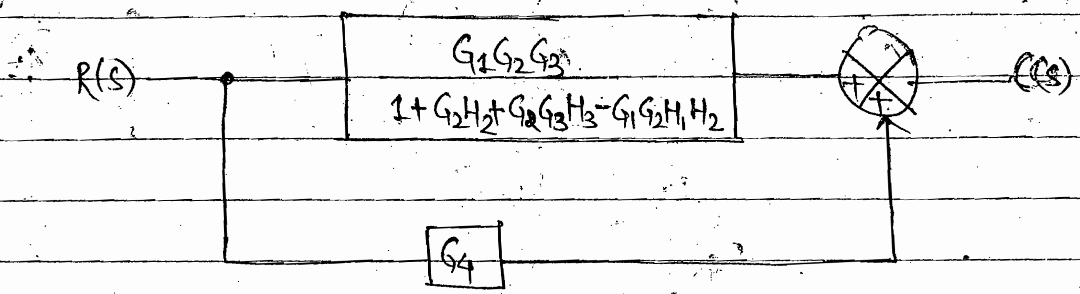




or



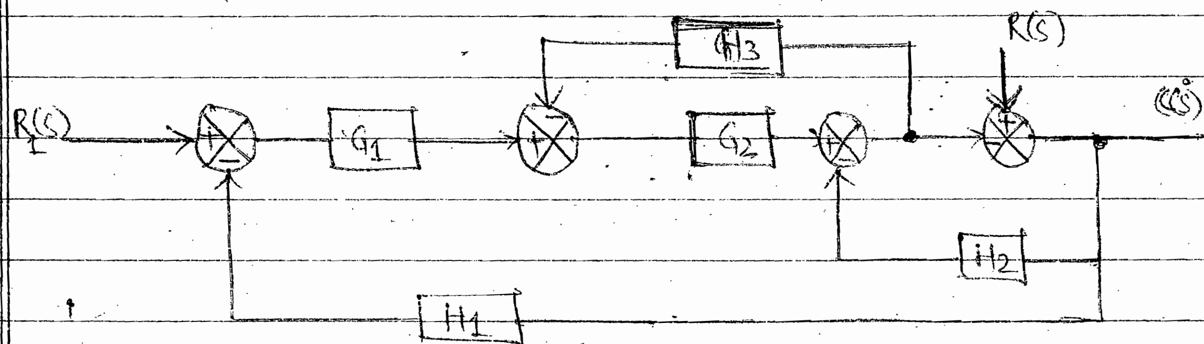
$$\therefore \frac{\dfrac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3}}{1+(-)\dfrac{G_1G_2\cancel{G_3}}{1+G_2H_2+G_2G_3H_3}\times\dfrac{H_1H_2}{\cancel{G_3}}} \Rightarrow \frac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3-G_1G_2H_1H_2}$$



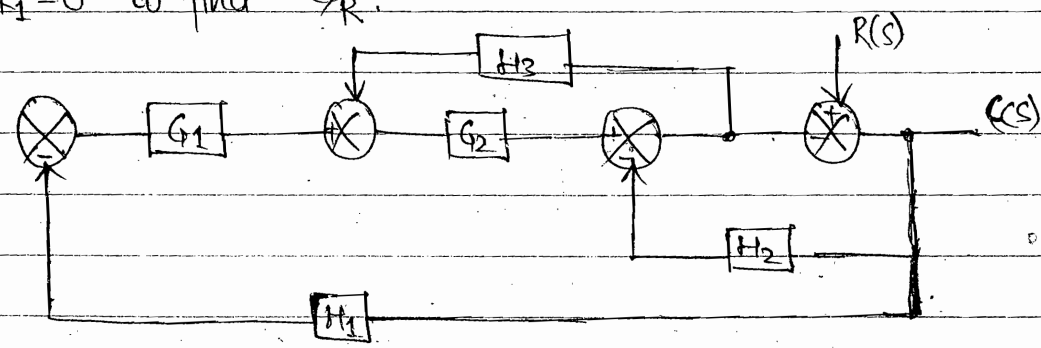
$\therefore$ They are in Parallel, Gain $= G_4 + \dfrac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3-G_1G_2H_1H_2}$

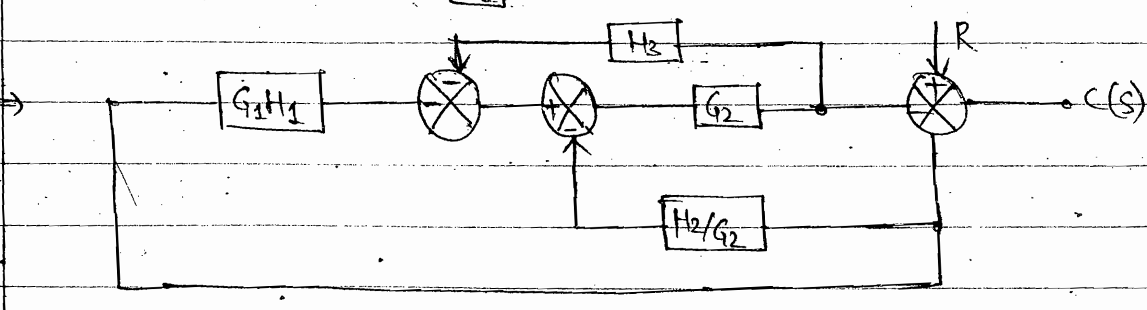
$$\therefore \boxed{\frac{C(s)}{R(s)} = G_4 + \frac{G_1 \cdot G_2 \cdot G_3}{1+G_2H_2+G_2G_3H_3-G_1G_2H_1H_2}} \ *$$

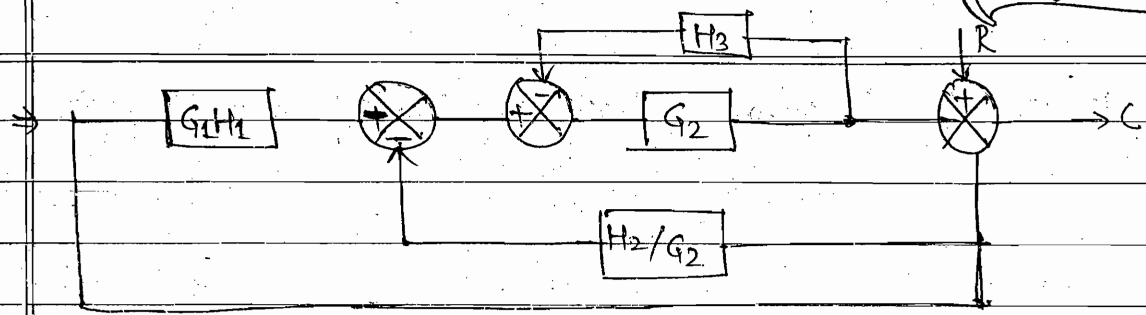
Ques:- (2) Obtain transfer function $\frac{C(s)}{R(s)}$ using Block Reduction Technique.

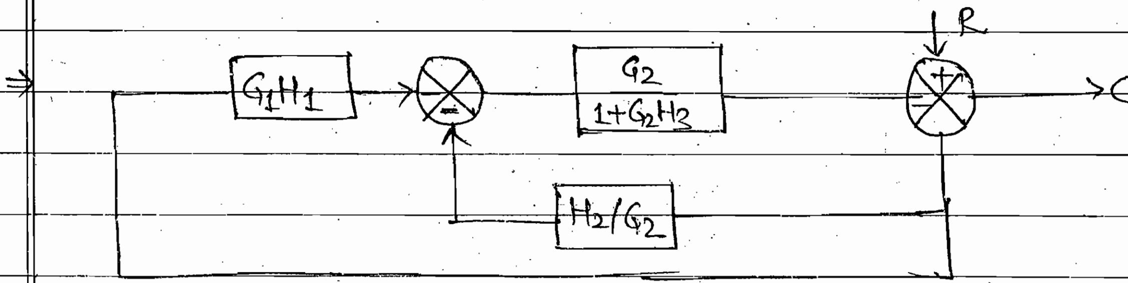


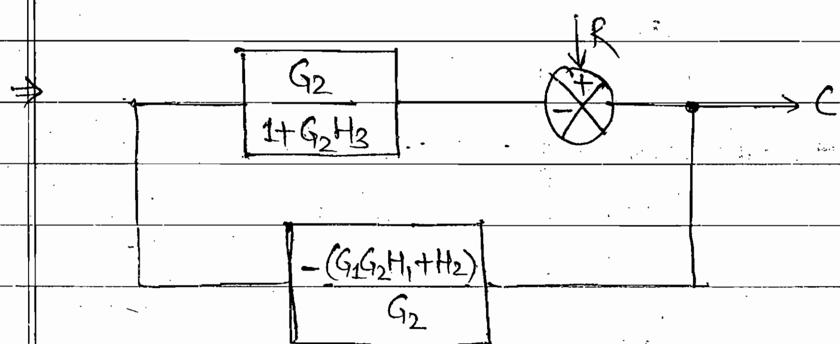
Ans:- (2) Put $R_1 = 0$ to find C/R.

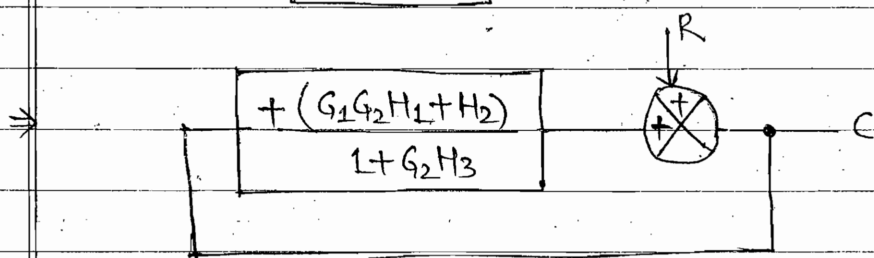








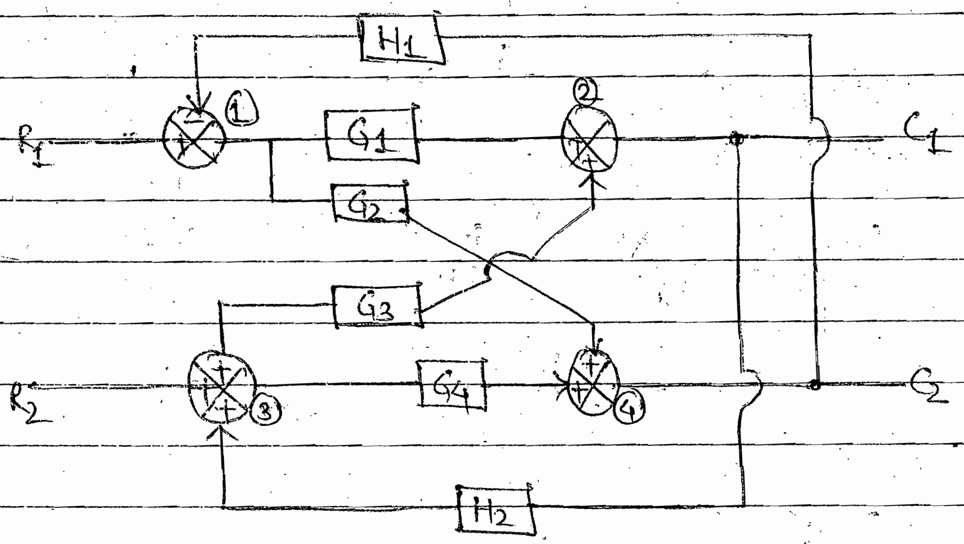




$$\therefore \frac{C}{R} = \frac{1+G_2H_3}{1+G_2H_3-(G_1G_2H_1+H_2)}$$

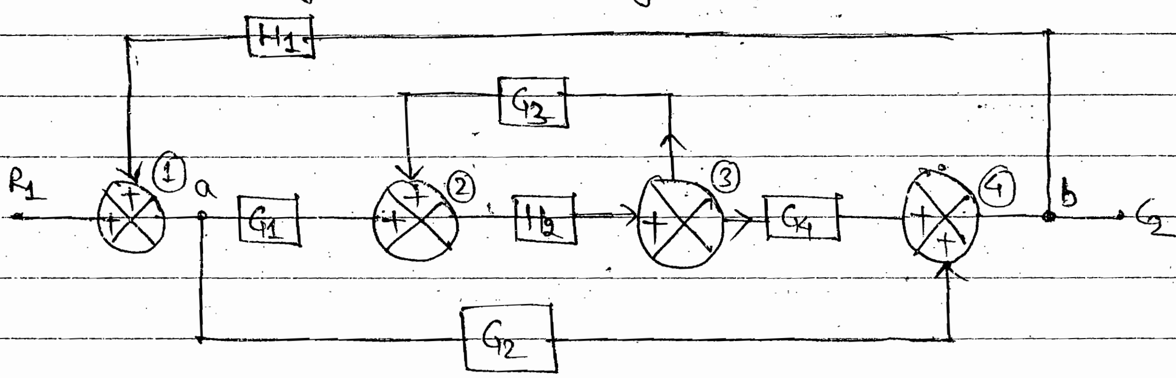
$$\therefore \boxed{\frac{C(s)}{R(s)} = \frac{1+G_2H_3}{1+G_2H_3-(H_2)-G_1G_2H_1}} *$$

Ques:-③ Calculate $\frac{C_2}{R_1}$ for following block using Block Reduction Technique.

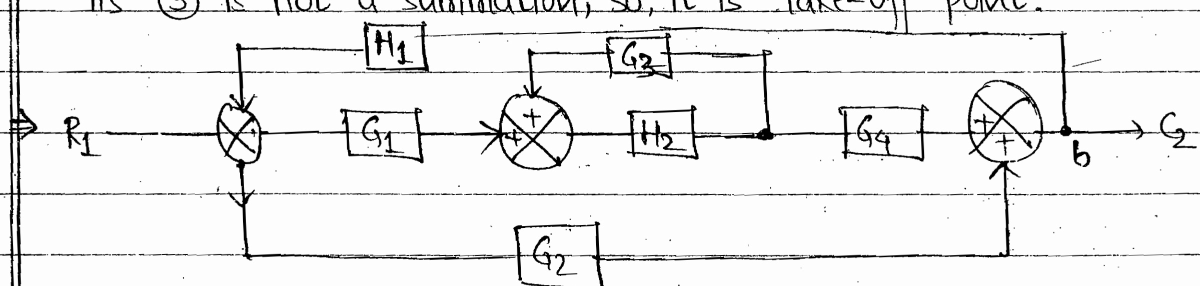


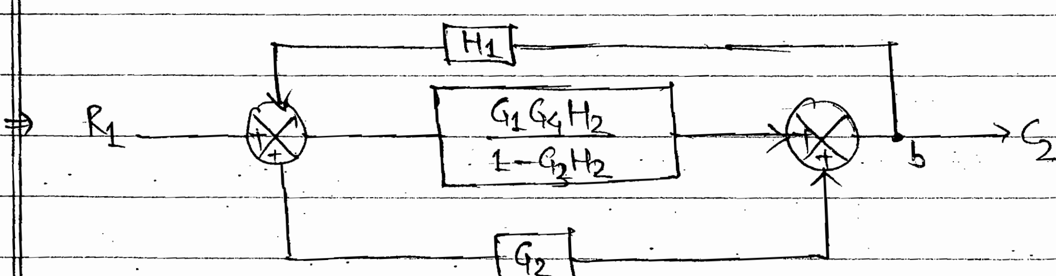
Ans:-③ Set $R_2 = 0$, just don't show C_1.

Converting into linear array of summations,



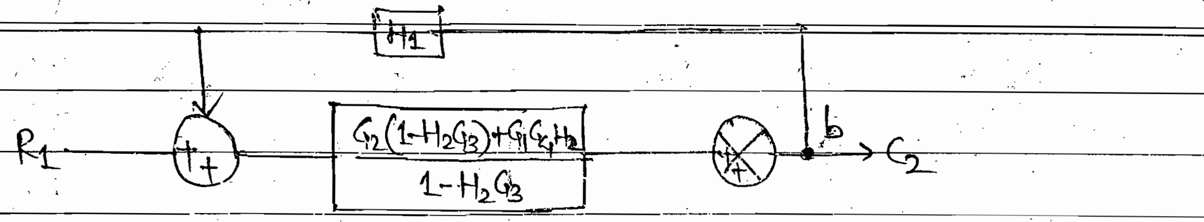
As ③ is not a summation, so, it is Take-off point.





$$\therefore \; C_2/R_1 = G_2 + \frac{G_1 G_4 H_2}{1-G_3 H_2} \Rightarrow \boxed{\frac{C_2}{R_1} = G_2 + \frac{G_1 G_4 H_2}{1-G_3 H_2}} *$$

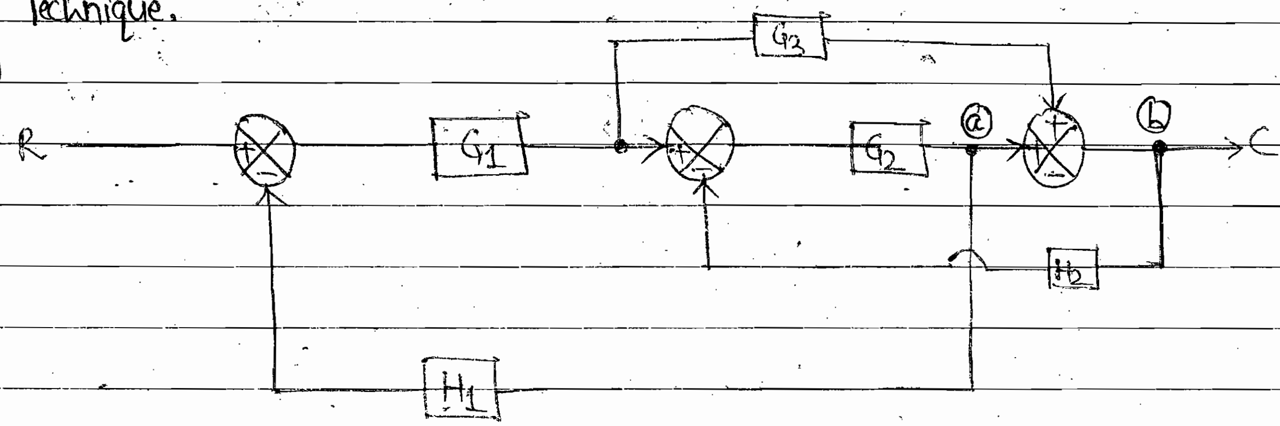
Then,



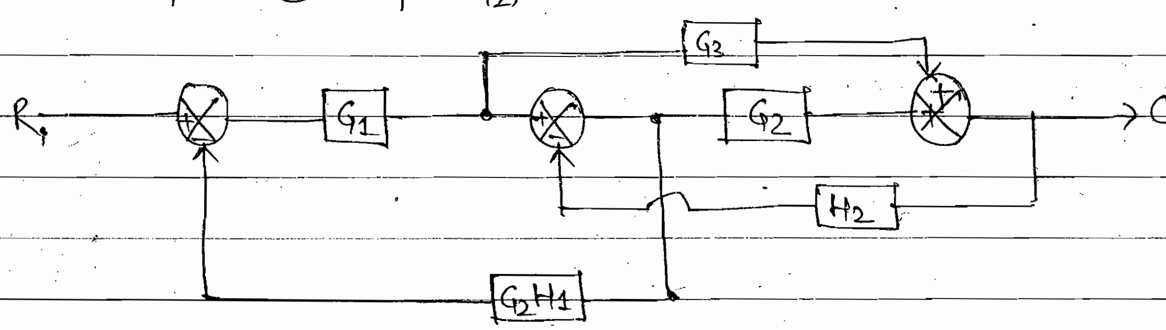
$$\therefore \boxed{\frac{C_2}{R_1} = \frac{G_2(1-G_3H_2)+G_1G_4H_2}{1-G_3H_2-H_1\left[G_2(1-G_3H_2)+G_1G_4H_2\right]}}$$

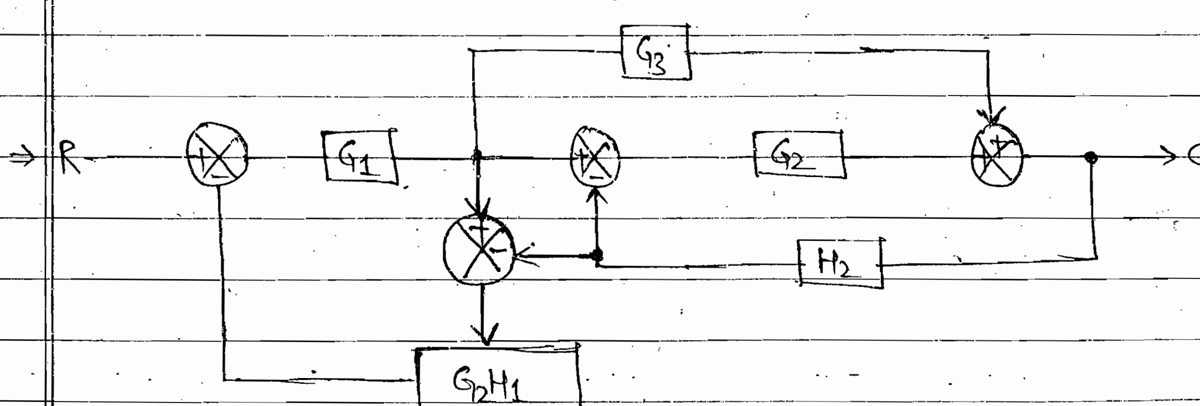
Ques:- (4) For the given block, obtain its transfer function C/R using block reduction Technique.

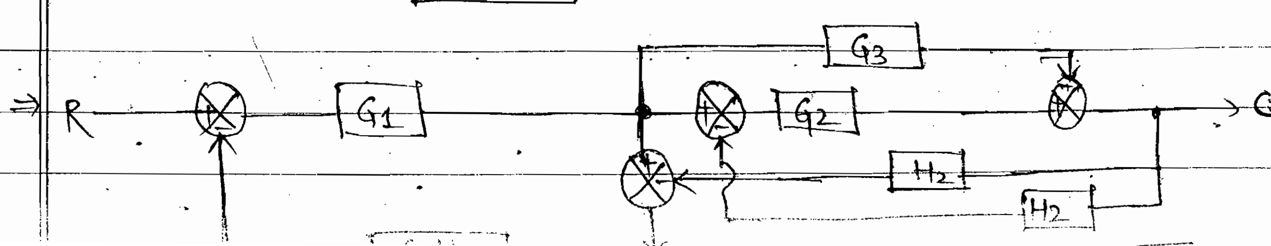
Ans:- (4)

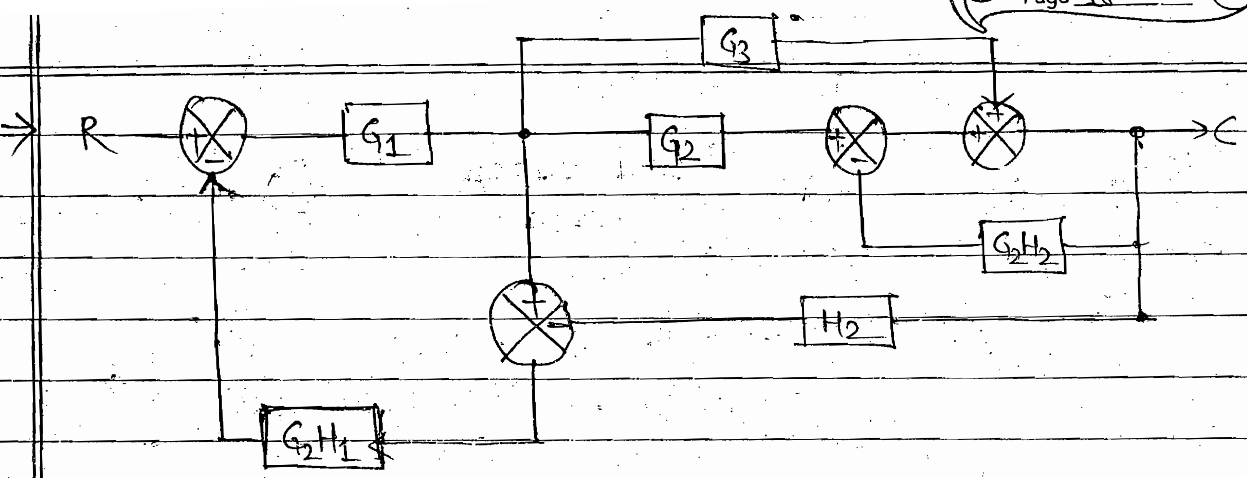


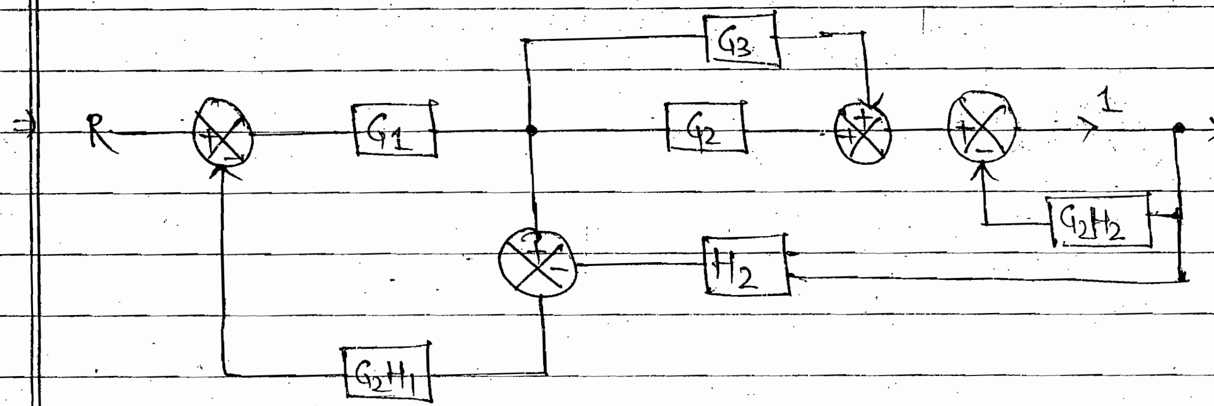
⇒ Take point ⓐ before G_2,

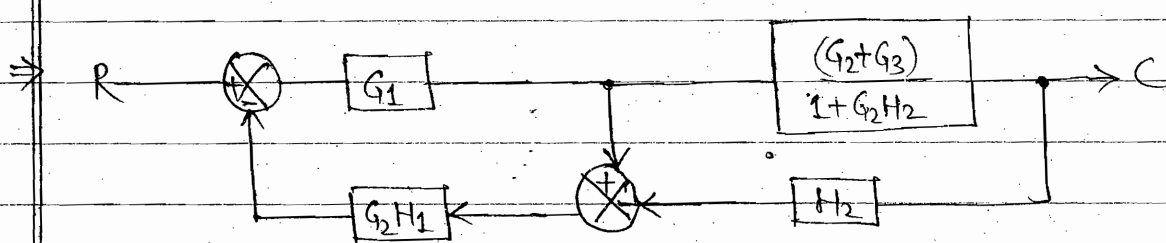




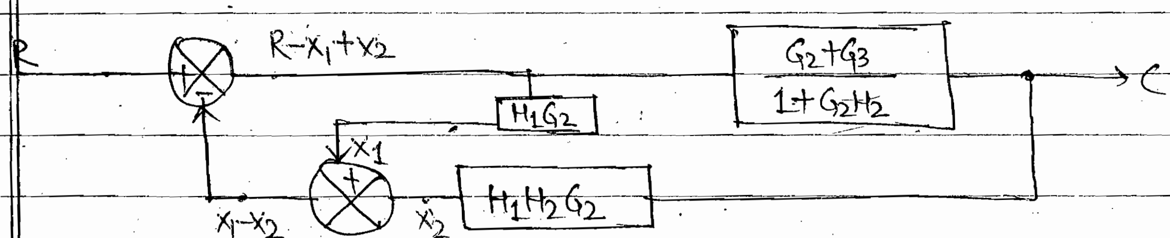


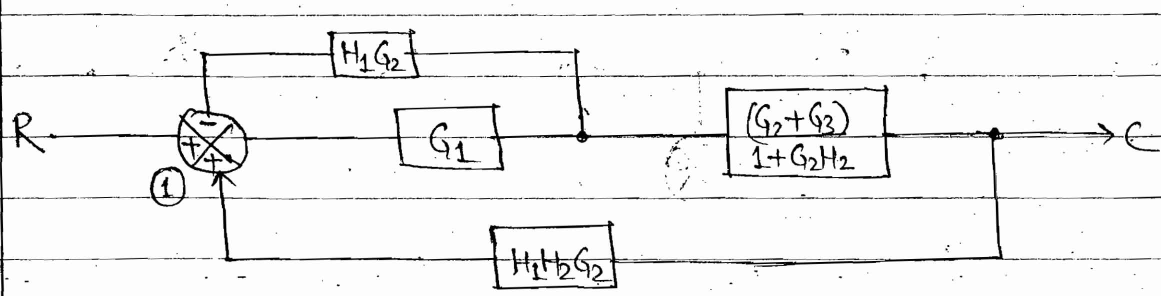




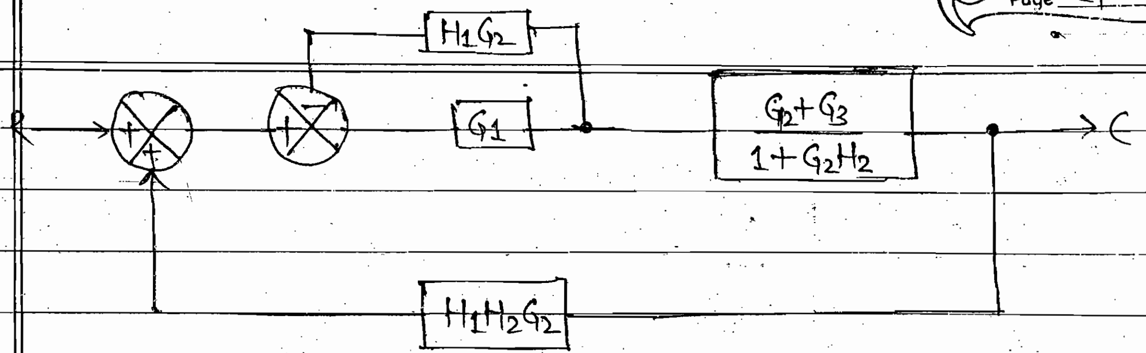


Moving summation before G_2H_1,

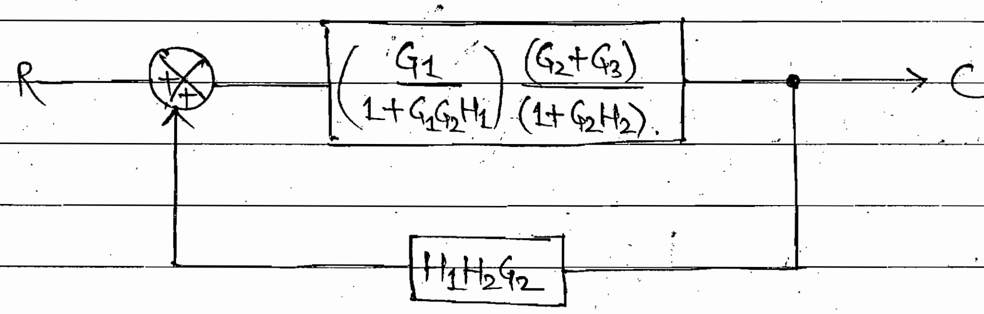




- Breaking (1) into 2 2-input summation,



$\Rightarrow$



$$\therefore \cancel{\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_2G_1H_1 + G_1G_2^2H_1H_2} + H_1H_2G_2}$$

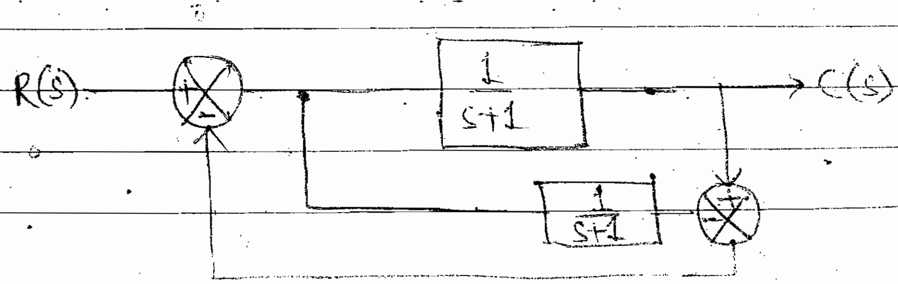
1

$$\Rightarrow \frac{C}{R} = \frac{\dfrac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1 + G_1G_2^2H_1H_2}}{1 - \dfrac{(G_1G_2 + G_1G_3)}{(1 + G_2H_2 + G_1G_2H_1 + G_1G_2^2H_1H_2)} \times H_1H_2G_2}$$

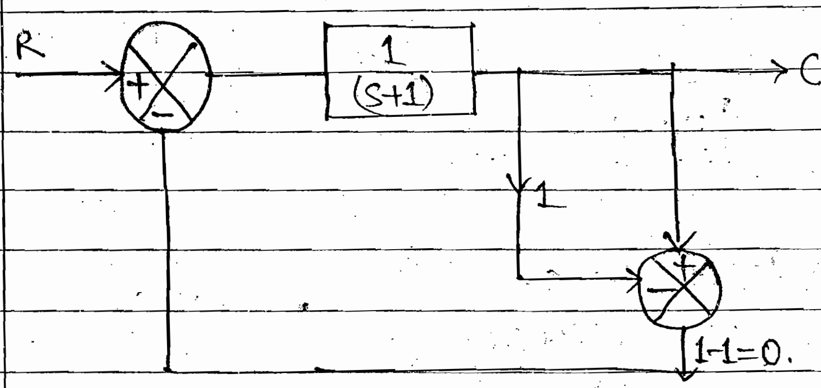
$$\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1 + G_1G_2^2H_1H_2 - G_1G_2^2H_1H_2 - G_1G_3G_2H_1H_2}$$

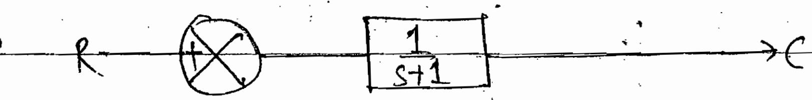
$$\Rightarrow \boxed{\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_2H_2 - G_1G_2G_3H_1H_2}} \ *$$

Ques :(5) For the given block, obtain its transfer function using block reduction technique.



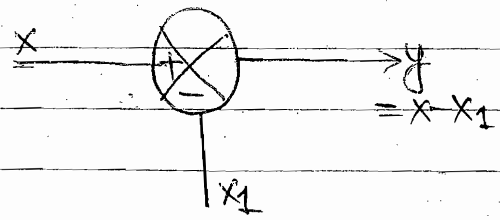
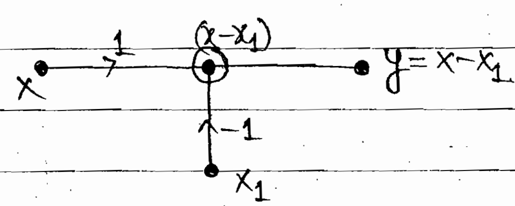
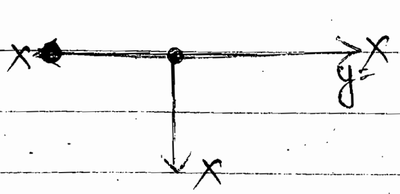
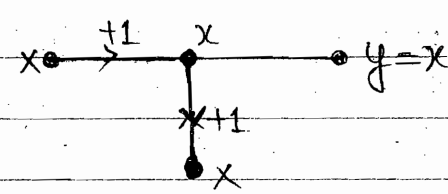
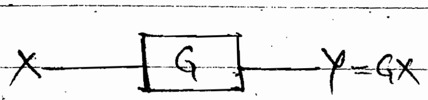
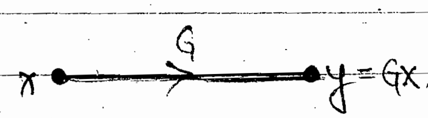
Ans:- 5

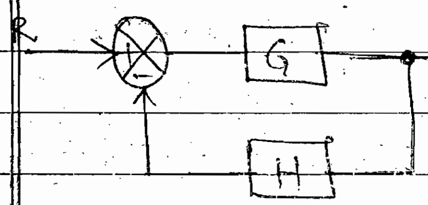


⇒ 

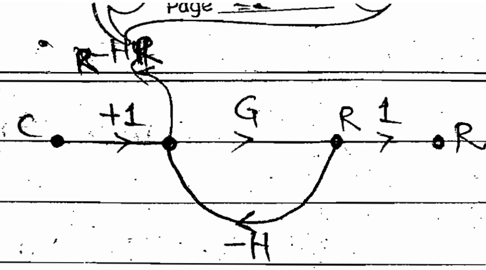
∴ $\boxed{\dfrac{C}{R} = \dfrac{1}{S+1}}$ *

*→ **Comparison between Block Diagram and Signal Flow Graph :-**

BLOCK DIAGRAM	SIGNAL FLOW GRAPH.
• Block Diagram is based on Lumped Parameter.	• Signal Flow graph is based on Distributed Parameter.
• In lumped Network, Gain exists only at particular point.	• In distributed parameter, Gain is uniformly distributed across the line.
	
	
	



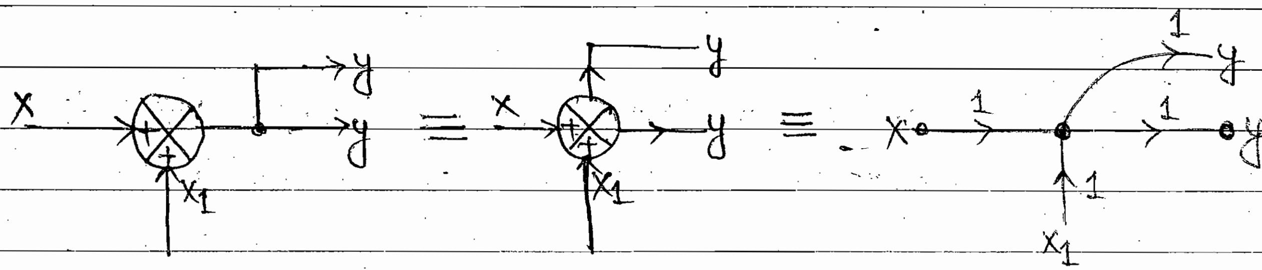
$$C = \frac{G}{1+GH}$$



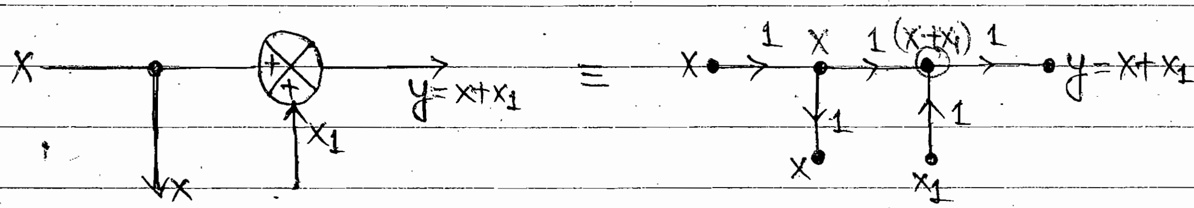
$$R = G[R - HR].$$

$$\Rightarrow \quad R = \left(\frac{G}{1+GH}\right)C.$$

*** Case:-I :- If summation is followed by Take-off point, we will use single node to represent both summation and take-off point.



*** Case:-II :- If take-off point is followed by summation, we will use two independent node, one for take-off point and second for summation.



Date
31-07-2014

→ * MASON'S GAIN FORMULA :-

- It is used to calculate Transfer Function using Signal Flow Graph.

* According to Mason's Gain Formula:

$$\boxed{\frac{Y}{X} = \sum_{K=1}^{n} \frac{P_K \cdot \Delta_K}{\Delta}} \quad *$$

where, n = Total number of Forward path.

P_K = K^{th} forward path.

$\Delta_K = 1 -$ [Sum of non-touching loops to K^{th} forward path] + [sum of product of two non-touching loop to K^{th} forward path] −

sum of three non-touching loops to K^{th} Forward path] + ---

$$\Delta = 1 - [\text{sum of all loops}] + [\text{sum of two non-touching loops}] - [\text{sum of three non-touching loops}] + \cdots$$

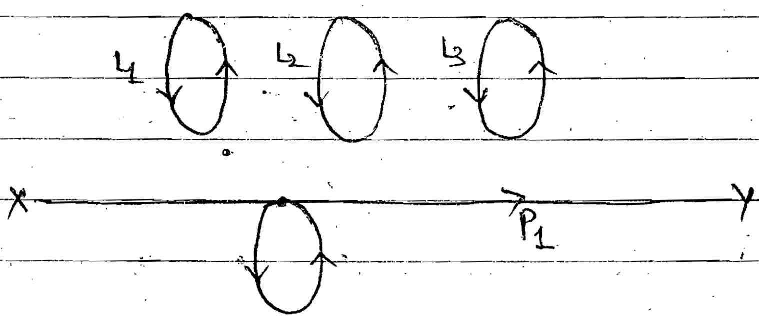
→ **Forward Path :-**

- It is the path which connects input with output and it will be not in loop form.

→ **Loop :-**

- In case of loop, origination node itself will be termination node and it won't traverse more than one node twice.

Example:-



Ans:-

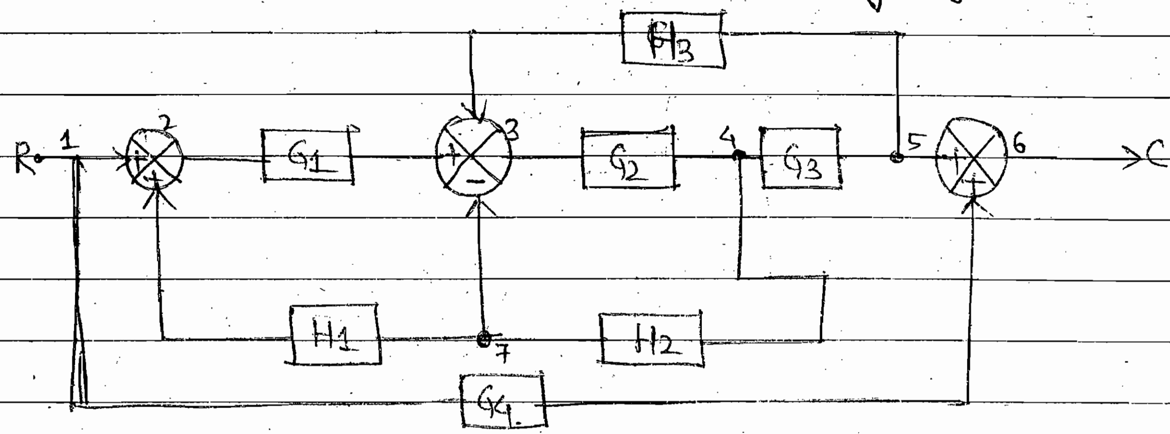
$$\Delta_1 = 1 - [L_1 + L_2 + L_3] + [L_1 \cdot L_2 + L_2 L_3 + L_3 L_1] - [L_1 L_2 L_3]$$

$$\Rightarrow \Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_2 + L_2 L_3 + L_3 L_1 + L_1 L_4 + L_2 L_4 + L_3 L_4] - [L_1 L_2 L_3 + L_2 L_3 L_4 + L_3 L_4 L_1 + L_4 L_1 L_2] + L_1 L_2 L_3 L_4.$$

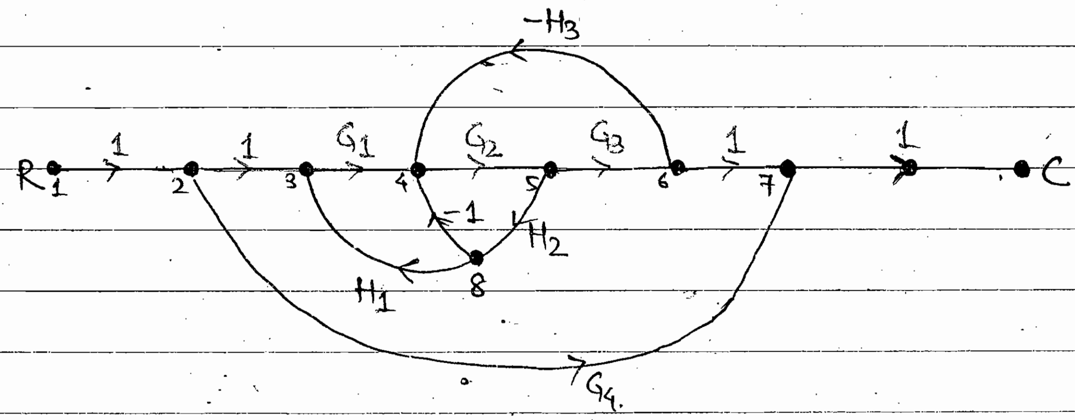
$$\therefore \quad Y/X = \frac{P_1 \Delta_1}{\Delta} \text{, Use this.}$$

Questions on Signal Flow Graph.

Ques:-(1) For the given block, obtain its transfer function using Signal Flow Graph.



Ans:- (1)



$P_1 = G_1 G_2 G_3$, $\Delta_1 = 1 - [0] = 1$

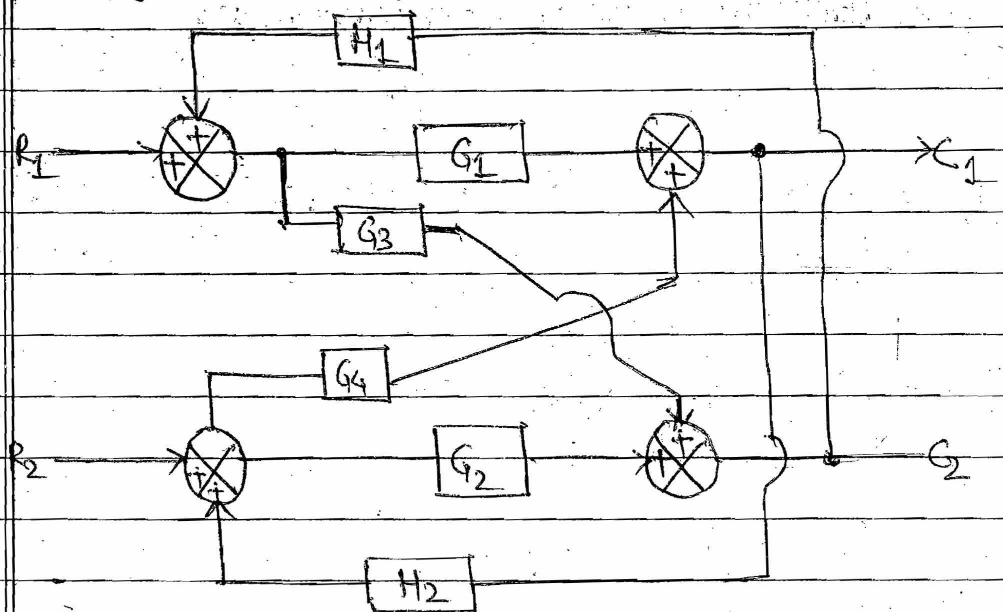
$P_2 = G_4$, $\Delta_2 = 1 - [G_1 G_2 H_1 H_2 - G_2 H_2 - G_2 G_3 H_3]$

$\Delta = 1 - [-G_2 H_2 - G_2 G_3 H_3 + G_1 G_2 H_1 H_2]$

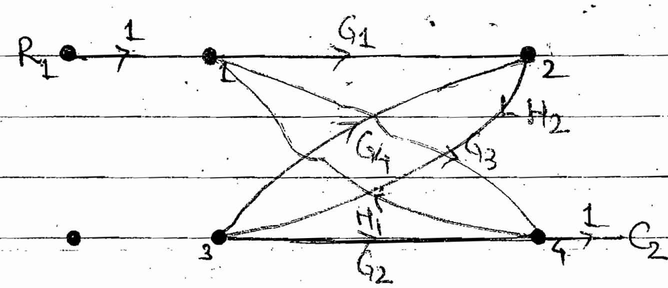
$$\therefore \frac{Y}{X} = \frac{G_1 G_2 G_3 + G_4 [1 + G_2 H_2 + G_2 G_3 H_3 - G_1 G_2 H_1 H_2]}{1 + G_2 H_2 + G_2 G_3 H_3 - G_1 G_2 H_1 H_2}.$$

or $$\boxed{\frac{Y}{X} = G_4 + \frac{G_1 G_2 G_3}{1 + G_2 H_2 + G_2 G_3 H_3 - G_1 G_2 H_1 H_2}}$$ *

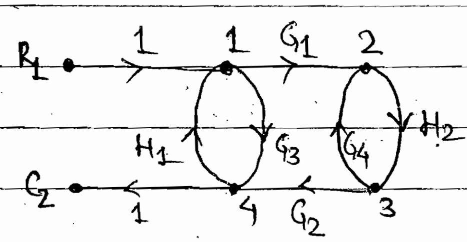
Ques:- (2) Using signal flow graph, find C_1/R_1.



Ans:- (2)



To simplify it:



$P_1 = G_3$, $\Delta_1 = 1 - [G_4 H_2]$

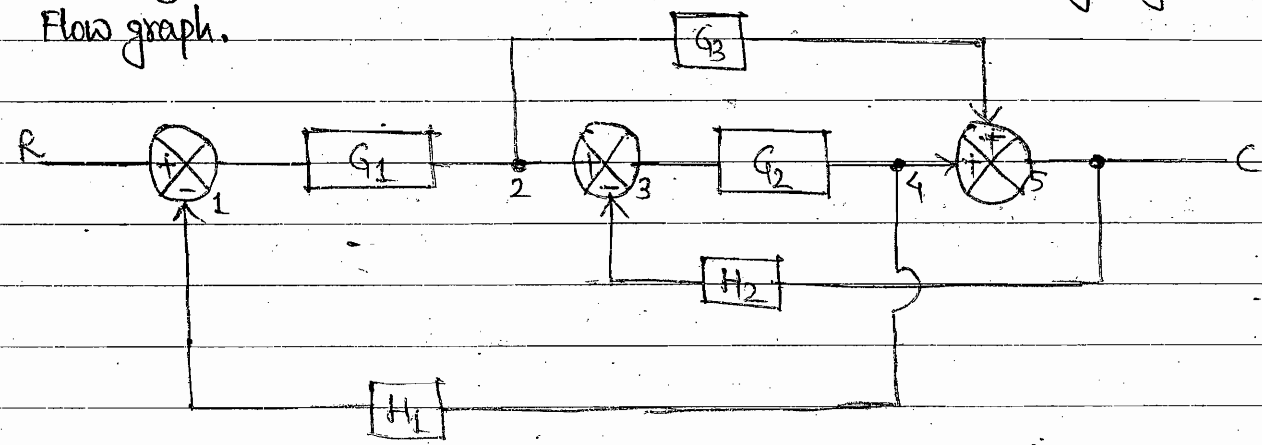
$\therefore$ $P_2 = G_1 G_2 H_2$, $\Delta_2 = 1 - [0] = 1$.

$\therefore$ $\Delta = 1 - [G_3 H_1 + G_4 H_2 + G_1 G_2 H_1 H_2] + [G_3 G_4 H_1 H_2]$.

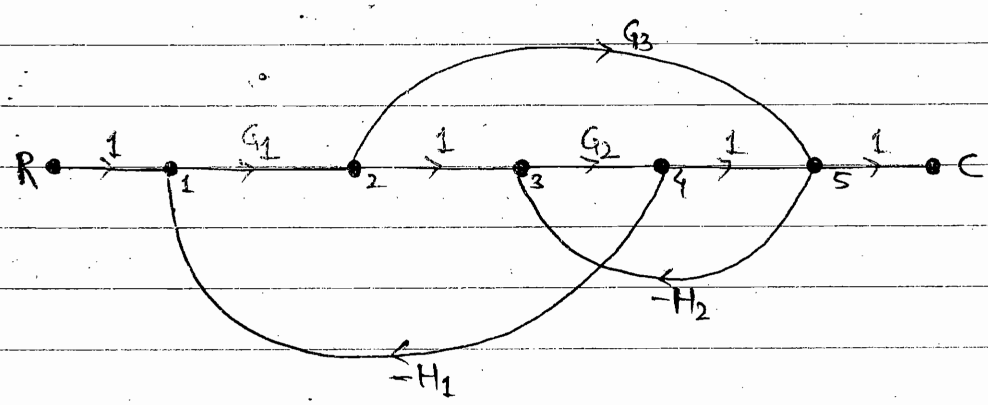
$$\therefore \frac{C_2}{R_1} = \frac{G_3[1-G_4H_2] + G_1G_2H_2}{1 - G_3H_1 + G_4H_2 + G_1G_2H_1H_2 + G_3G_4H_1H_2}$$

$$\therefore \boxed{\frac{C_2}{R_1} = \frac{G_3[1-G_4H_2] + G_1G_2H_2}{1 - G_3H_1 - G_4H_2 - G_1G_2H_1H_2 + G_3G_4H_1H_2}} \; *$$

Ques: (3) For the given block, obtain its transfer Function (C/R) using signal Flow graph.



Ans: (3)



$P_1 = G_1G_2$, $\Delta_1 = 1 - [0] = 1$

$P_2 = G_1G_3$, $\Delta_2 = 1 - [0] = 1$.

$\Delta = 1 - [-G_1G_2H_1 - G_2H_2 + G_1G_2G_3H_1H_2]$

$$\therefore \boxed{\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_2H_2 - G_1G_2G_3H_1H_2}} \; *$$

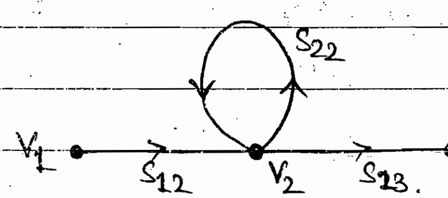
MASON'S GAIN LIMITATION :-

- Mason Gain Limitation exists on Self-sustaining Loop.

Self-Sustaining Loop :-

- It contains only single node and origination node itself will be termination node.

Example:-



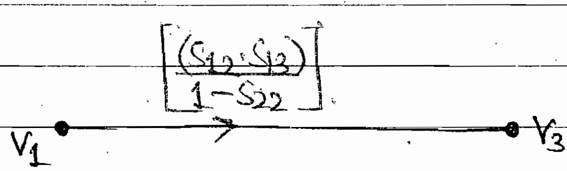
Ans:- Potential at V_2, $V_2 = S_{12}V_1 + V_2 S_{22}$.

$$\therefore \boxed{V_2 = \frac{S_{12}.V_1}{1-S_{22}}} *$$

Now, potential at V_3, $V_3 = V_2 \cdot S_{23}$

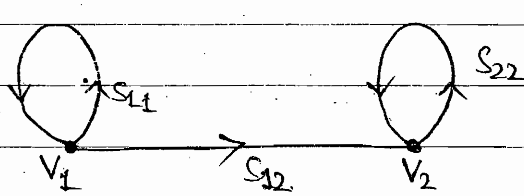
$$\text{or} \quad V_3 = \left(\frac{S_{12} \cdot S_{23}}{1-S_{22}}\right) V_1$$

$\therefore$ Given signal flow graph can be replaced with:



✓ Here, S_{22} must not be zero.

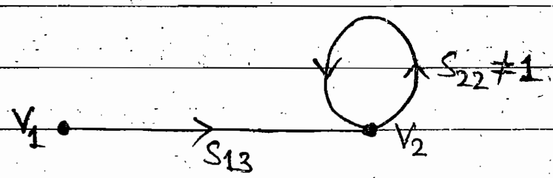
Other Limitation:-



• Potential at V_1, $V_1 = S_{11}V_1$

Here, necessarily, $S_{11} = 1$, to satisfy Node Equation.

∴



∴ $V_2 = V_1 \cdot S_{13} + S_{22} \cdot V_2$.

∴ $\boxed{V_2 = \dfrac{S_{12} \cdot V_1}{1 - S_{22}}}$ *

Chapter:-2

TIME RESPONSE ANALYSIS

*→ **Open Loop System :-**

- In case of Open Loop system, the location of pole is independent of any parameter which means its pole will not locate on Right side of s-plane. Hence, system will be highly stable.

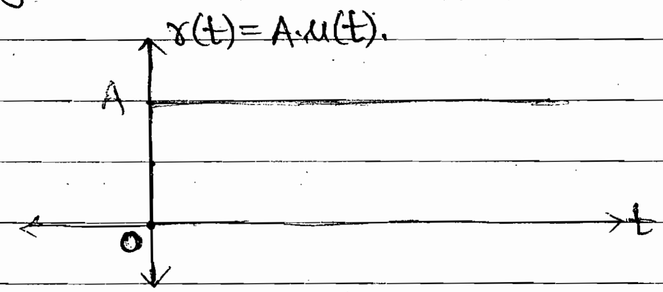
*→ **Closed Loop System with Unity Feedback :-**

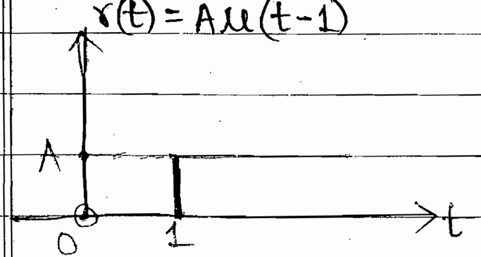
- Here, the location of pole is parameter-dependent, so under specific condition, there is possible chances that pole will go towards right side of s-plane thereby making the system unstable.

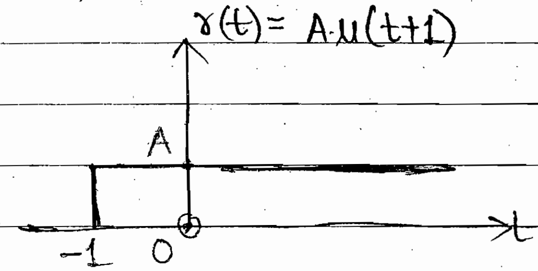
** Hence, Basic Open Loop system is more stable than Closed Loop system.

te
08-2014

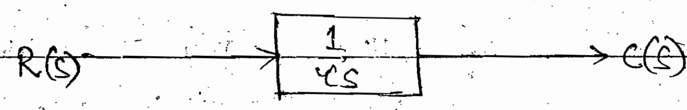
*→ **Basic Open Loop system with Step Input :-**





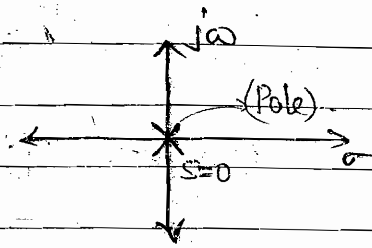


* For block given below :-



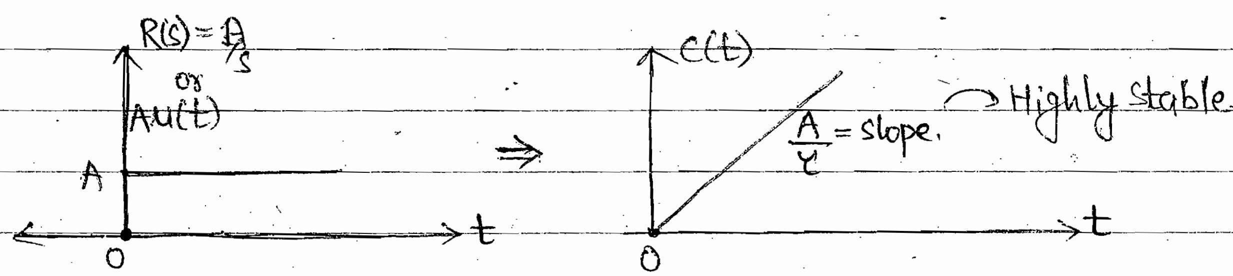
$\therefore \quad \dfrac{C(s)}{R(s)} = \dfrac{1/\tau}{s}$

or $\quad C(s) = \dfrac{1}{\tau s} \cdot R(s)$



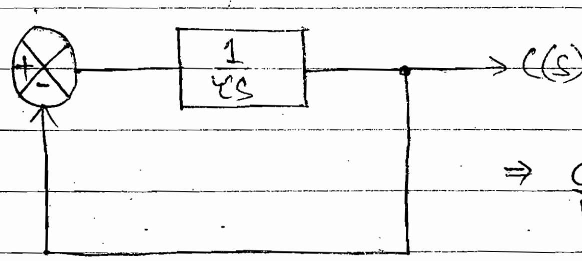
- For step input, $R(s) = \dfrac{A}{s}$

$\therefore \quad C(s) = \dfrac{A}{\tau s^2} \Rightarrow \boxed{c(t) = \left(\dfrac{A}{\tau}\right) t}$ * [Ramp as output].

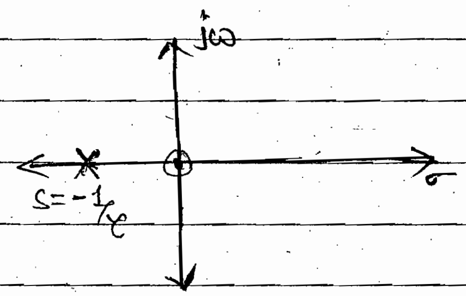


* BIBO stability condition is not applicable here as it applies only on Voltage and current gains and not on Impedances.
i.e input and output must be of same nature.

* First order closed loop system for step input :-



$\Rightarrow \dfrac{C(s)}{R(s)} = \dfrac{(1/\tau)}{(s + 1/\tau)}$



- Here, Closed loop system depends upon τ and can be unstable for some specific value of τ.

As Input $R(s) = A/s$

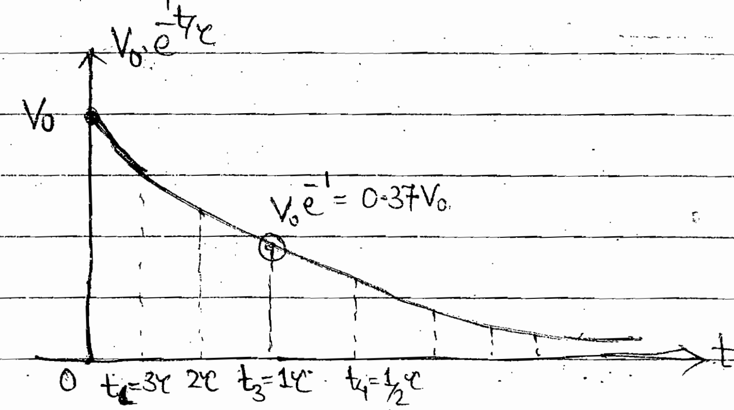
$$\therefore \quad C(s) = \frac{(A/\tau)}{s(s+1/\tau)}.$$

$$\text{or} \quad C(s) = \frac{A}{s} - \frac{A}{(s+1/\tau)}. \quad \text{[Partial Fractions]}.$$

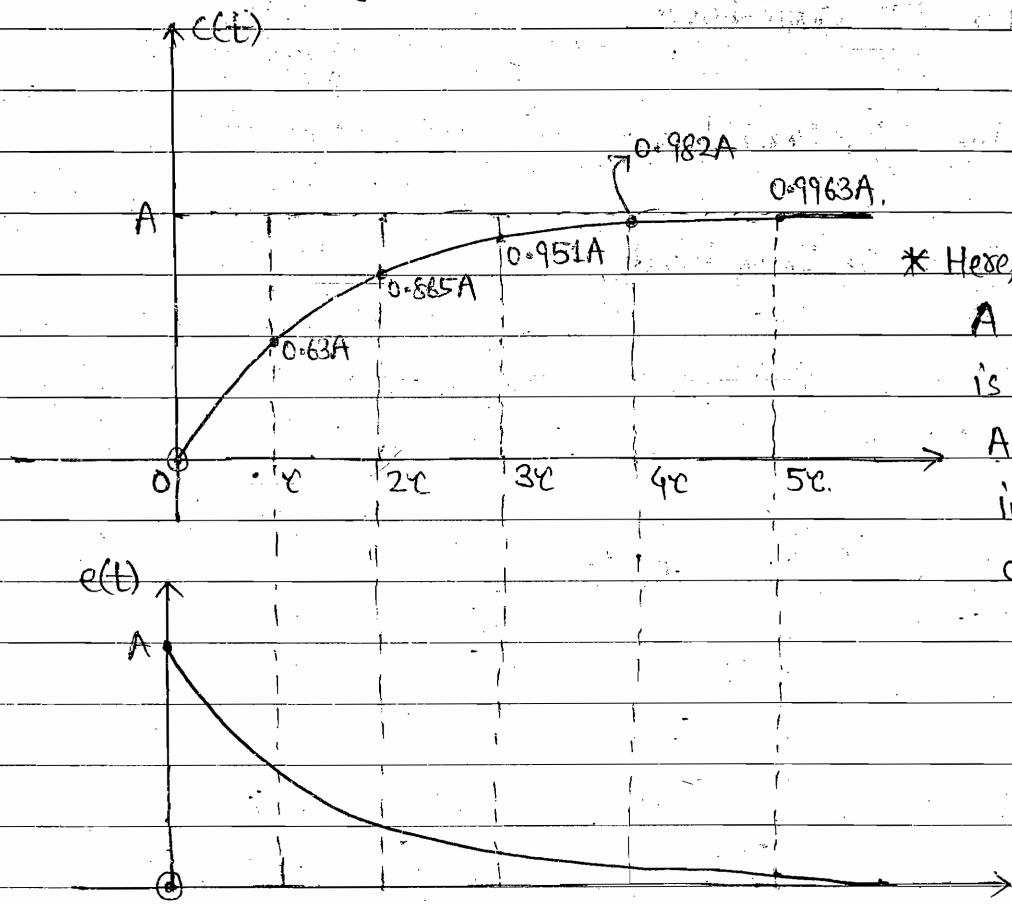
$$\therefore \quad \boxed{c(t) = A\left[1 - e^{-t/\tau}\right] \cdot u(t)} \; * \quad \text{——— (1)}$$

- Time Constant is inverse of the location of Real part of pole.
- If any system contains 'n' number of poles, then due to 'n' number of Exponential terms will exist in the output response and Time constant will be inverse of real part of respective pole location.
- Time constant is defined as, "that time at which Amplitude decreases by 63% of Maximum Amplitude.

$\boxed{\text{Time Constant } (\tau) = t_3}$ *



• Output as given by eq-① can be drawn as :-



* Here, final point is A and output is trying to reach A which is input itself and also pre-defined value.

		Error %
•	At $t=0$; $c(0) = A[1-1] = 0$.	100%
•	At $t=\tau$; $c(\tau) = A[1-e^{-1}] = A[1-0.37] = 0.63A$.	37%
•	At $t=2\tau$; $c(2\tau) = A[1-e^{-2}] = A[1-0.135] = 0.865A$.	13.5%
•	At $t=3\tau$; $c(3\tau) = A[1-e^{-3}] = A[1-0.049] = 0.951A$.	4.9%
•	At $t=4\tau$; $c(4\tau) = A[1-e^{-4}] = A[1-0.018] = 0.982A$.	1.8%
•	At $t=5\tau$; $c(5\tau) = A[1-e^{-5}] = A[1-0.0037] = 0.9963A$.	0.37%
•	At $t=6\tau$; $c(6\tau) = A[1-0.0022] = 0.9978A$.	0.22%
•	At $t=\infty$; $c(\infty) = A$	

- For 1% Error-band, $t_s = 5\tau$
- For 2%, Error band, $t_s = 4\tau$.
- For 5% Error band, $t_s = 3\tau$.

Now, from eq-(1), t_s at ~~opu~~ output = 98% with error band of 2%,

$$\therefore \quad 0.982A = A[1-e^{-t/\tau}]$$

$$\text{or} \quad e^{-t/\tau} = 0.018$$

$$\text{or} \quad -t_s/t = \ln(0.018)$$

$$\text{or} \quad \boxed{t_s = 3.99\tau}* \quad \text{or} \quad \boxed{t_s = 4\tau}*$$

* For 1% Error band :-

$$0.99A = A[1-e^{-t_s/\tau}]$$

$$\Rightarrow \quad \boxed{t_s = 4.62\tau}*$$

* For 5% Error band :- $0.95A = A[1-e^{-t_s/\tau}]$

$$\Rightarrow \boxed{t_s = 2.99\tau}* \quad \text{or} \quad \boxed{t_s = 3\tau}*$$

Error Response :-

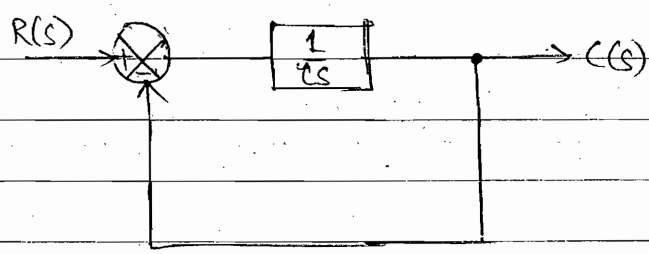
$$\boxed{e(t) = \text{Desired Output} - \text{Actual Output}} \ *$$

or $$\boxed{e(t) = r(t) - c(t)} \ *$$

$$\therefore \ e(t) = Au(t) - A[1 - e^{-t/\tau}] = Ae^{-t/\tau}$$

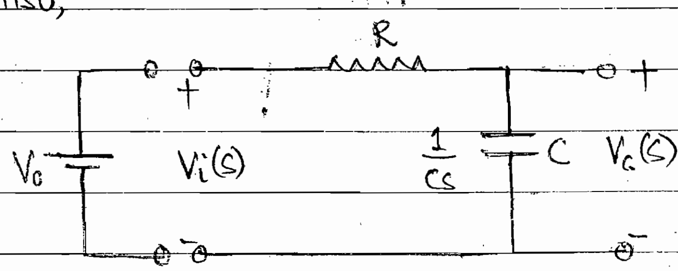
- At $t=0$; $e(0) = A$
- At $t=\tau$; $e(\tau) = 0.37A$
- At $t=2\tau$; $e(2\tau) = 0.135A$
- At $t=3\tau$; $e(3\tau) = 0.049A$
- At $t=4\tau$; $e(4\tau) = 0.018A$ (2% error-band).
- At $t=5\tau$; $e(5\tau) = 0.0033A$
- At $t=\infty$; $e(\infty) = 0$.

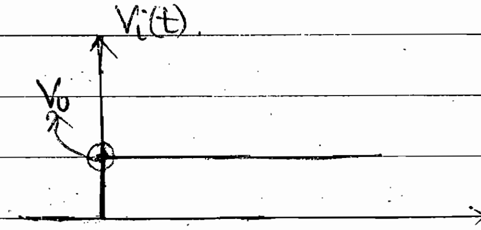
* Again,



$$\frac{C(s)}{R(s)} = \frac{(1/\tau)}{(s+1/\tau)} \quad \text{——— (1)}$$

Also,





$$V_i(s) = \frac{V_0}{s} \quad \text{[Laplace Transform]}$$

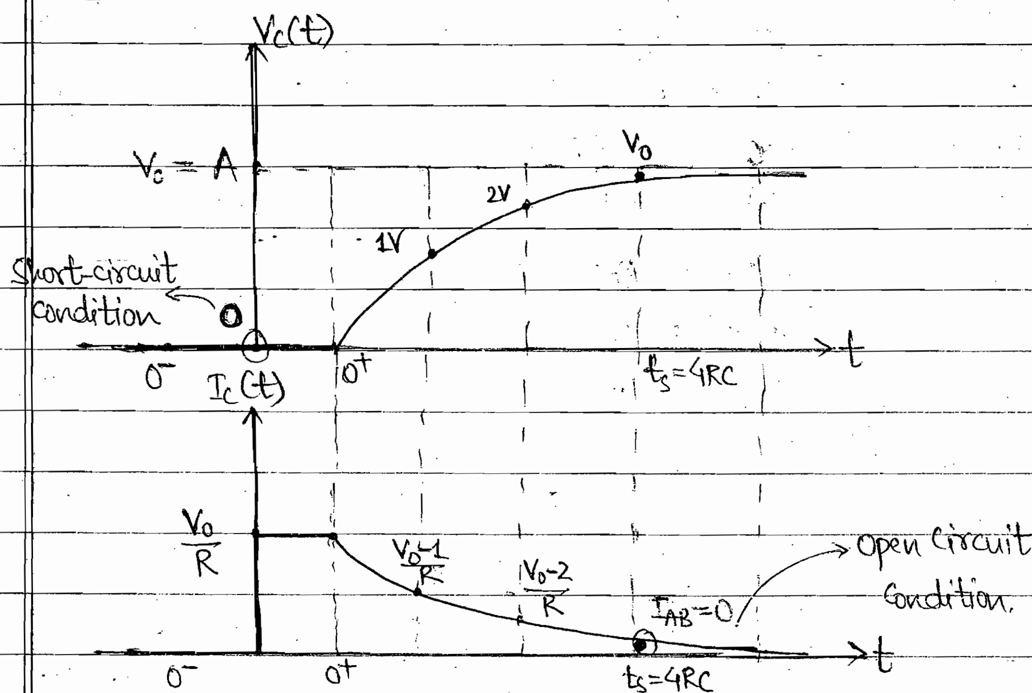
Now, $$\frac{V_c(s)}{V_i(t)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs+1} = \frac{(1/RC)}{(s+1/RC)} \quad \text{——— (2)}$$

Comparing (1) and (2), $\boxed{\tau = RC}$ *

$$\therefore \boxed{V_c(t) = V_0\left[1 - e^{-t/RC}\right]} \; *$$

or in terms of block diagram,

$$\boxed{c(t) = A\left[1 - e^{-t/\tau}\right]} \; *$$

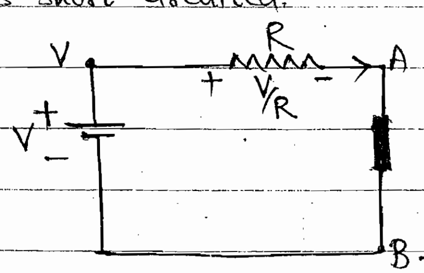


(I) Short circuit condition :-

* Capacitor is short-circuited.

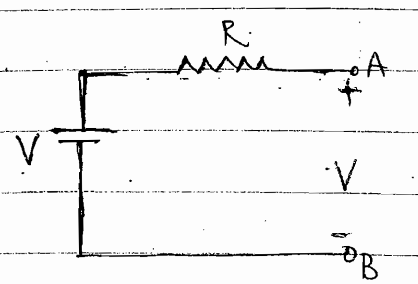
$\therefore I_{AB} = \frac{V}{R}$ (Maximum)

$V_{AB} = 0$ (Minimum).



(II) Open Circuit condition :-

* Capacitor is open-circuited.



$V_{AB} = V$ (Maximum)

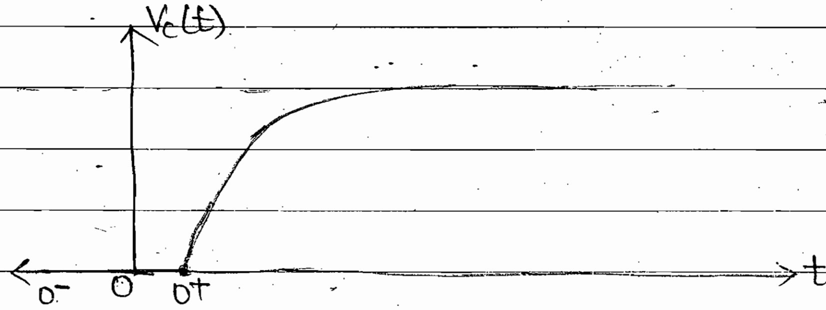
$I_{AB} = 0$ (Minimum).

- For Capacitor, Voltage cannot change instantaneously but current does. i.e.

$$\boxed{V_c(0^-) = V_c(0^+)} \; *$$

But $\boxed{I_c(0^-) \neq I_c(0^+)}$ *

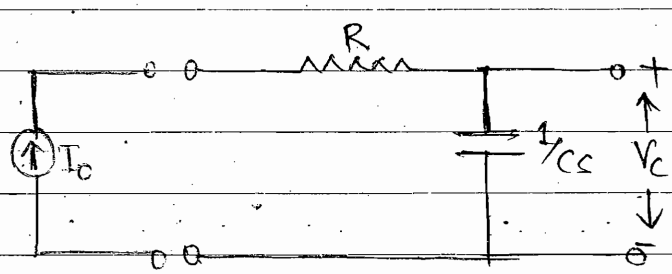
- Now, by decreasing Resistance or more accurately Time constant of the system, I_{AB} will increase i.e more charge will flow across plates of Capacitor and it charges more rapidly, thus time of voltage decreases from $t=0^+$ to some other decreased value.



- If Time constant is made 0, then $0^- = 0 = 0^+ = \tau$ and Capacitor voltage increases instantaneously. (Not possible generally).

* Here, voltage V_0 is given to RC circuit and output is also a voltage i.e. $V_c(t)$, so it will be a Closed Loop System but in general, RC circuit will not always be Open or closed loop system and it depends on Input to be applied.

* <u>For example</u> :- If instead of V_0, I_0 is given, then :



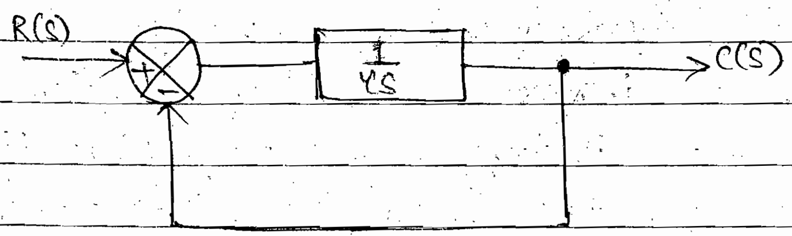
$$\Rightarrow \frac{V_c(s)}{I_i(s)} = \frac{1/C}{s}$$

$$\Rightarrow V_c(s) = \frac{1}{C \cdot s} \times \frac{I_0}{s}$$

$$\therefore \boxed{V_c(t) = \left(\frac{I_0}{C}\right) \cdot t} \; *$$

- Here, Capacitor will charge to infinite time and current will not become 0. It will be highly stable.
- It is also an Open Loop System.

2014 * → Closed Loop First order system for Ramp Input :-



$$\frac{C(S)}{R(S)} = \frac{(1/\tau)}{(s+1/\tau)}.$$

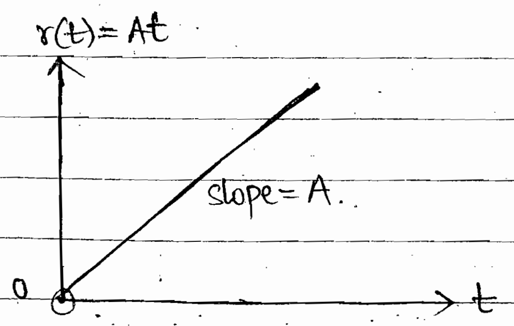
As $R(S) = 1/s^2$ (For Ramp Input).

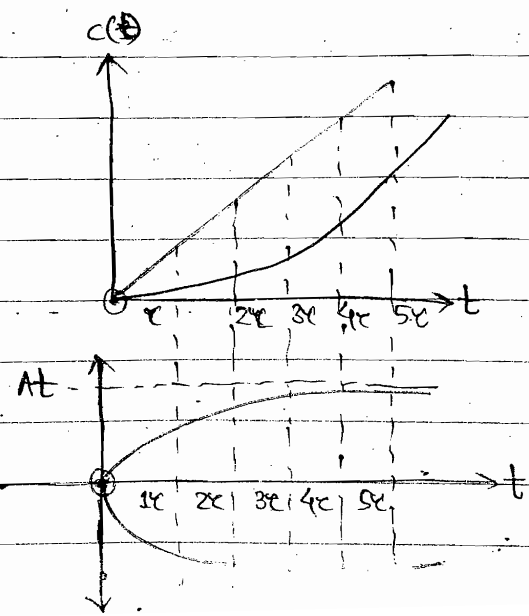
$$\therefore C(S) = \frac{1/\tau}{s^2(s+1/\tau)}$$

$$\therefore C(S) = \frac{A}{s^2} - \frac{A\tau}{s} + \frac{A\tau}{(s+1/\tau)}.$$

$$\therefore c(t) = At - A\tau u(t) + A\tau e^{-t/\tau}$$

or $$\boxed{c(t) = At - A\tau + A\tau e^{-t/\tau}} *$$





Now, Error Response, $e(t) = r(t) - c(t)$

$$\therefore e(t) = At - \{At - A\tau + A\tau . e^{-t/\tau}\}$$

$$\therefore \boxed{e(t) = A\tau[1 - e^{-t/\tau}]} *$$

- At $t=0$, $c(t) = 0$; $e(0) = 0$
- At $t = 1\tau$; $c(\tau) = 0.37 A\tau$; $e(\tau) = 0.63 A\tau$.
- At $t = 2\tau$; $c(2\tau) = 1.135 A\tau$; $e(2\tau) = 0.865 A\tau$.
- At $t = 3\tau$; $c(3\tau) = 2.049 A\tau$; $e(3\tau) = 0.951 A\tau$.
- At $t = 4\tau$; $c(4\tau) = 3 A\tau$; $e(4\tau) = A\tau$.
- At $t = \infty$; $c(\infty) = At - A\tau$ $[t \geq t_s = \infty]$
- At $t = 5\tau$; $c(5\tau) = 4A\tau$; $e(5\tau) = A\tau$.

* System Error ; $e_s(t)$ = Slope of Input − Slope of Output.

$$\therefore e_s(t) = A - A[1 - e^{-t/\tau}]$$

[As, $r(t) = At$

$$\therefore \frac{dr(t)}{dt} = A$$

and $c(t) = At - A\tau[1 - e^{-t/\tau}]$.

$$\therefore \frac{dc(t)}{dt} = A - Ae^{-t/\tau}.$$]

$$\therefore e_s(t) = A - A[1 - e^{-t/\tau}].$$

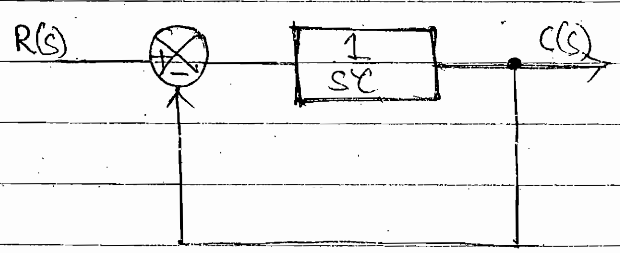
$$\therefore \boxed{e_s(t) = Ae^{-t/\tau}} *$$

- At $t=0$; $e_s(t=0) = A$ (100%)
- At $t=\tau$; $e_s(\tau) = 0.\cancel{0}37A$. (37%)
- At $t=2\tau$; $e_s(2\tau) = 0.135A$ (13.5%)

- At $t=3\tau$; $e_s(3\tau) = 0.049A$ (4.9%)
- At $t=4\tau$; $e_s(4\tau) = 0.018A$. (1.8%)
- At $t=5\tau$; $e_s(5\tau) = 0.0033A$ (0.33%)

- At $t=\infty$; $e(\infty) = 0$ (0%).

* Closed Loop First Order system with Impulse Input :-



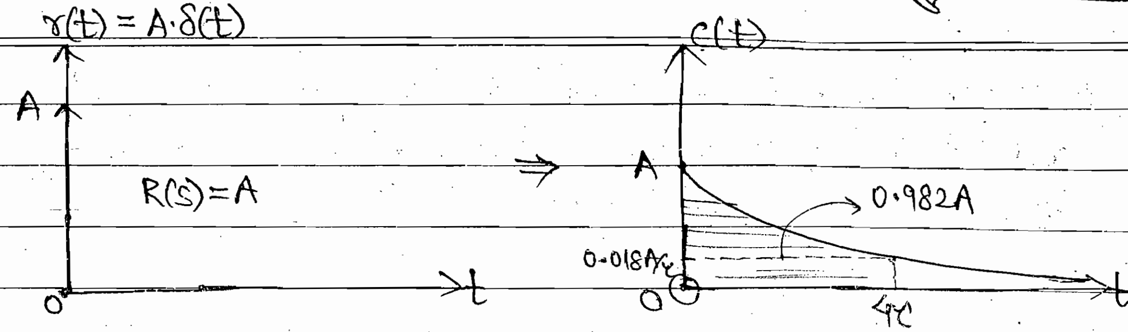
$$\frac{C(s)}{R(s)} = \frac{1/\tau}{s+1/\tau}$$

As $R(s) = A$ [Impulse Input].

$$\therefore \quad C(s) = \frac{(A/\tau)}{(s+1/\tau)}$$

$$\therefore \quad \boxed{c(t) = \frac{A}{\tau}.e^{-t/\tau}} \; *$$

- At $t=4\tau$, $c(4\tau) = \frac{A}{\tau} \times 0.018$.



Now, Area under the curve at 4τ,

$$\therefore \quad \text{Area} = \int_0^{4\tau} c(t)\cdot dt = \int_0^{4\tau} \frac{A}{\tau}\cdot e^{-t/\tau}\cdot dt.$$

$$\text{Area} = -A\left[e^{-4} - e^{0}\right] = -A\left[0.018 - 1\right].$$

$$\therefore \quad \boxed{\text{Area} = 0.982A} * \quad [1.8\% \text{ error}].$$

But for limits upto $t=0$ to $t=\infty$,

$$\boxed{\text{Area} = A} *$$

* In case of Impulse input, output error is not defined but system error is defined in terms of Area.

$$\therefore \quad e_s(t) = \text{Input Area} - \text{Output Area}.$$

$$\Rightarrow \quad e_s(t) = A - \int_0^t c(t)\cdot dt$$

$$\therefore \quad \boxed{e_s(t) = Ae^{-t/\tau}} *$$

- At $t=0$; $e_s(0) = 0$.
- At $t=\tau$; $e_s(\tau) = 0.37A$.
- At $t=2\tau$; $e_s(2\tau) = 0.135A$
- At $t=3\tau$; $e_s(3\tau) = 0.049A$.
- At $t=4\tau$; $e_s(4\tau) = 0.018A$.

- In case of Parabolic input, rate of change of slope will be constant i.e system error will depends now on rate of change of slope.

Now,

(i) $C_R(t) = At - A\tau\left[1 - e^{-t/\tau}\right]$ $r_R(t) = At$

$\frac{d}{dt}(C_R(t)) = A - Ae^{-t/\tau}$. $\frac{d}{dt}(r_R(t)) = A = r_s(t)$

(ii) $C_s(t) = A\left[1 - e^{-t/\tau}\right]$. $r_s(t) = Au(t)$.

$\frac{d}{dt}(C_s(t)) = \frac{A}{\tau} \cdot e^{-t/\tau}$ $\frac{d}{dt}(r_s(t)) = A\delta(t) = r_i(t)$.

As In laplace transform:

(i) $C_R(s) = \frac{1}{s}\left[\frac{(A/\tau)}{s(s+1/\tau)}\right]$ $R_R(s) = \frac{1}{s}\left[\frac{A}{s}\right]$.

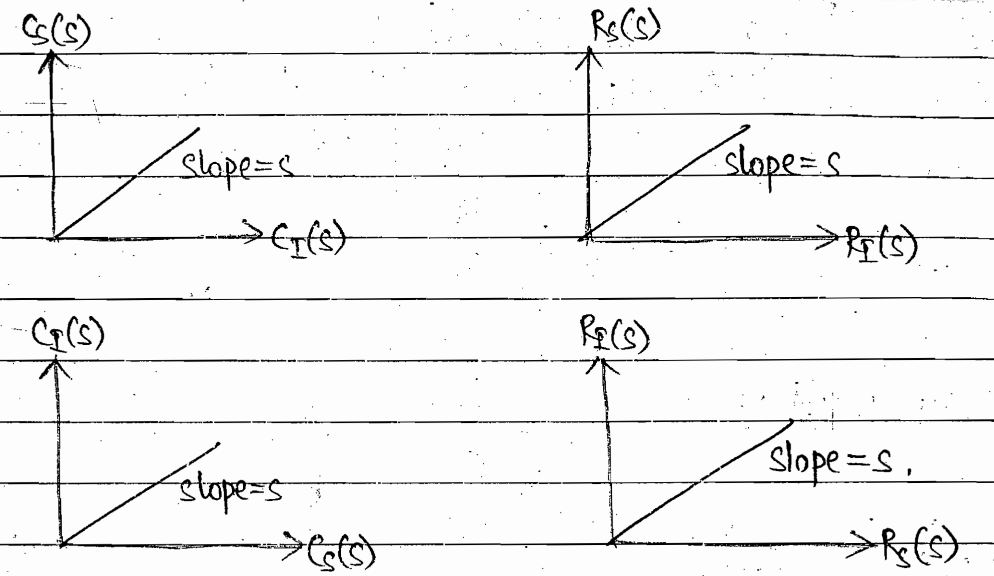
$C_s(s) = sC_R(s) = \frac{(A/\tau)}{s(s+1/\tau)}$ $R_s(s) = sR_R(s) = A/s$.

$\boxed{C_s(t) = \frac{d}{dt}[C_R(t)]}$ * $\boxed{r_s(t) = \frac{d}{dt}(r_R(t))}$ *

(ii) $C_s(s) = 1/s\left[\frac{A/\tau}{(s+1/\tau)}\right]$ $R_s(s) = 1/s\,[A]$.

$C_I(s) = sC_s(s) = \frac{A\tau}{s+1/\tau}$ $R_I(s) = sR_s(s) = A$.

$\boxed{C_I(t) = \frac{d}{dt} \cdot [C_s(t)]}$ * $\boxed{r_I(t) = \frac{d}{dt}[r_s(t)]}$ *



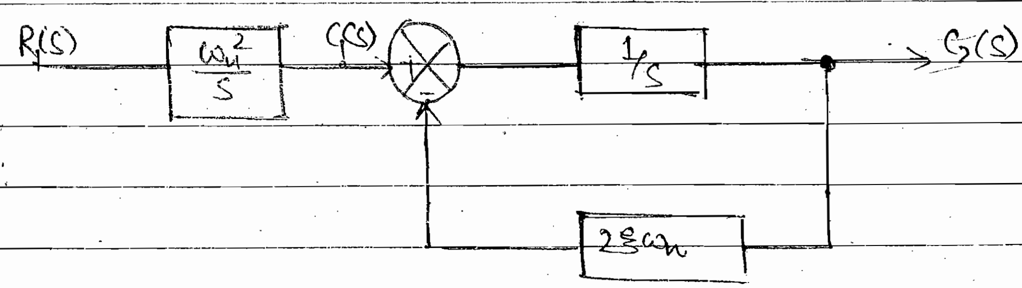
* → For Parabolic Input :- $r_p(t) = At^2$

$$\therefore \quad R_p(s) = \frac{A.2!}{s^3} = \frac{2.A}{s^3}$$

or $\quad sR_p(s) = 2R_R(s) \Rightarrow \boxed{R_R(s) = \frac{s}{2}.R_p(s)}$ *

$\therefore$ To maintain Linearity, given input as :- $r_p(t) = \frac{At^2}{2}$

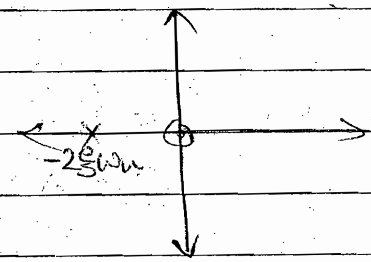
* → 'OPEN LOOP SECOND ORDER SYSTEM' :-



$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s} \times \frac{1}{(s+2\xi\omega_n)}$$

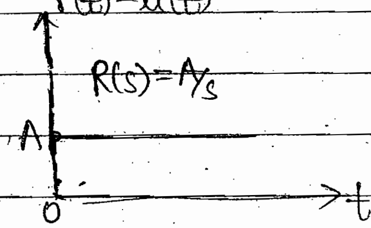
$$\Rightarrow \quad \boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}} \ *$$

$$R(s) \longrightarrow \boxed{\frac{\omega_n^2}{s(s+2\xi\omega_n)}} \longrightarrow C(s)$$



$$\boxed{\tau = \frac{1}{2\xi\omega_n}} \quad *$$

(i) For step input :-



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

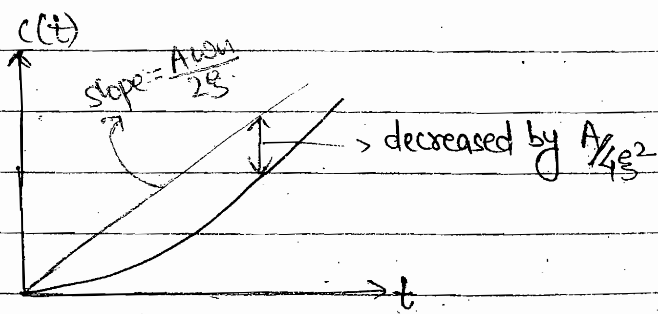
$\Rightarrow$ $R(s) = A/s$ (For step input)

$$\therefore \quad C(s) = \frac{A\cdot\omega_n^2}{s^2(s+2\xi\omega_n)}$$

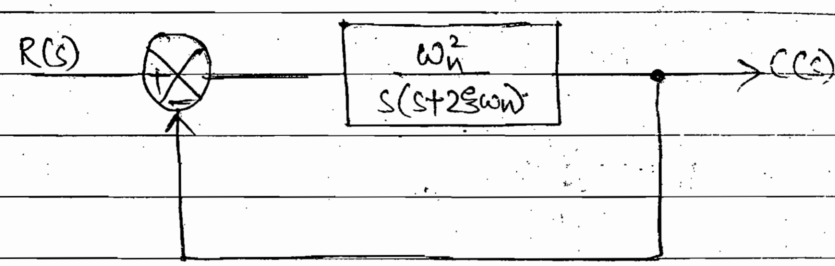
$$\therefore \quad C(s) = \frac{\frac{A\cdot\omega_n}{2\xi}}{s^2} - \frac{A/4\xi^2}{s} + \frac{A/4\xi^2}{(s+2\xi\omega_n)}$$

$$\therefore \quad \boxed{c(t) = \frac{A\cdot\omega_n\cdot t}{2\xi} - \frac{A}{4\xi^2} + \frac{A}{4\xi^2}e^{(-2\xi\omega_n t)}} \quad *$$

Also, $c\left[t \geq t_s = 4\tau = \frac{4}{2\xi\omega_n}\right] = \frac{A\cdot\omega_n\cdot t}{2\xi} - \frac{A}{4\xi^2}$



* CLOSED LOOP SECOND ORDER SYSTEM :



$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

so, for location of pole, $s^2+2\xi\omega_n s+\omega_n^2 = 0$.

$$\therefore \boxed{s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}} \; *$$

where, ξ = Damping Ratio, ω_n = Natural Frequency.

- In Time Response Analysis, ω_n is constant and ξ is variable.

** Case:- I :- Underdamped System :

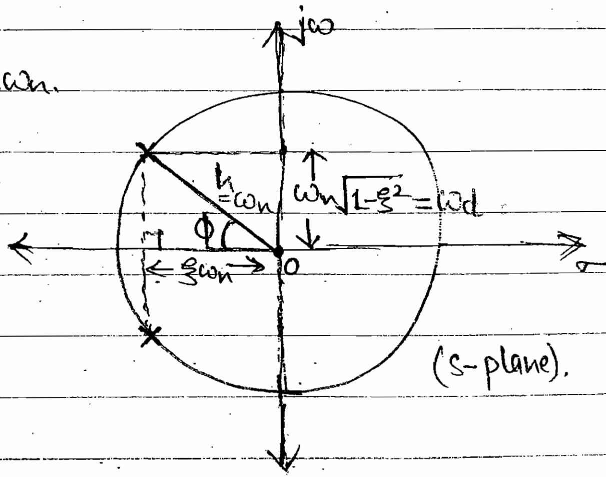
- In this case,

$$\boxed{0<\xi<1} \; *$$

$$\boxed{s_{1,2} = -\xi\omega_n \pm j\sqrt{1-\xi^2}\cdot\omega_n} \; *$$

Here, as real part of pole is : $\xi\omega_n$.

$$\therefore \boxed{\text{Time Constant } (\tau) = \frac{1}{\xi\omega_n}} \; *$$



Now, $h = \sqrt{\omega_n^2(1-\xi^2)+\xi^2\omega_n^2}$

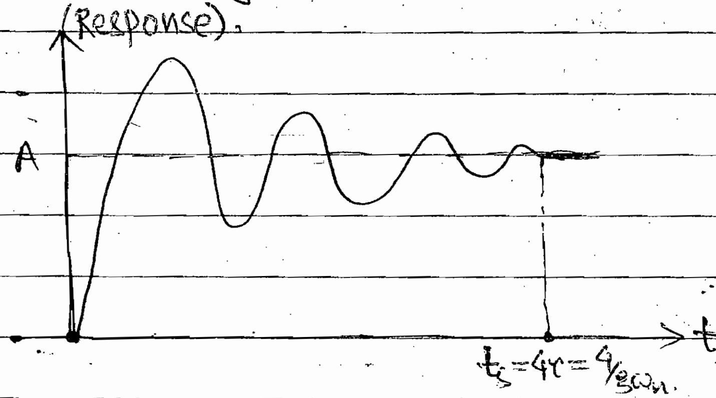
$$\therefore \boxed{h = \omega_n} \; *$$

Now, $\cos\phi = \dfrac{\xi\omega_n}{\omega_n}$ $\qquad$ $\sin\phi = \dfrac{\omega_n\sqrt{1-\xi^2}}{\omega_n}$

$\boxed{\cos\phi = \xi}$ * $\qquad$ $\boxed{\sin\phi = \sqrt{1-\xi^2}}$ *

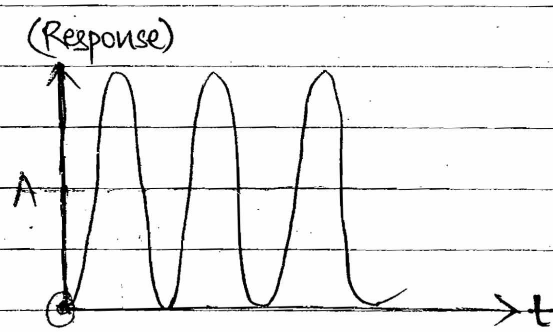
$\therefore$ $\boxed{\tan\phi = \dfrac{\sqrt{1-\xi^2}}{\xi}}$ *

- If ξ is now changed, then location of pole will be on locus of circle formed by constant radius ω_n.



$\therefore$ $\boxed{\text{settling time} = 4\tau = \dfrac{4}{\xi\omega_n}}$ *

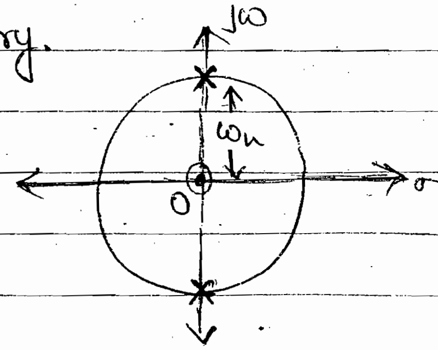
* Case-II :- Undamped System :-



Here, $\boxed{\xi = 0}$ *

$\boxed{s_{1,2} = \pm j\omega_n}$ *

- Roots will be purely Imaginary.

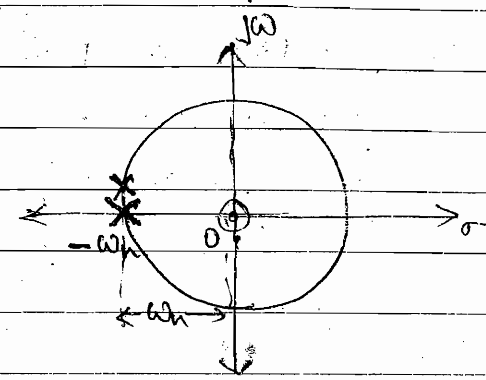


- $\boxed{\tau = \infty}$ *
- $\boxed{t_s = \infty}$ *

- Undamped system will oscillate between two finite values continuously. This type of system is Marginally stable system.
- In this system, output contains no exponential terms but contains sinusoidal terms.

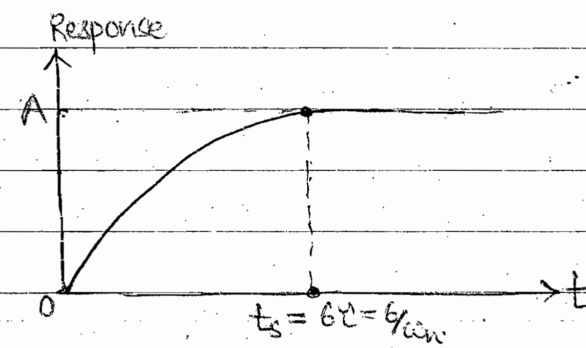
Case:-III :- Critically Damped system :-

Here, $\boxed{\xi = 1}$ *



∴ $\boxed{s_{1,2} = \pm\omega_n}$ *

- Roots are purely real and equal.
- $\boxed{\tau = 1/\omega_n}$ *
- $\boxed{t_s = 6\tau}$ *.



- Relating Critically and Underdamped systems :-

$$t_{s(C.D)} = t_{s(U.D)} \Rightarrow \frac{6}{\omega_n} = \frac{4}{\xi\omega_n}$$

$$\Rightarrow \boxed{\xi = \frac{2}{3}} \; *.$$

∴ If $\xi = 2/3$; $t_{s(C.D)} = t_{s(U.D)}$

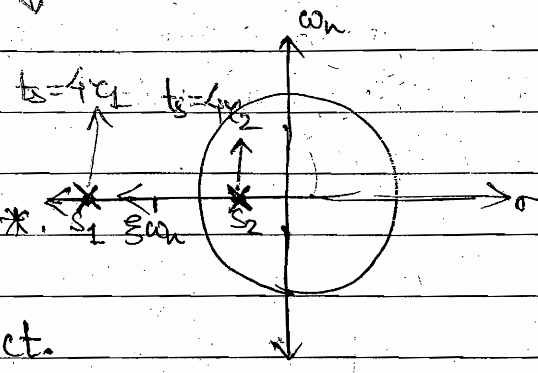
If $\xi < 2/3$; $t_{s(C)} < t_{s(D)}$

If $\xi > 2/3$; $t_{s(C)} > t_{s(D)}$

Case-IV: Overdamped System :-

Here, $\boxed{\xi > 1}$ *

$$\boxed{S_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}}$$ *



- Roots are Real and Distinct.
- There will be two Time constants in overdamped system i.e.

$$\tau_1 = \frac{1}{\xi\omega_n + \omega_n\sqrt{\xi^2-1}}, \quad \tau_2 = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$$

$\therefore$ As $\tau_2 > \tau_1$ or $\frac{\tau_2}{\tau_1} > 1$

or $\boxed{\frac{\tau_1}{\tau_2} < 1}$ *

- Settling time of output will be : $4\tau_2$

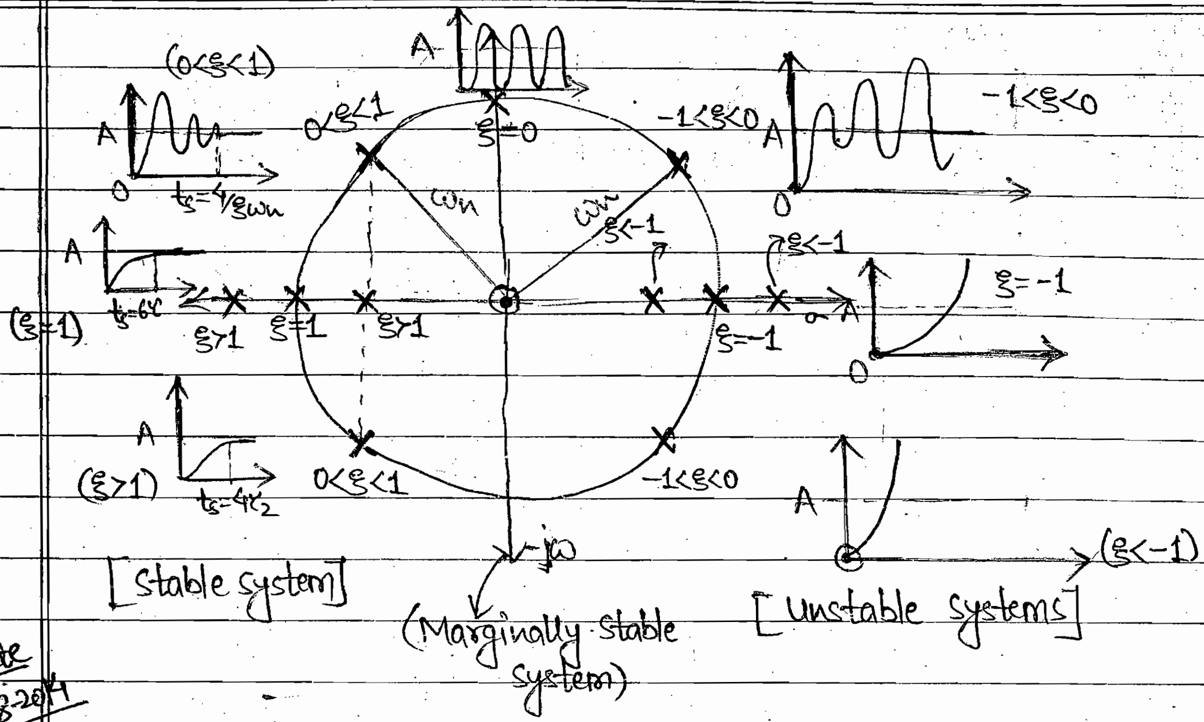
As with input as Unit step,

$$c(t) = A\left[1 - C_1 e^{-t/\tau_1} - C_2 e^{-t/\tau_2}\right].$$

- At $t = 4\tau_1$, $c(4\tau_1) = A\left[1 - C_1 \cdot e^{-4} - C_2 \cdot e^{-4\left(\frac{\tau_1}{\tau_2}\right)}\right]$
- At $t = 4\tau_2$, $c(4\tau_2) = A\left[1 - C_1 \cdot e^{-4\left(\frac{\tau_2}{\tau_1}\right)} - C_2 e^{-4}\right]$ [settled entirely].

$$\therefore \boxed{t_s = 4\tau_2 = \frac{4}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}}$$ *

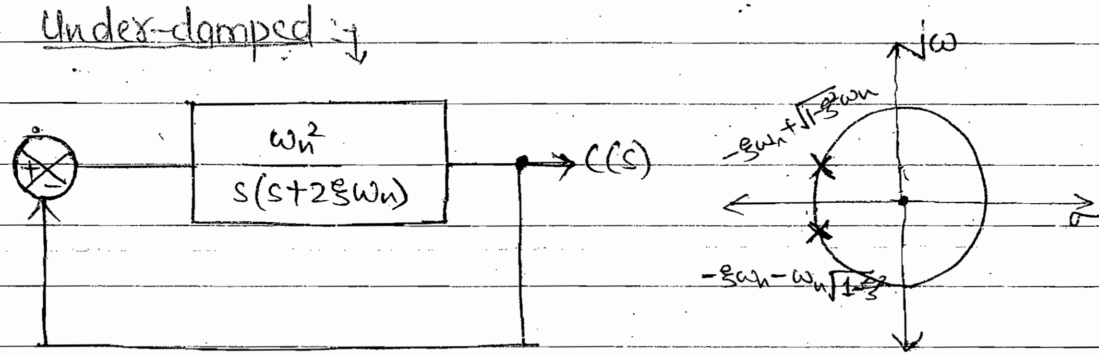
* t_s will depend on the pole nearer to Origin or simply Dominant Pole.



Date
06-08-2014

* CLOSED LOOP SECOND ORDER SYSTEM FOR STEP INPUT :-

Case:-I: Under-damped :-

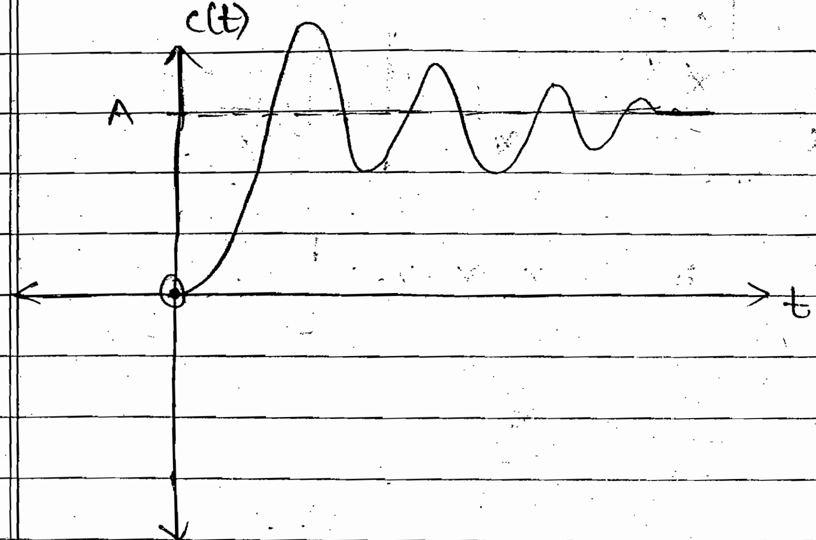


$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+\xi\omega_n+j\sqrt{1-\xi^2}\,\omega_n)(s+\xi\omega_n-j\sqrt{1-\xi^2}\cdot\omega_n)}$$

$\therefore$ For $R(s) = 1/s$

$$\therefore \boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot\sin\left(\omega_n\sqrt{1-\xi^2}\,t+\phi\right)} \; *$$

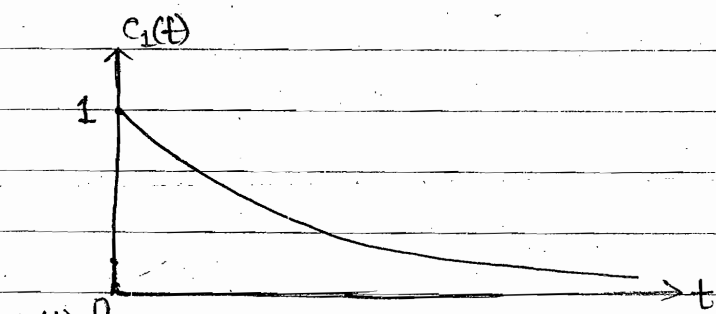
where, $\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$.

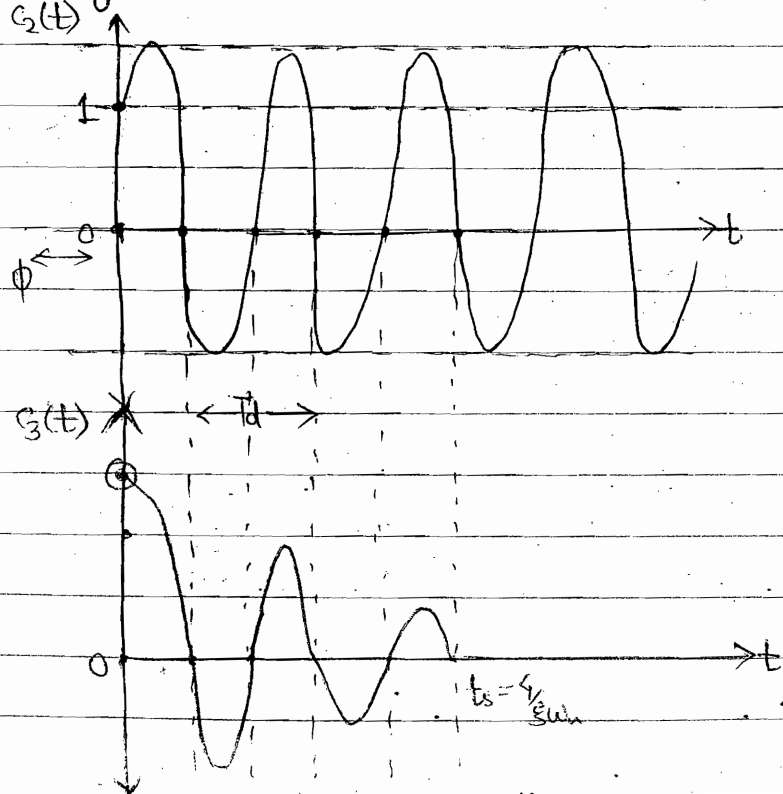


• Suppose :-

$$c_1(t) = e^{-\xi\omega_n t} \quad ; \quad c_2(t) = \frac{1}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t+\phi\right]$$

At $t=0$, $c_1(0) = 1$; $c_2(0) = \frac{1}{\sqrt{1-\xi^2}} \times \sqrt{1-\xi^2} = 1$





$\therefore\ G_3(t) = G_1(t)\cdot G_2(t) \Rightarrow G_3(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)$.

or $G_3(t=0) = \frac{1}{\sqrt{1-\xi^2}} \times \sqrt{1-\xi^2} = 1$.

or $G_3(t=4/\xi\omega_n) = \frac{1}{\sqrt{1-\xi^2}} \sin\left\{\omega_n\sqrt{1-\xi^2} \times \frac{4}{\xi\omega_n} + \phi\right\} e^{-\xi\omega_n \times 4/\xi\omega_n}$

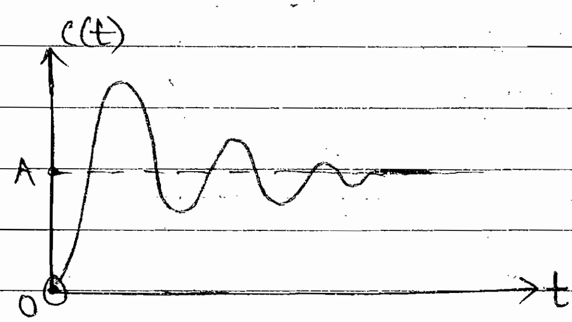
$G_3(4/\xi\omega_n) = \frac{1}{\sqrt{1-\xi^2}} \sin\left(\frac{4\sqrt{1-\xi^2}}{\xi} + \phi\right) e^{-4}$

e^{-4} is assumed to be 0 under 2% error-band.

$\therefore\ \boxed{G_3(t_s) = 0}$ *.

Now, $c(t) = A[1 - G_3(t)]$

$\therefore$ As $G_3(t)$ decreases, $c(t)$ increases as shown.



- Always Exponential term is responsible for settling of Response.

① RISE TIME (t_r) :-

- It is defined as time when output reaches 0 to 100% of desired output for under-damped system and 10% to 90% of desired output of overdamped systems.
- Rise time (t_r) is Non-periodic in nature.
- It exists only in First oscillation cycle.

So, when ouput is 100%, time will be Rise time.

$$\therefore \quad c(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\{\omega_d t + \phi\}\right]$$

$$\Rightarrow \quad A = A\left[1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)\right].$$

$$\Rightarrow \quad \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t_r + \phi) = 0.$$

$$\therefore \quad \sin(\omega_d t_r + \phi) = 0$$

$$\Rightarrow \quad \sin(\omega_d t_r + \phi) = \sin(0^\circ).$$

$$\Rightarrow \quad \omega_d t_r + \phi = n\pi.$$

And as t_r exists only for First oscillation cycle, so, $\boxed{n=1}$ *

$$\therefore \quad \boxed{t_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}}} \; * \quad (\phi \text{ in Radians}).$$

② PEAK TIME (t_p) :-

- It is defined as, time in which slope of output response is zero and output is either Maxima or Minima.
- If slope is 0 and output is Maximum, then that time will be known as Peak Overshoot time.
- If slope is 0 and output is Minimum, then that time will be known as Peak Undershoot time.

* Both Peak overshoot time and Peak undershoot time are Periodic in nature and exist in each oscillation cycle alternately.

- To get t_p, Differentiate $c(t)$ and equate to 0,

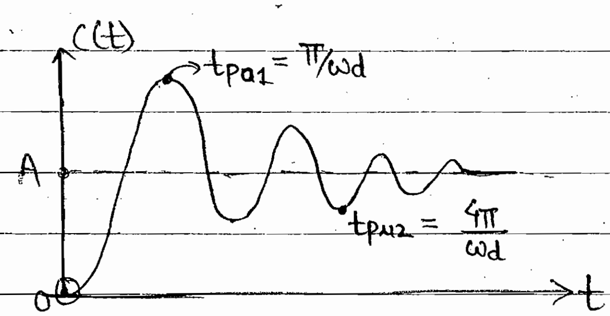
$$\therefore \frac{d}{dt} c(t) = 0.$$

$$\therefore t_{pon} = \frac{(2n-1)\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{(2n-1)\pi}{\omega_d}$$

$$\therefore \boxed{t_{pon} = \frac{(2n-1)\cdot\pi}{\omega_d}} \; * \; \text{(For overshoots).}$$

* n = Odd (1,3,5---) for Peak Overshoots.
* n = Even (2,4,6---) for Peak Undershoots.
} for $\boxed{t_p = \frac{n\pi}{\omega_d}}$ *

* For Undershoot : $\boxed{t_{pun} = \frac{(2n)\cdot\pi}{\omega_d}}$ *



(3) PERCENTAGE OVERSHOOT ($M_p\%$) :-

- It exists at Peak Overshoot time.
- As peak overshoot time is periodic, $\% M_{po}$ is also Periodic in nature.
- It will exist in each oscillation.

$$\boxed{\% M_{pon} = \frac{c(t)_{max} - c(t)_{desired}}{c(t)_{desired}} \times 100\,\%} \; * \quad \% M_{pon} = e^{\frac{-(2n-1)\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$$

$\therefore$ For first Maximum overshoot, $\% M_{po1} = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$

(4) PERCENTAGE UNDERSHOOT :-

- It exists at peak undershoot time.
- It is periodic in nature.

$$\boxed{\%M_{pun} = \frac{c(t)_{desired} - c(t)_{min}}{c(t)_{desired}} \times 100\% = e^{-2n\pi\xi/\sqrt{1-\xi^2}} \times 100\%} \ *$$

* Note :- $\boxed{\frac{\%M_{po1}}{\%M_{u1}} = \frac{\%M_{pu1}}{\%M_{po2}} \text{ and so on..}}$ *

- Between Peak overshoot time and peak undershoot time, time difference will always remain constant and is equal to $\left(\pi/\omega_d\right)$ and it is half of the Time period of Damped sinusoid.

so, note * $\boxed{\text{First Undershoot} = \text{Time period of Damp Sinusoid} = T_d = \frac{2\pi}{\omega_d}}$ *

(5) SETTLING TIME (t_s) :-

$$\boxed{t_s = \frac{4}{\xi\omega_n} = 4\tau} \ *$$

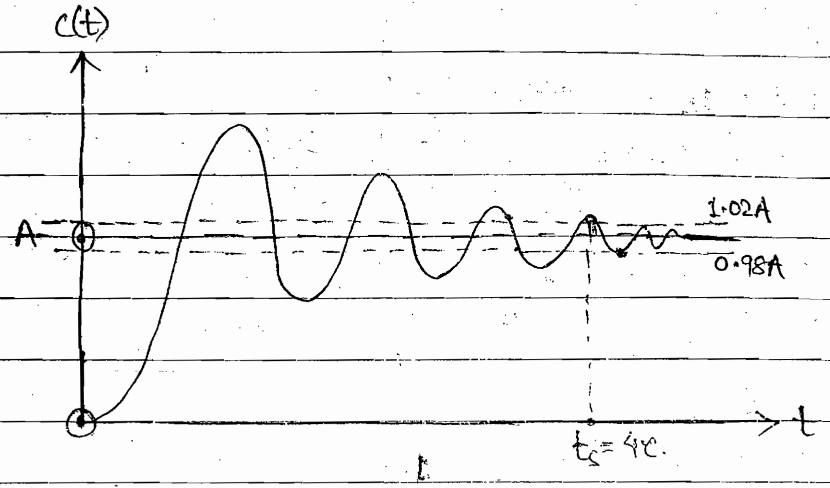
- In case of Settling time, in Underdamped system, it is defined as time when output will reach 98% of desired value or 102% of desired value.

$t_s = 4\tau$; For 2% error-band

$t_s = 3\tau$; For 5% error-band.

$t_s = 4.62\tau$; For 1% error band.

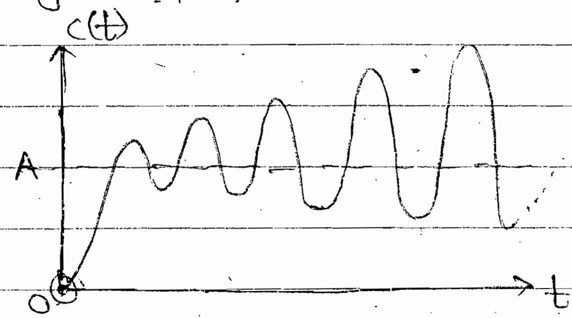
$$\boxed{\text{No. of cycles} = \frac{t_s}{T_d}} \ *$$



* **Subcase :-** If exponential term is Positive :-

$$\text{for } c(t) = A\left[1 - \frac{e^{\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)\right].$$

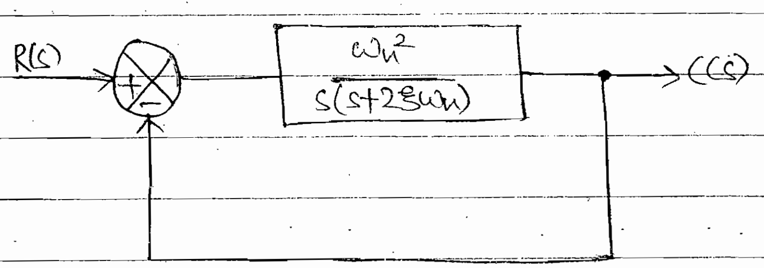
- Here, roots will lie on Right-half of s-plane and output response will be as shown :



* If Exponential term is negative, then with time, output will settle.

* If Exponential term is positive, with time output never settles.

Case :- II :- Undamped system :-



Here, $\boxed{\xi = 0}$ *

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \Rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

- For $R(s) = A/s$, $C(s) = \dfrac{A\omega_n^2}{s(s+j\omega_n)(s-j\omega_n)}$

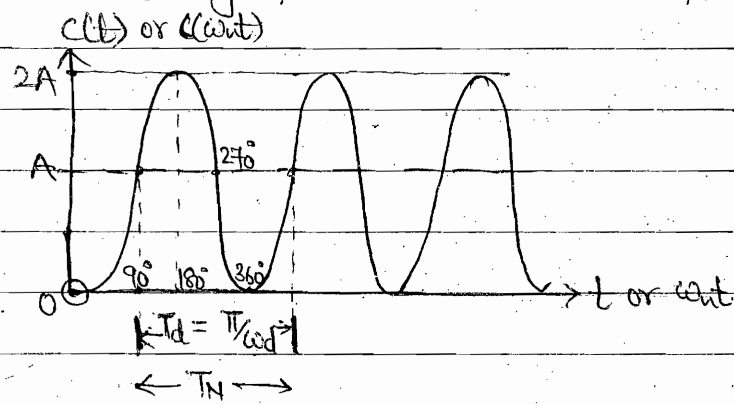
$$\therefore \quad C(s) = \frac{A}{s} - \frac{A/2}{(s-j\omega_n)} - \frac{A/2}{(s+j\omega_n)}$$

$$\Rightarrow \quad c(t) = A\left[1 - \left(\frac{e^{j\omega_n t} + e^{-j\omega_n t}}{2}\right)\right]$$

$$\boxed{c(t) = A\left[1 + \cos(\omega_n t)\right] \cdot u(t)} \; *$$

- Here,

** ω_n = Frequency of oscillation = Location of Imaginary part of pole



$$\boxed{\omega_n = \frac{2\pi}{T_N}} \; *$$

- Now, $\% M_{po} = \dfrac{2A - A}{A} \times 100 = 100\%$

Also, $\% M_{pu} = \dfrac{A - 0}{A} \times 100 = 100\%$

** In case of Undamped systems, Frequency of oscillation of output is : ω_n [as $\omega_n = \omega_d$ here].

** But in underdamped systems, frequency of oscillation of output is : ω_d. [$\omega_d = \omega_n\sqrt{1-\xi^2}$].

- In undamped systems, %Mpo and %Mpu remain constant i.e 100%.

** Case: III> Critically Damped System :-

Here, $\boxed{\xi = 1}$ * $\quad \dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$\tau = 1/\omega_n \; ; \; t_s = 6\tau = 6/\omega_n$

$$\therefore C(s) = \frac{A}{s} - \frac{A\omega_n}{(s+\omega_n)^2} - \frac{A}{(s+\omega_n)} \quad \left[\text{For } R(s) = 1/s\right].$$

$$\therefore c(t) = A - A\omega_n t \cdot e^{-\omega_n t} - A e^{-\omega_n t}$$

$$\therefore \boxed{c(t) = A\left[1 - e^{-\omega_n t}\{1+\omega_n(t)\}\right]} \; *$$

Now, $c(t_s = 4/\omega_n) = A\left[1 - e^{-\omega_n \times 4/\omega_n}\{1 + \omega_n \times \frac{4}{\omega_n}\}\right] = A\left[1 - e^{-4}(5)\right]$

$$\boxed{c(4/\omega_n) = 0.91A} \; *$$

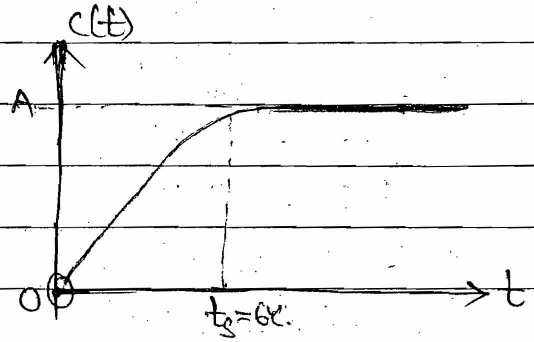
- Here, at $t = 4/\omega_n$, output has only reached to 91% with 9% error. We can't apply 2% error-band criterion here, so in critically damped system, $t_s \neq 4\tau \neq 4/\omega_n$.

So, $c(t = 6/\omega_n) = A\left[1 - e^{-\omega_n \times 6/\omega_n}\{1 + \omega_n \times 6/\omega_n\}\right] = A\left[1 - e^{-6}(7)\right]$

$$\therefore \boxed{c(6/\omega_n) = 0.983A} \; *$$

$\therefore$ At $t = 6\tau = 6/\omega_n$, output has reached 98.3%. So, using 2% error-band criterion, we can say that output has got settled.

$$\therefore \text{So, } \boxed{t_s = 6\tau = \frac{6}{\omega_n}} \; *$$



* Case: IV :- Overdamped system :-

Here, $\boxed{\xi > 1}$ * $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

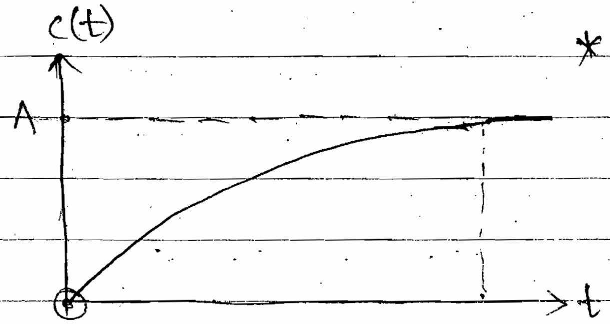
$\therefore \dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})}$.

or for $R(s) = A/s$.

$\therefore C(s) = \dfrac{A\omega_n^2}{s(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})}$

$\therefore C(s) = \dfrac{A}{s} + \dfrac{A/2\sqrt{\xi^2-1}\,(\xi + \sqrt{\xi^2-1})}{(s + \xi\omega_n + \omega_n\sqrt{1+\xi^2})} + \dfrac{A/2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})}{(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})}$

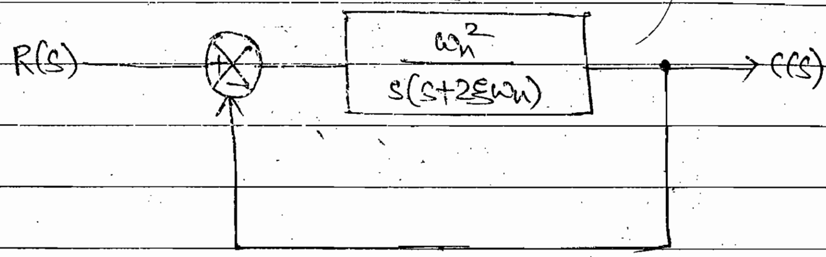
$\therefore c(t) = A\left[1 - \dfrac{e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}}{2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})} + \dfrac{e^{-(\xi\omega_n + \xi\omega_n)t}}{2\sqrt{\xi^2-1}\,(\xi + \sqrt{\xi^2-1})}\right]$.



* Sluggish response than Critically damped system response.

$t_s = 4\tau_2 = \dfrac{4}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$

* CLOSED LOOP SECOND ORDER SYSTEM FOR IMPULSE INPUT :-

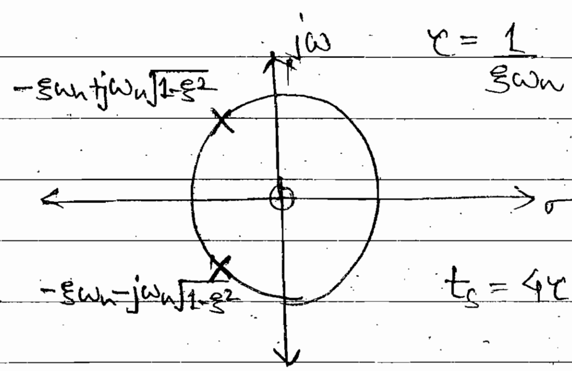


$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \text{--- (1)}$$

** Case:-I, Under-damped System :-

- $\boxed{\xi<1}$ *

$$\tau = \frac{1}{\xi\omega_n}$$



For $R(s) = A$.

$\therefore$ From eq-(1),

$$t_s = 4\tau = \frac{4}{\xi\omega_n}$$

$$C(s) = \frac{A\cdot\omega_n^2}{(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})}$$

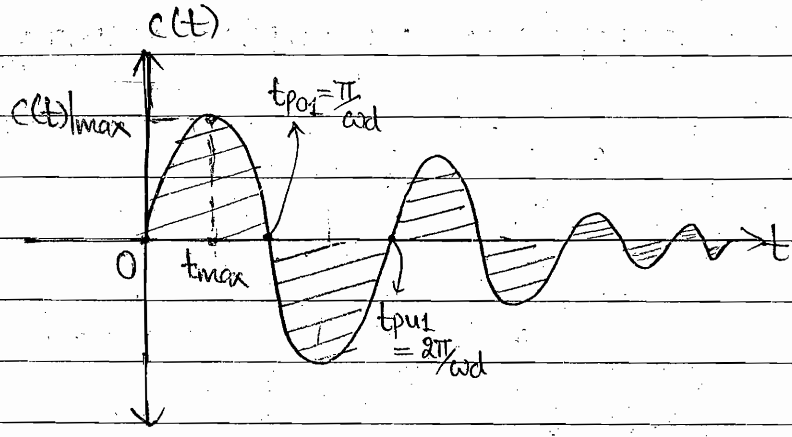
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore \quad C(s) = \frac{\dfrac{A\omega_n}{j2\sqrt{1-\xi^2}}}{(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})} - \frac{\dfrac{A\omega_n}{(j2)\sqrt{1-\xi^2}}}{(\xi\omega_n + j\omega_n\sqrt{1-\xi^2}+s)}.$$

$$\Rightarrow \quad c(t) = \frac{A\omega_n}{\sqrt{1-\xi^2}}\left[\frac{e^{-\xi\omega_n t}\cdot e^{j\omega_n\sqrt{1-\xi^2}t} - e^{-(\xi\omega_n)t}\cdot e^{-j\omega_n\sqrt{1-\xi^2}t}}{2j}\right]$$

$$\Rightarrow \quad c(t) = \frac{A\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\frac{e^{j\omega_d t} - e^{-j\omega_d t}}{2j}\right]$$

$$\boxed{c(t) = \frac{A\cdot\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\cdot \sin(\omega_d t)\cdot u(t)} \; *$$



$$\boxed{t_{max} = \frac{\tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_n\sqrt{1-\xi^2}} = \frac{\phi}{\omega d}} \ *$$

$$\boxed{c(t)|_{max} = A \cdot \omega_n \cdot e^{-\xi/\sqrt{1-\xi^2}\,\phi}} \ *$$

$\therefore$ $\boxed{t_{poN} = \frac{(2n-1)\pi}{\omega d}}$ * $\boxed{t_{pun} = \frac{2n\pi}{\omega d}}$ * } In terms of Area.

- Mpo and Mpu are system's characteristics.

Here, $\% Mp(on) = \frac{\text{Maximum Area} - \text{Desired Value}}{\text{Desired Area}} \times 100\%$

$$\% Mp(on) = e^{-(2n-1)\pi\xi/\sqrt{1-\xi^2}} \times 100\%$$

and $\% Mp(un) = \frac{\text{Desired Area} - \text{Minimum Area}}{\text{Desired Area}} \times 100\%$

$$\% Mp(un) = e^{-2n\pi\xi/\sqrt{1-\xi^2}} \times 100\% .$$

- Mpo and Mpu percentage will remain constant and equal.

* % Mpo and % Mpu in both for Step as well as Impulse inputs will remain equal as they are system parameters and don't depend on Input applied.

* Their time constant and settling time will also be same.

$\boxed{t_s = 4\tau}$ * $\boxed{\tau = \dfrac{1}{\xi\omega_n}}$ *

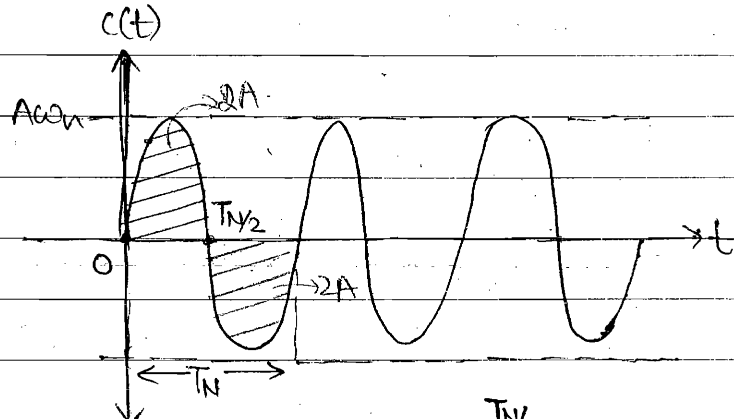
Case-II: Undamped system:-

$\boxed{\xi = 0}$ * From eq-(1), $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2+\omega_n^2}$

For $R(s) = A$ [Impulse input].

$\therefore \quad C(s) = \dfrac{A\omega_n^2}{s^2+\omega_n^2} = A\omega_n \cdot \left\{ \dfrac{\omega_n}{s^2+\omega_n^2} \right\}$

$\boxed{c(t) = A\omega_n \cdot \sin(\omega_n t)\cdot u(t)}$ * $\quad \omega_n = \omega_d = \dfrac{2\pi}{T_N}$



Area under curve $= \displaystyle\int_0^{T_N/2} c(t)\cdot dt = \int_0^{T_N/2} A\cdot\omega_n \cdot \sin(\omega_n t)\cdot dt.$

$\Rightarrow \quad \text{Area} = -\dfrac{A\omega_n}{\omega_n}\left[\cos\omega_n t\right]_0^{T_N/2} = -A\left[\cos\left(\dfrac{2\pi}{T_N}\times\dfrac{T_N}{2}\right) - \cos(0)\right]$

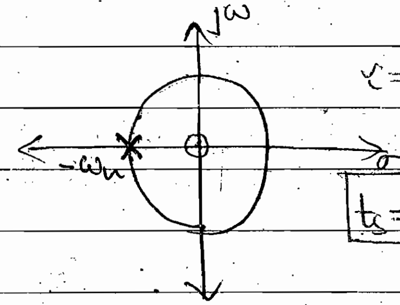
$\therefore$ $\boxed{\text{Area} = 2A = \text{Same as in Step Input case}}$ *.

Case:- III. Critically Damped System :-

$\boxed{\xi = 1}$ * From eq-(1), $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \dfrac{\omega_n^2}{(s+\omega_n)^2}$

For Impulse input, $R(s) = A$.

$\therefore \quad C(s) = \dfrac{A \cdot \omega_n^2}{(s+\omega_n)^2}$



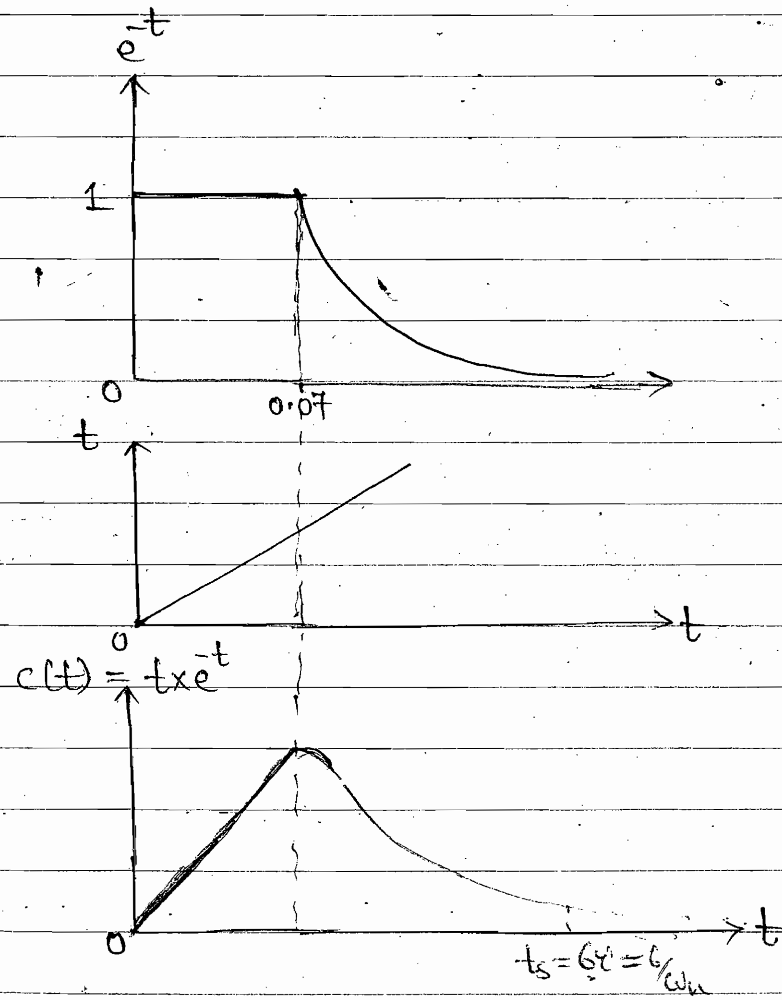
$\tau = 1/\omega_n$

$\boxed{t_s = 6\tau}$ *.

$$\boxed{c(t) = A \cdot \omega_n^2 \cdot t \cdot e^{-\omega_n t} \cdot u(t)} \; *$$

—* Plotting output :-

Let, $A = \omega_n = 1$, $\quad c(t) = t \cdot e^{-\omega_n t} \cdot u(t)$.



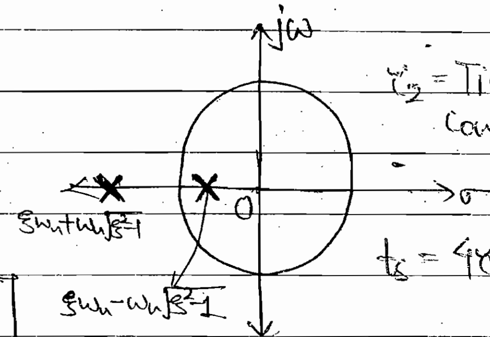
Case:- IV :- Over-damped system :-

$\boxed{\xi > 1}$ * From eq-(1), $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

For Impulse input of $R(s) = A$,

$$\therefore \quad C(s) = \frac{A\cdot\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{A\cdot\omega_n^2}{(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1})(s+\xi\omega_n-\omega_n\sqrt{\xi^2-1})}$$

$$C(s) = \frac{\frac{A\cdot\omega_n}{2\sqrt{\xi^2-1}}}{(s+\xi\omega_n-\omega_n\sqrt{\xi^2-1})} - \frac{\frac{A\cdot\omega_n}{2\sqrt{\xi^2-1}}}{(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1})}$$



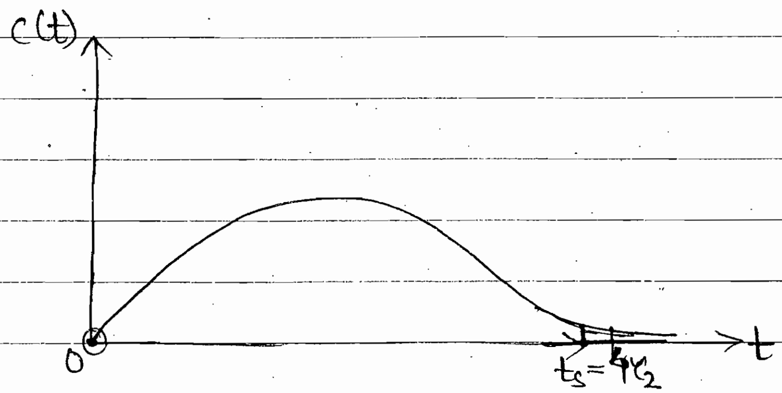
τ_2 = Time constant

$t_s = 4\tau_2$

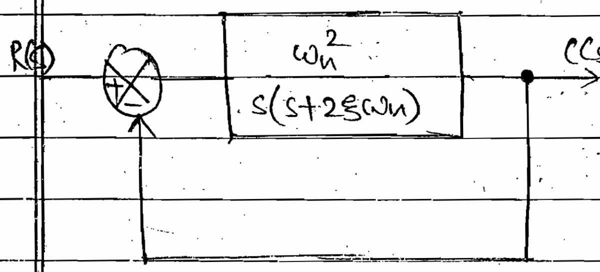
$$\therefore \quad c(t) = \frac{A\cdot\omega_n}{2\sqrt{\xi^2-1}}\left[e^{-\xi\omega_n t}\cdot e^{\omega_n\sqrt{\xi^2-1}t} - e^{-\xi\omega_n t}\cdot e^{-\omega_n\sqrt{\xi^2-1}t}\right]$$

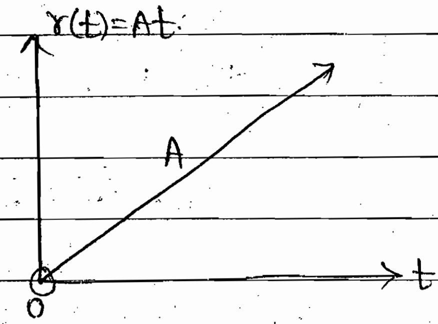
$$\therefore \quad c(t) = \frac{A\cdot\omega_n\cdot e^{-\xi\omega_n t}}{2\sqrt{\xi^2-1}}\cdot\left[\frac{e^{\omega_n\sqrt{\xi^2-1}t} - e^{-\omega_n\sqrt{\xi^2-1}t}}{2}\right]$$

$$\therefore \quad \boxed{c(t) = \frac{A\cdot\omega_n\cdot e^{-\xi\omega_n t}}{2\sqrt{\xi^2-1}}\left[e^{\omega_n\sqrt{\xi^2-1}t} - e^{-\omega_n\sqrt{\xi^2-1}t}\right]} \; *$$



*→ CLOSED LOOP SECOND ORDER SYSTEM FOR RAMP INPUT :-





$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \text{ —— (1)}$$

** case :- Under-damped systems :-

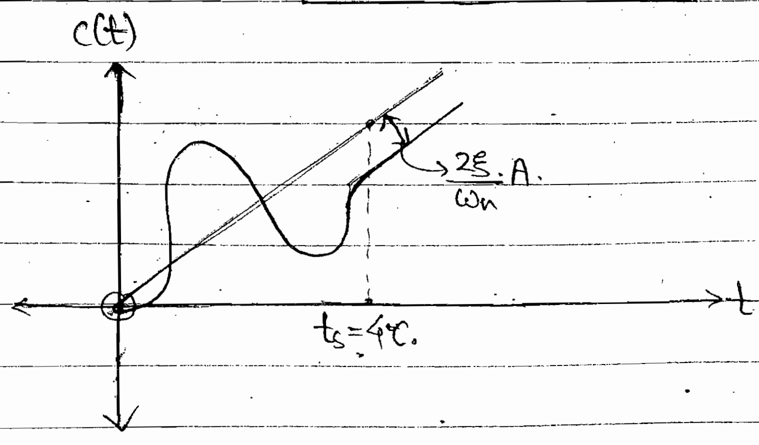
$\boxed{\xi<1}$ * From eq-(1), Also, $R(s) = \frac{A}{s^2}$ [For Ramp Input].

$$\therefore C(s) = \frac{A\cdot\omega_n^2}{(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})}$$

$$\text{or } C(s) = \frac{A\cdot\omega_n^2}{(s+\xi\omega_n - j\omega_d)(s+\xi\omega_n+j\omega_d)}$$

$$\therefore \boxed{c(t) = A\left[1-\frac{2\xi}{\omega_n}+\frac{e^{-\xi\omega_n t}}{\omega_d}\sin(\omega_d t+\phi)\right]\cdot u(t)} \; *$$

where, $$\boxed{\phi = \tan^{-1}\frac{2\xi\sqrt{1-\xi^2}}{2\xi^2-1}} \; *$$



→ Reason for Graph :-

$$\tan\phi = \frac{2\xi\sqrt{1-\xi^2}}{2\xi^2-1} = \frac{P}{B}.$$

$$\therefore \quad P = 2\xi\sqrt{1-\xi^2} \quad ; \quad B = 2\xi^2-1 ;$$

$$H = \sqrt{P^2+B^2} \quad \Rightarrow \quad H = 1$$

$$\therefore \quad \sin\phi = 2\xi\sqrt{1-\xi^2} \quad ; \quad \cos\phi = 2\xi^2-1 \; ; \; t_s = \frac{4}{\xi\omega_n} \, , \quad \tau = \frac{1}{\xi\omega_n}$$

Now, $c(t) = \left[A - \frac{2\xi \cdot A}{\omega_n} + \frac{A \cdot e^{-\xi\omega_n t}}{\omega_d} \cdot \sin(\omega_d t + \phi)\right] . u(t)$

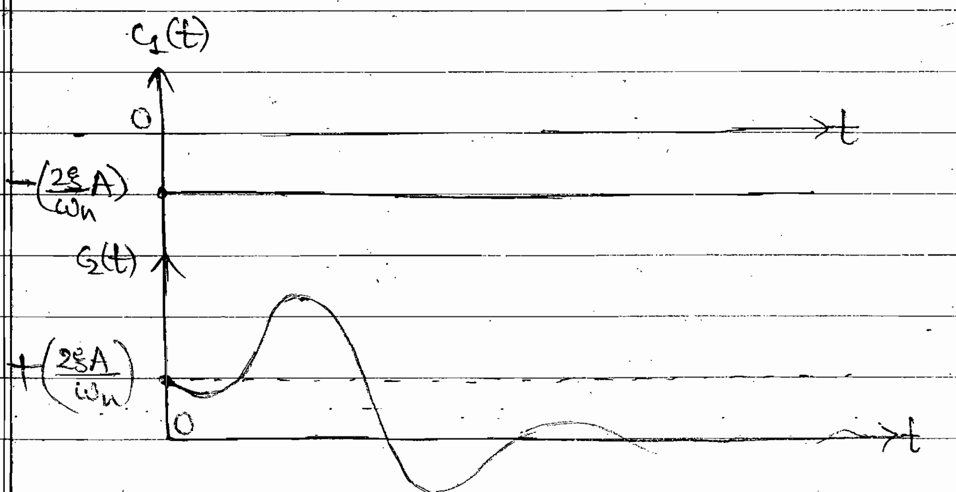
$$\therefore \quad c_1 = -\frac{2\xi}{\omega_n} A .$$

$$c_2 = \frac{A}{\omega_d} \cdot e^{-\xi\omega_n t} \sin(\omega_d t + \phi)$$

Now, $c_2(t=0) = \frac{2\xi}{\omega_n} . A .$

$$c_2(t = 4\tau) = \frac{A}{\omega_d} \times \sin\left[\omega_d \times \frac{4}{\xi\omega_n} + \phi\right] . e^{-4}$$

$$\boxed{c_2(t_s = 4\tau) = 0} \; *$$



Now, $c(t) = At - \frac{2\xi}{\omega_n} \cdot A + C_2(t)$.

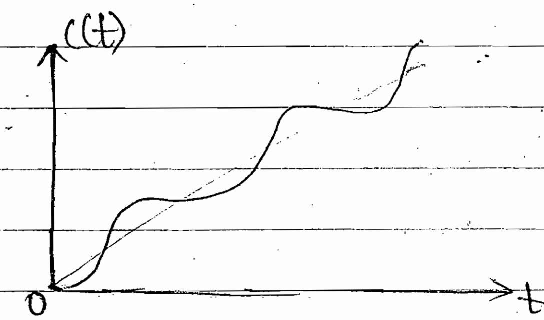
At $t=0$, $c(t=0) = 0 - \frac{2\xi \cdot A}{\omega_n} + \frac{2\xi \cdot A}{\omega_n} = 0$.

** Case:- II: Undamped System :-

$\boxed{\xi = 0}$ *.

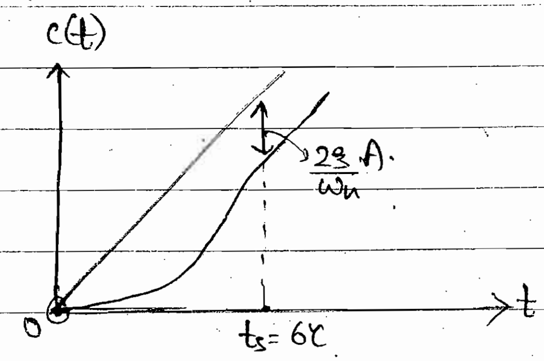
$$\boxed{c(t) = At + \frac{A}{\omega_n} \sin\{\omega_n t + 180°\}} \ast \quad \phi = 180° = \tan^{-1}\left(\frac{0}{-1}\right).$$

$$\therefore \boxed{c(t) = At - \frac{A}{\omega_n} \sin(\omega_n t)} \ast$$



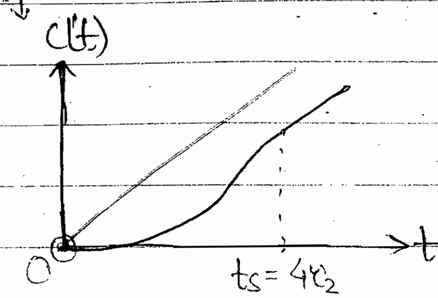
** Case: III:- Critically Damped System :-

$\boxed{\xi = 1}$ *



** Case:- IV: Overdamped Systems :-

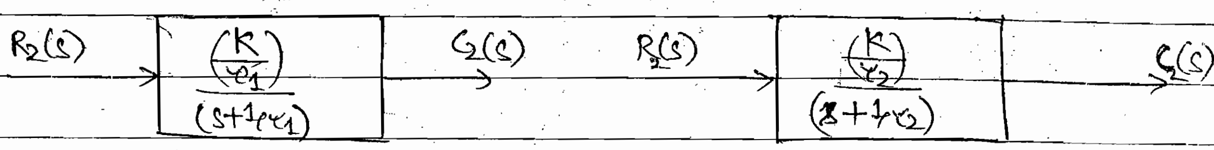
$\boxed{\xi > 1}$ *



HIGHER ORDER SYSTEMS :-

1. Cascading of First order System with First order system :-

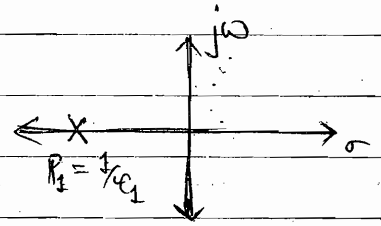
- If systems are not cascaded, then

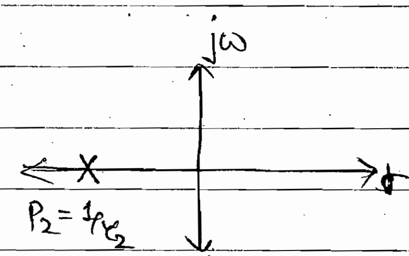


$$\frac{C_1(s)}{R_1(s)} = \frac{(K/\tau_1)}{(s+1/\tau_1)} \qquad \frac{C_2(s)}{R_2(s)} = \frac{1/\tau_2}{(s+1/\tau_2)}$$

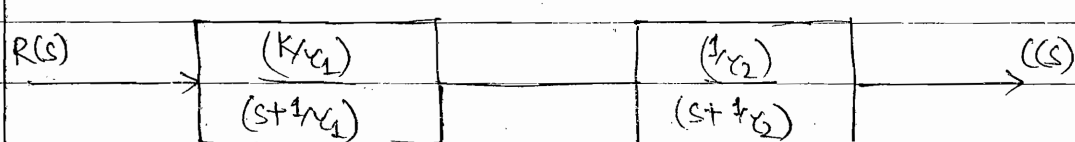
For Impulse Input,

$$c_1(t) = K \cdot P_1 \cdot e^{-P_1 t} \qquad c_2(t) = P_2 e^{-P_2 t}$$



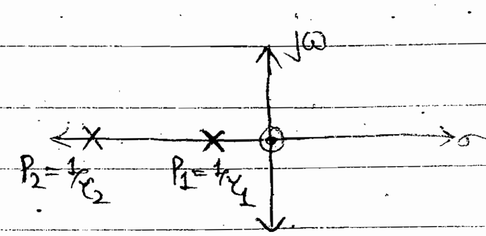


- Now, we cascade the two systems. So,



$$\therefore \quad \frac{C(s)}{R(s)} = \frac{(K/\tau_1)\cdot(1/\tau_2)}{(s+1/\tau_1)(s+1/\tau_2)} = \frac{(K\cdot P_1)\cdot P_2}{(s+P_1)(s+P_2)}$$

- Here, system has become second order system with P_1 as Dominant Pole.



- For Impulse input;

$$C(s) = \frac{(K \cdot P_1) \cdot P_2}{(s + 1/\tau_1)(s + 1/\tau_2)} = \frac{(K \cdot P_1 \cdot P_2)/(P_2 - P_1)}{(s+P_1)} - \frac{(K \cdot P_1 \cdot P_2)/(P_2 - P_1)}{(s+P_2)}$$

$$\therefore \boxed{c(t) = \frac{K \cdot P_1 \cdot P_2}{P_2 - P_1} \cdot \left[e^{-P_1 t} - e^{-P_2 t} \right]}$$ * (Exact Analysis). — (a)

Dominant Pole output

- We can convert Second order system into First order by Approximation Method.
- If $\boxed{\frac{P_2}{P_1} > 5}$ * P_1 being Dominant pole, then 's' term of P_2 can be equated to zero.

$$\therefore \frac{C(s)}{R(s)} = \frac{K \cdot P_1 \cdot P_2}{(s+P_1)(s+P_2)}$$, so, put 's' term of $P_2 = 0$.

$$\therefore \frac{C_{approx}(s)}{R(s)} = \frac{K \cdot P_1 \cdot P_2}{(s+P_1)(P_2)}$$

$$\boxed{\frac{C_{approx}(s)}{R(s)} = \frac{K \cdot P_1}{(s+P_1)}}$$ *

* For Example :- $K = 100$ $P_1 = -2$, $P_2 = -5$.

* By Exact Analysis : $c(t) = \frac{100 \times 2 \times 5}{3} \left[e^{-2t} - e^{-5t} \right]$.

$$\boxed{c(t) = 42.85}$$ * — (1)

With $P_1 = -2$, $P_2 = -5$,

$$\frac{C_{approx}(s)}{R(s)} = 100 \times 2 e^{-2t} = 27.06.$$ — (2)

- As by approximation, error exceeds the error band of 20%, so Approximation analysis can't be used.

Similarly, if $K = 100$, $P_1 = -2$, $P_2 = -12$.

$$c(t) = \frac{100 \times 24}{10}\left[e^{-2t} - e^{-12t}\right] = 240\left[e^{-2t} - e^{-12t}\right].$$

$$\boxed{c(t) = 32.48} * \quad \text{—— (3)}$$

As, $\frac{P_2}{P_1} > 5$, we can use Approximation,

$$\therefore C_{app}(t) = 100 \times 2\, e^{-2t} = 200\, e^{-2t}$$

$$\therefore \boxed{C_{app}(t) = 27.06} * \quad \text{—— (4)}$$

- Here, error is below 20% error band, So we can use Approximate Analysis.

* From Exact Analysis eq-(a), divide it by $P_1 \cdot P_2$,

$$\therefore c(t) = \frac{K}{\frac{1}{P_1} - \frac{1}{P_2}}\left[e^{-P_1 t} - e^{-P_2 t}\right].$$

If P_2 will be at Infinity, then

$$\boxed{c(t) = P_1 \cdot K\left[e^{-P_1 t}\right]} * \text{. (equation of First order system).}$$

** As pole in any second order system moves towards Infinity, the second order systems will transform into First order systems.

→* Consider a Fourth order System :-

$$\frac{C(s)}{R(s)} = \frac{288}{(s+2)(s+5)(s+12)(s+24)}.$$

As $R(s) = 1$ [Impulse].

$$c(t) = a_0 e^{-2t} + a_1 \cdot e^{-5t} + a_2 \cdot e^{-12t} + a_3 e^{-24t}$$ [Exact Analysis].

$$c(t=1/6) = a_0 e^{-1/3} + a_1 e^{-5/6} + \underbrace{a_2 e^{-2} + a_3 e^{-4}}_{\to = 0 \text{ (settles)}}$$

$$c(t=1/3) = a_0 e^{-2/3} + a_1 e^{-5/3} + \underbrace{a_2 e^{-4} + a_3 e^{-8}}_{\to = 0}$$

$$c(t=4/5) = a_0 e^{-8/5} + \underbrace{a_1 e^{-4} + a_2 e^{-48/5} + a_3 e^{-\frac{96}{5}}}_{\to = 0.}$$

$$c(t=2) = \underbrace{a_0 e^{-4} + a_1 e^{-10} + a_2 e^{-24} + a_3 e^{-48}}_{\to = 0.}$$

Now, using Approximation,

$P_2 = 5$ can't be neglected.

But $P_3 = 12$ and $P_4 = 24$ can be neglected as: $\frac{P_4}{P_1} = \frac{P_3}{P_1} \ngtr 5.$

$$\therefore \frac{C(s)}{R(s)} = \frac{\cancel{288}}{(s+2)(s+5) \times \cancel{12 \times 24}} = \frac{1}{(s+2)(s+5)}$$

∴ For $R(s) = 1$ [Impulse input].

$$\therefore c(s) = \frac{1/3}{(s+2)} - \frac{1/3}{(s+5)}$$

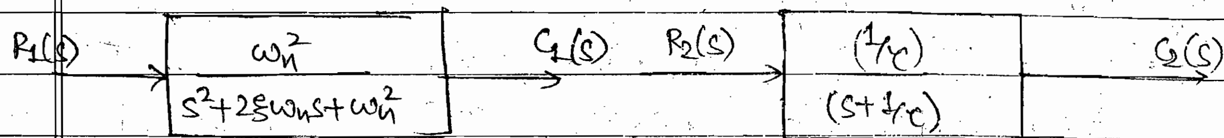
$$\Rightarrow \boxed{c(t) = \frac{1}{3}\left[e^{-2t} - e^{-5t}\right]}$$ * (Approximate Analysis).

Date 08-08-2014

→ Higher order systems (Continued...).

2) Cascading of second order with First order system :-

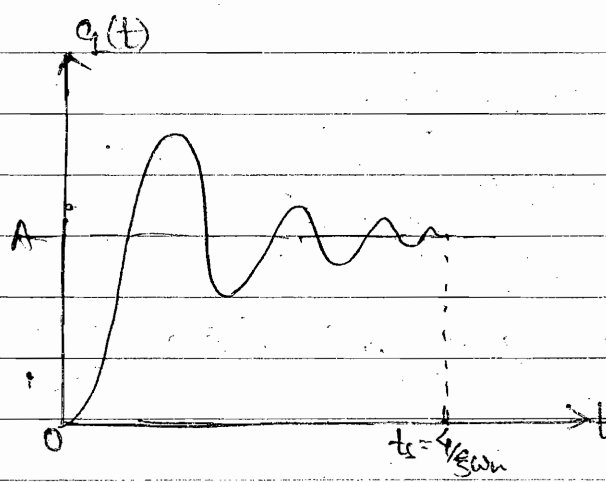
$r_1(t) = A\cdot u(t)$. $\qquad\qquad r_2(t) = A\,u(t)$

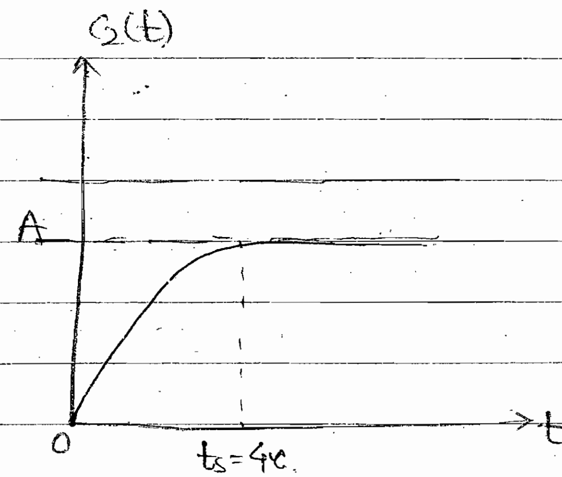


$R_1(s) = A/s \qquad\qquad R_2(s) = A/s$.

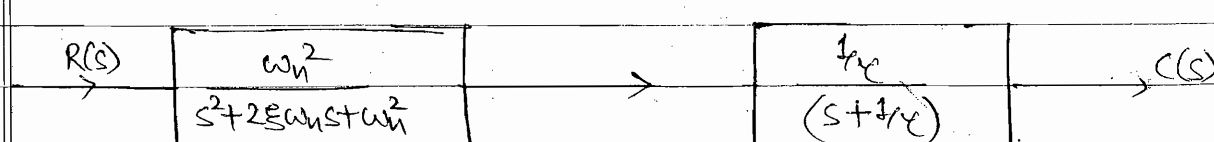
$$\frac{C_1(s)}{R_1(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad\qquad \frac{C_2(s)}{R_2(s)} = \frac{(1/\tau)}{(s+1/\tau)}$$

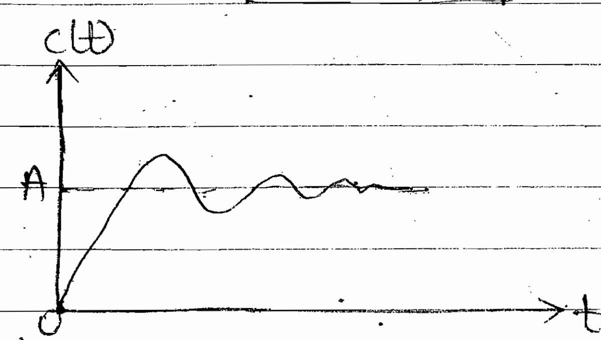
$$c_1(t) = A\left[1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)\right] \qquad\qquad c_2(t) = A\left[1-e^{-t/\tau}\right].$$





Now, if the two systems are cascaded, then,





* It becomes Third order system, oscillation reduces but not fully.
* t_s will remain same as that of IInd order system.

**** Case:- I: Second order Pole is Dominant Pole:**

- In this case, when II^{nd} order pole will come towards First order system, the % overshoot of III^{rd} order system will be less than pure II^{nd} order system.
- t_r of III^{rd} order system will be greater than t_r of pure II^{nd} order system, while t_s of III^{rd} and II^{nd} both systems will remain same.

* When first order pole approaches closer towards II^{nd} order pole, % overshoot will further increase, t_r will also increase, t_s will remain same.

Now, $\frac{1}{\tau} > \xi\omega_n$ or $1 > \tau(\xi\omega_n)$.

$$\therefore c(t) = A\left[1-(\ )e^{-\xi\omega_n t}\sin(\omega_d t+\phi)\right] + e^{-t/\tau}$$

$$\text{For } c(t=4\tau) = A\left[1-(\ )\underbrace{e^{-(<4)}}_{\text{does not settle}}\cdot\sin(\omega_d t+\phi)\right] + e^{-4}$$

$$c(t=4/\xi\omega_n) = A\left[1-(\ )\underbrace{e^{-4}}\sin(\omega_d t+\phi)\right] + \underbrace{e^{-4}}_{\text{Both settle}}$$

**** Case:- II: Second order as well as First order pole are dominant:**

- When both the poles are dominant poles, the output response of third order system will be analogous to Critically Damped Response of pure II^{nd} order system.

  * t_s will remain same.

**** Case:- III: First order Pole is Dominant Pole:**

- In this case, when second order pole is cascaded with first order pole and I^{st} order pole is dominant pole, then output of III^{rd} order system will be analogous to Overdamped system of pure II^{nd} order system and its t_s will be 4τ.

Qu

A

Q

which is greater than t_s of pure II^{nd} order system.

→* Approximate Analysis For Case-I :-

As $P_2 = 1/\tau$, $P_1 = \xi\omega_n$.

$$\therefore \frac{1/\tau}{\xi\omega_n} > 5.$$

$$\therefore T_{approx}F = \frac{\omega_n^2 \times 1/\tau}{(s^2+2\xi\omega_n s+\omega_n^2)(s+1/\tau)}$$, put $s=0$ in $(s+1/\tau)$.

$$\therefore T.F_{approx} = \frac{\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2)}$$ [second order system]

→* Approximate Analysis for case: III :-

$P_2 = \xi\omega_n$, $P_1 = 1/\tau$.

$$\therefore \frac{P_2}{P_1} > 5 \Rightarrow \frac{\xi\omega_n}{1/\tau} > 5$$

$$\therefore TF_{approx} = \frac{\omega_n^2 \times 1/\tau}{(s^2+2\xi\omega_n s+\omega_n^2)(s+1/\tau)}$$, put $s=0$ in II^{nd} order.

$$T.F_{approx} = \frac{1/\tau}{(s+1/\tau)}.$$ [First order system]

Ques:-(1) A third order system is approximated into second under-damped order system. What will be its effect on t_r of II^{nd} order system?
(a) t_r will increase (b) t_r will decrease (c) remains same.

Ans:-(1) t_r will decrease, % overshoot will increase, t_s will remain same.
Ans (b).

Ques:-(2) $T(s) = \frac{100}{(s^2+2\xi\omega_n s+1)(s+10)}$, approximation into II^{nd} order system will be :-

(a) $\frac{10}{(s^2+2s+1)}$ (b) $\frac{100}{(s+10)}$ (c) $\frac{100}{(s^2+2s+1)}$ (d) None.

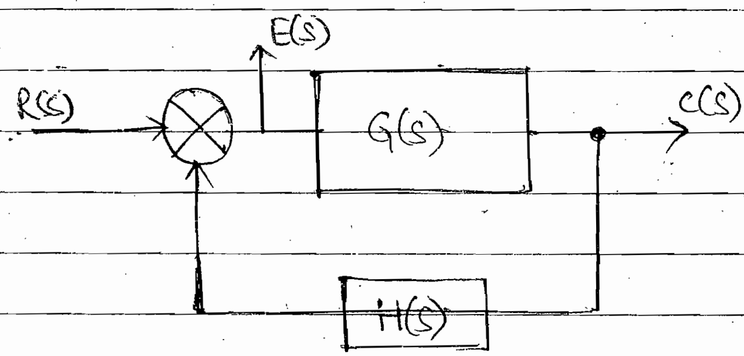
Ans:- (2) $\xi\omega_n = 1$, $\tau = 1/10$

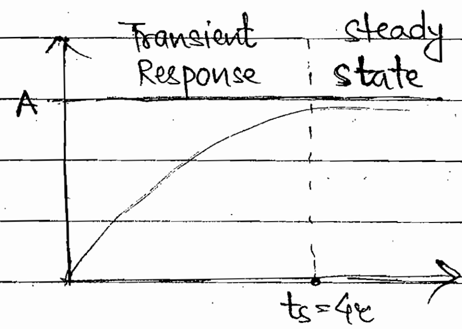
$\therefore \frac{10}{1} > 5 \Rightarrow$ Approximation can be used.

$$\therefore T = \frac{100}{(s^2+2s+1)(10)}$$

$$\Rightarrow \boxed{T_{app}(s) = \frac{10}{s^2+2s+1}} \quad * \quad \text{Ans (a).}$$

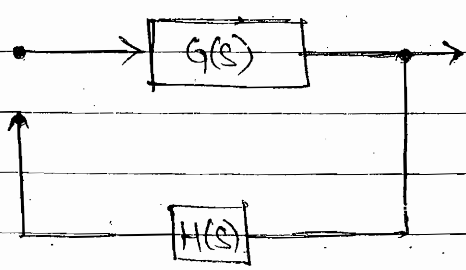
* → STEADY STATE ERROR :-





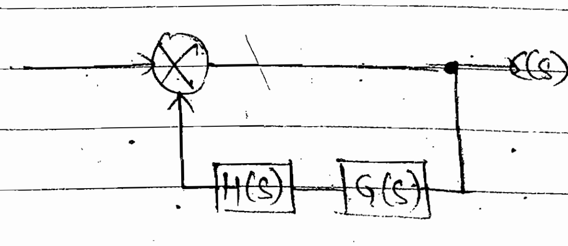
- If we withdraw summation from closed loop system, then closed loop system will convert into open loop system whose Transfer function is : G(s)·H(s). and steady state error of closed loop system solely depends on its open loop transfer function.

ie



OLTF = G(s) H(s).

Now, for block given below:



$$\frac{E(s)}{R(s)} = \frac{1}{1+\{G(s)\cdot H(s)\}}$$

$$\boxed{E(s) = \frac{R(s)}{1+G(s)H(s)}} \; *$$

$$\boxed{\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}} \; *$$

Case-I: For Step Input :-

$$R(s) = A/s.$$

$$e_{ss} = \lim_{s\to 0} \frac{A\times s\times 1/s}{1+G(s)H(s)} = \frac{A}{1+\lim_{s\to 0}\{G(s)H(s)\}}$$

or $\boxed{e_{ss} = \frac{A}{1+K_p}}$ * K_p = Positional Error Constant or Co-efficient.

$$\boxed{K_p = \lim_{s\to 0} G(s)H(s)} \; *$$

Case-II: For Ramp Input :-

$$R(s) = A/s^2$$

$$\therefore e_{ss} = \lim_{s\to 0} \frac{A/s}{1+G(s)H(s)} = \frac{A}{\lim_{s\to 0}(s+sG(s)H(s))}$$

$\therefore \boxed{e_{ss} = \frac{A}{K_v}}$ * ~~K_v = Positio~~

K_v = Velocity Error Constant or Coefficient

$$\boxed{K_v = \lim_{s\to 0} s\cdot G(s)H(s)} \; *.$$

** Case:-III: For Parabolic Input :-

$$R(s) = A/s^3 \qquad \left(r(t) = At^2/2\right).$$

$$\therefore e_{ss} = \lim_{s\to 0} \frac{A/s^2}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{A}{s^2 + s^2G(s)H(s)} = \frac{A}{0 + \lim_{s\to 0} s^2G(s)H(s)}$$

$$\therefore \boxed{e_{ss} = \frac{A}{K_a}} \ *$$

K_a = Acceleration Error Constant or Co-efficient.

$$\boxed{K_a = \lim_{s\to 0} s^2 \cdot G(s)H(s)} \ *$$

*→ Type and order of the System :-

** Type :-

- It is defined by total number of poles locating at origin and Order is defined by total number of poles in a system.
- Type and order of a system is defined by its open loop transfer function.

** Type 0 System :-

Example: $G(s)H(s) = \dfrac{K(sT_1+1)(sT_2+1)}{(sT_3+1)(sT_4+1)}$,

- Here K = DC Gain of the system here.

$S_z = -1/T_1, -1/T_2$

$S_p = -1/T_3, -1/T_4$;

1> For Step Input :-

$$\therefore K_p = \lim_{s\to 0} G(s)H(s) = K \text{ (DC gain)}.$$

$$\therefore \boxed{K_p = K = \text{DC Gain}} * \qquad \boxed{e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+K} = \text{Finite}} *$$

2> For Ramp Input :-

$$K_v = \lim_{s\to 0} s\cdot G(s)H(s) = \lim_{s\to 0} \frac{\cancel{s} \times (sT_1+1)(sT_2+1)}{(sT_3+1)(sT_4+1)}$$

$$\therefore \boxed{K_v = 0} * \qquad \boxed{e_{ss} = \infty} *.$$

3> For Parabolic Input :-

$$K_a = \lim_{s\to 0} s^2\cdot G(s)H(s) \Rightarrow \boxed{K_a = 0} * \quad \boxed{e_{ss} = \infty} *.$$

** Type 1 system :-

* Example: $G(s)H(s) = \dfrac{K(sT_1+1)(sT_2+1)}{s(sT_3+1)(sT_4+1)}$ [Order = 3, Type = 1]

1> For Step Input :-

$$K_p = \lim_{s\to 0} G(s)H(s) = \frac{1}{0} \Rightarrow \boxed{K_p = \infty} * \quad \boxed{e_{ss} \simeq 0} *$$

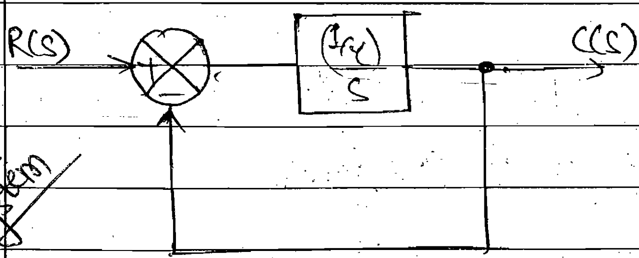
2> For Ramp Input :-

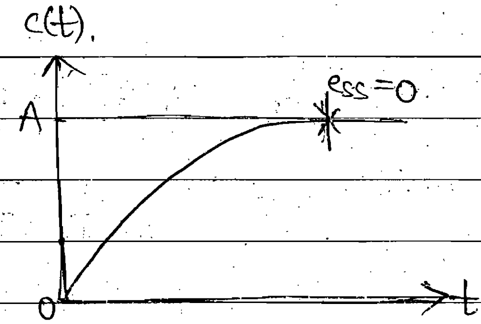
$$K_v = \lim_{s\to 0} s\cdot G(s)H(s) = K \Rightarrow \boxed{K_v = K = \text{DC Gain}} * \quad \boxed{e_{ss} = \frac{A}{K}} *$$

3> For Parabolic Input :-

$$K_a = \lim_{s\to 0} s^2\cdot G(s)\cdot H(s) = 0 \Rightarrow \boxed{K_a = 0} * \quad \boxed{e_{ss} = \infty} *$$

First order system

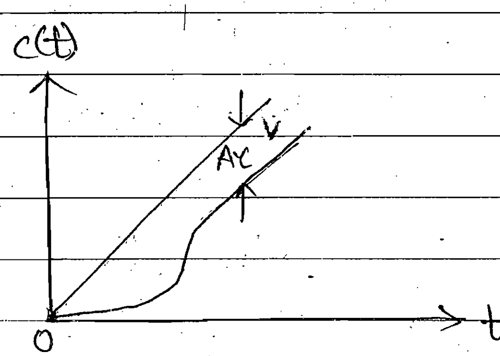




$$G(s)H(s) = \frac{(1/\tau)}{s}$$

$$\therefore K_p = \lim_{s\to 0} \frac{1/\tau}{s} = \infty$$

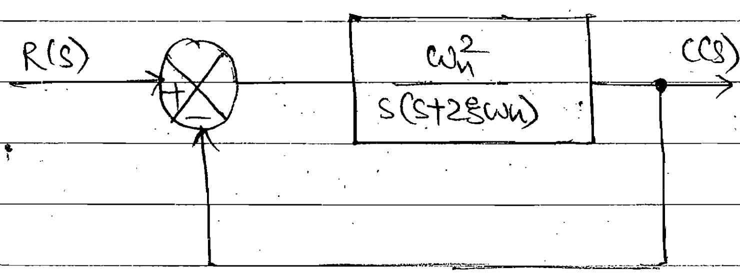
$$\therefore \boxed{e_{ss} = 0} \ *$$



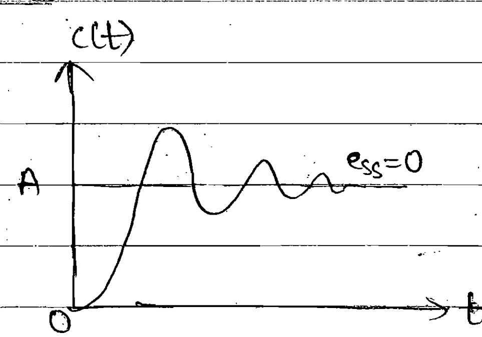
$$\therefore K_v = \lim_{s\to 0} s \cdot \frac{1/\tau}{(s)} = \frac{1}{\tau}$$

$$\therefore \boxed{e_{ss} = A\tau} \ *$$

Second order system

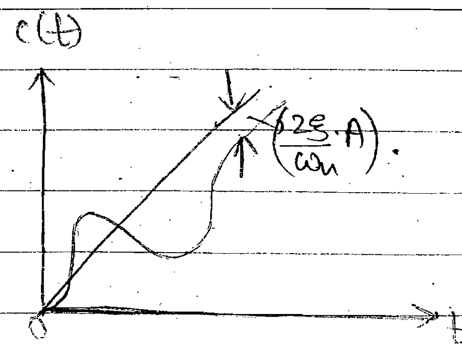


$$G(s)H(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s}$$



$$\therefore K_p = \lim_{s\to 0} \frac{\omega_n^2}{s(s+2\xi\omega_n)} = \infty$$

$$\boxed{e_{ss} = 0} \ *$$



$$\therefore K_v = \lim_{s\to 0} s \cdot \frac{\omega_n^2}{s^2+2\xi\omega_n s} = \frac{\omega_n}{2\xi}$$

$$\boxed{e_{ss} = \frac{2\xi}{\omega_n}\cdot A} \ *$$

** Type 2-system :-

Example:- $G(s)H(s) = \frac{K(sT_1+1)(sT_2+1)}{s^2(sT_3+1)(sT_4+1)}$ [Type→2, Order→4]

1) For Step Input :-

$K_p = \lim_{s\to 0} G(s)H(s) \Rightarrow \boxed{K_p = \infty}$ * $\boxed{e_{ss} = 0}$ *

2) For Ramp Input :-

$K_v = \lim_{s\to 0} s\, G(s)H(s) \Rightarrow \boxed{K_v = \infty}$ * $\boxed{e_{ss} \simeq 0}$ *

3) For Parabolic Input :-

$K_a = \lim_{s\to 0} s^2 G(s)H(s) \Rightarrow \boxed{K_a = K = \text{DC Gain}}$ * $\boxed{e_{ss} = \frac{A}{K_a} = \frac{A}{K}}$ *

** • Increasing type of a system will reduce steady state error but it also reduces stability of the system.

* → DYNAMIC ERROR CO-EFFICIENT :-

As $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$

Using Long Division,

$1+G(s)H(s) \overline{)\;1\;} \;\; (C_0 + C_1 s + C_2 s^2$

$\therefore \boxed{\frac{E(s)}{R(s)} = C_0 + C_1(s) + C_2(s)}$ * where C_0, C_1 and C_2 are dynamic error coefficients.

$\therefore \frac{E(s)}{R(s)} = C_0 + C_1(s) + C_2(s) = \frac{1}{1+K_p} + \frac{s}{K_v} + \frac{s^2}{K_a}$

because, $c_0 = \frac{1}{1+K_p}$; $G_1(s) = \frac{s}{K_v}$, $G_2(s) = \frac{s^2}{K_a}$.

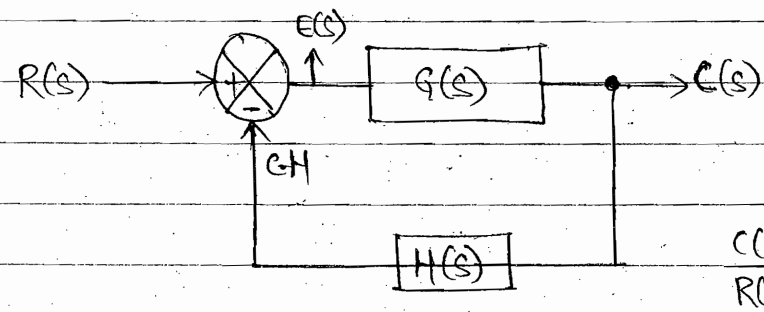
$$\therefore \quad E(s) = \frac{R(s)}{1+K_p} + \frac{s \cdot R(s)}{K_v} + \frac{s^2 \cdot R(s)}{K_a}$$

$$\Rightarrow \quad e(t) = \left[\frac{r(t)}{1+K_p} + \frac{\{ d/dt\, r(t) \}}{K_v} + \frac{\{ d^2/dt^2 (r(t)) \}}{K_a} \right]$$

Taking time $\to \infty$ under limits,

$$\therefore \quad \boxed{e_{ss} = \lim_{t \to \infty} \left[\frac{r(t)}{1+K_p} + \frac{d/dt(r(t))}{K_v} + \frac{d^2/dt^2(r(t))}{K_a} \right]} \; *$$

* → **UNITY FEEDBACK SYSTEM :-**



$$OLTF = G(s) \cdot H(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

•, Now, we will convert this non-unity feedback system to unity feedback system having same Closed Loop Transfer function.

$$* \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{G'(s)}{1+G'(s)}$$

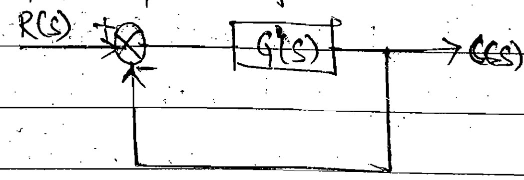
$$\Rightarrow \quad G(s)\left[1+G'(s)\right] = G'(s)\left[1+G(s)H(s)\right].$$

$$\Rightarrow \quad \boxed{G'(s) = \frac{G(s)}{1+G(s)H(s)-G(s)}} \; *$$

$$\therefore \quad \boxed{\text{OLTF of u.f.b system} = G'(s) = \frac{G(s)}{1+G(s)H(s)-G(s)}} \; *$$

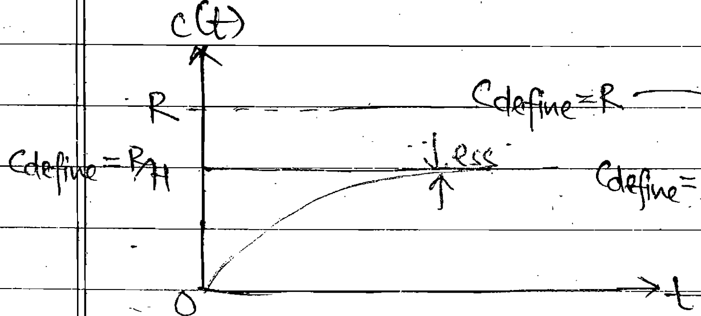
• Point to be noted here is that:

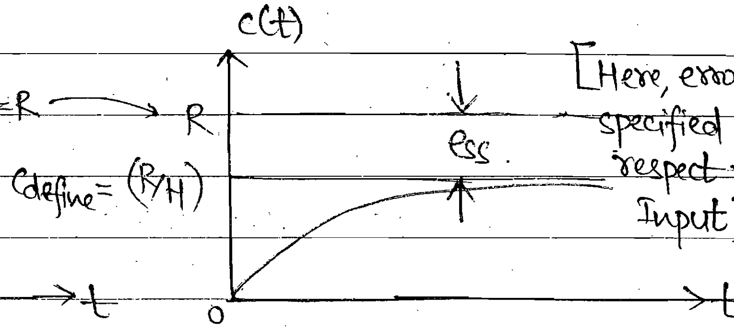
Both non-unity feedback system and u.f.b ~~s~~ will have same closed Loop transfer function but open Loop Transfer Functions of both will be different.



$$C(S) = \frac{R(S)\cdot G(S)}{1+G(S)H(S)}$$

$$C(S) = \frac{G'(S)\cdot R(S)}{1+G'(S)} = \frac{G(S)\cdot R(S)}{1+G(S)H(S)}.$$





[Here, error is specified with respect to Input]

$E = R - CH$

for $E = 0$,

$$\boxed{C_{define} = \frac{R}{H}} \ *$$

$E = R - C$

for $E = 0$,

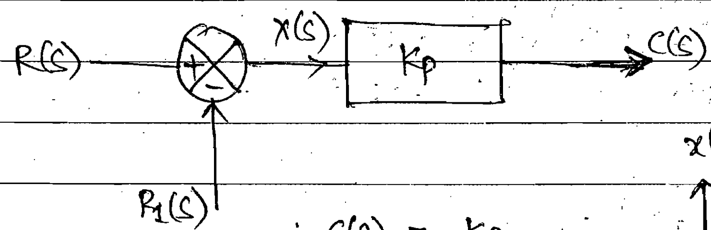
$$\boxed{R = C_{define}} \ *$$

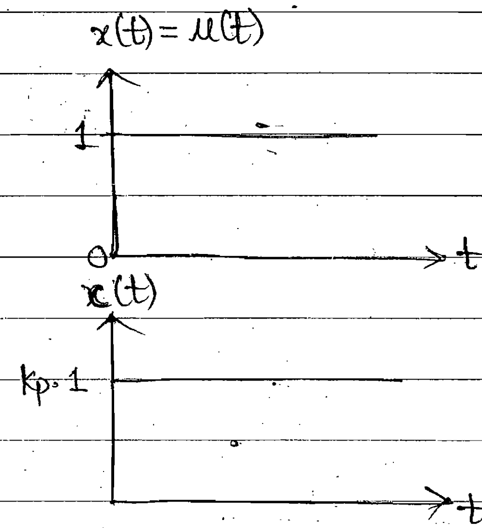
Chapter: 3

CONTROLLERS

1) Proportional Controller :-

- It represents Gain of the system. It is zero-order system because it does not contain either pole or zero.

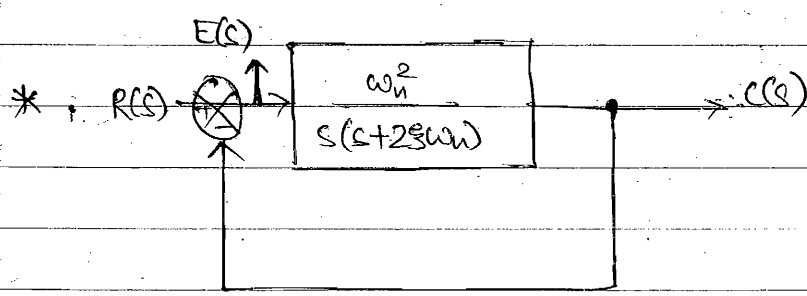




$$\therefore \frac{C(s)}{X(s)} = K_p$$

$$C(s) = K_p X(s)$$

$$\boxed{c(t) = K_p \cdot x(t)}\ *$$



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\Rightarrow \quad s^2+2\xi\omega_n s+\omega_n^2 = 0.$$

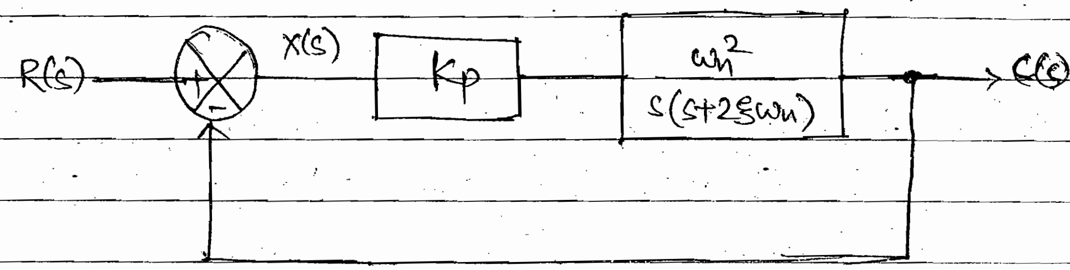
$$\therefore \quad s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

Now, Open Loop T.F $= \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

$\therefore\ K_v = \lim_{s\to 0} \dfrac{\cancel{s}\cdot\omega_n^2}{\cancel{s}(s+2\xi\omega_n)} \Rightarrow K_v = \dfrac{\omega_n}{2\xi}$

$\therefore\ \boxed{e_{ss} = \dfrac{2\xi\cdot A}{\omega_n}}$ *

- Now, when Proportional controller is applied in system:



$G(s)\cdot H(s) = \dfrac{K_p\cdot\omega_n^2}{s(s+2\xi\omega_n)}$

Now, $\dfrac{C(s)}{R(s)} = \dfrac{K_p\cdot\omega_n^2}{s^2+2\xi\omega_n s+K_p\cdot\omega_n^2}$

$\therefore\ s^2+2\xi\omega_n s+K_p\cdot\omega_n^2 = 0$

$\therefore\ s^2+2\xi'\omega_n' s+(\omega_n')^2 = 0.$

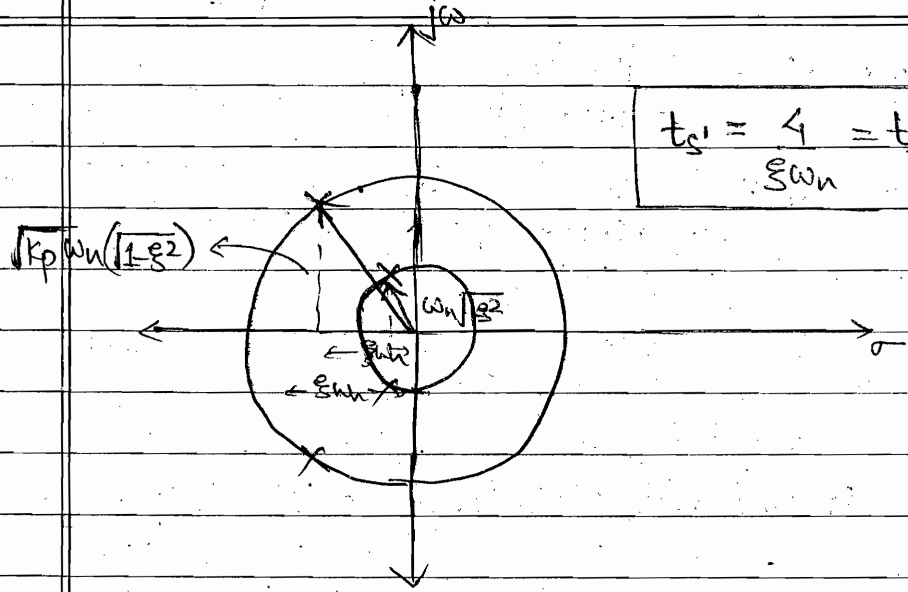
$\therefore\ \boxed{\omega_n' = \sqrt{K_p}\cdot\omega_n}$ *, $\boxed{\xi' = \dfrac{\xi}{\sqrt{K_p}}}$ *

$S_{1,2} = -\xi'\omega_n' \pm j\,\omega_n'\sqrt{1-\xi'^2}$

$\therefore\ S_{1,2} = \dfrac{-\xi}{\sqrt{K_p}}\cdot\sqrt{K_p}\cdot\omega_n \pm j\cdot\sqrt{K_p}\cdot\omega_n\sqrt{1-\dfrac{\xi^2}{K_p}}$

$\boxed{S_{1,2} = -\xi\omega_n \pm j\sqrt{K_p}\cdot\omega_n\sqrt{\dfrac{1-\xi^2}{K_p}}}$ * Location of poles for Controller used control system.

So, Real part will be same but imaginary part will change.

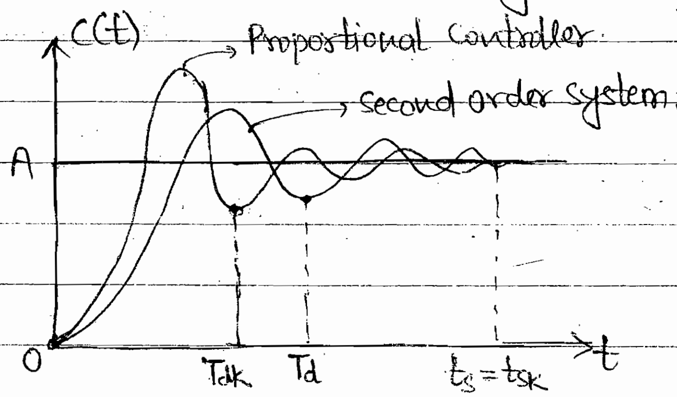


$$\boxed{t_s' = \frac{4}{\xi\omega_n} = t_s} \quad *$$

** • With application of Proportional Controller :-

* ξ will decrease by factor of $\sqrt{K_p}$, as a result, % M_p will increase, t_r will decrease while settling time will remain same.

* Stability will remain unaffected because stability is defined in terms of, " Distance of poles from Imaginary axis" and that distance is same for both the system.



$$\omega_d = \omega_n\sqrt{1-\xi^2} \quad ; \quad \omega_{dk} = \sqrt{K_p}\cdot\omega_n\sqrt{1-\frac{\xi^2}{K_p}}$$

$$\therefore \frac{2\pi}{\omega_{dk}} = T_{dk} \text{ or } \frac{2\pi}{T_{dk}} = \omega_{dk} > \omega_d = \frac{2\pi}{T_d}$$

$$\therefore \boxed{T_{dk} < T_d} \quad *$$

** • With application of Proportional Controller :-

* Damped Angular frequency will increase. As a result, Time period of Damped Oscillation will decrease. Hence, number of oscillation cycles upto t_s will also increase.

Now, e_{ss} will be :-

$$K_v = \lim_{s \to 0} s \times \frac{K_p \cdot \omega_n^2}{s(s+2\xi\omega_n)} \Rightarrow \boxed{K_v = \frac{K_p \cdot \omega_n}{2\xi}} \ *$$

$$\boxed{e_{ss} = \frac{2\xi \cdot A}{K_p \cdot \omega_n}} \ *$$

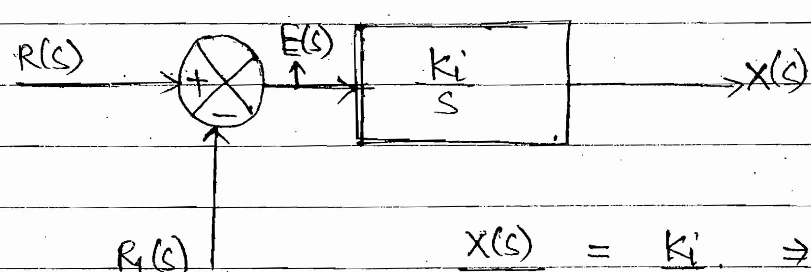
**• With application of Proportional controller :
  * Steady State Error will decrease by a factor of Kp.

• Proportional controller does not affect Time Constant and it affects only frequency parameter which is Undamped Natural Frequency (ω_n).

2) Integral Controller :-
  • Integral controller is used to suppress Offset Error.

* Offset Error :- It is defined as, " Error in output when input is 0".

Thus, whenever we want to reduce e_{ss}, we use Integral controller but the drawback of it is that, to reduce e_{ss}, number of oscillation cycles gets increased. As a result, t_s will increase and stability will decrease.



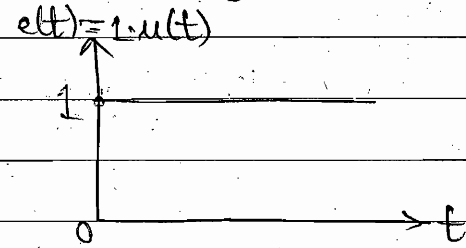
$$\frac{X(s)}{E(s)} = \frac{K_i}{s} \Rightarrow X(s) = K_i \left[\frac{E(s)}{s} \right]$$

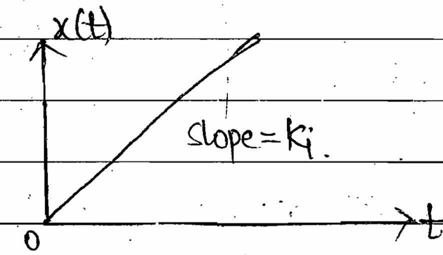
$$\Rightarrow x(t) = K_i \int e(t) \cdot dt$$

$$\therefore \boxed{x(t) = K_i \int e(t)\, dt} \ *$$

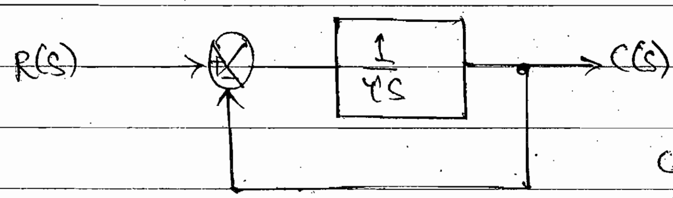
$\therefore$ For $e(t) = 1 \cdot u(t)$. $\therefore$ $x(t) = K_i \int 1 \cdot dt$.

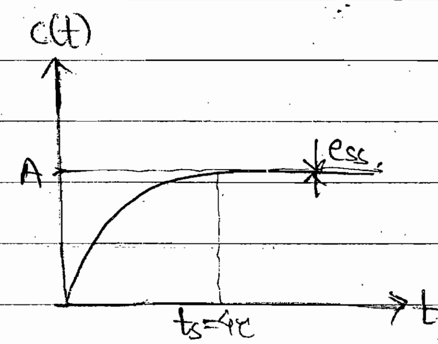
$\therefore$ $\boxed{x(t) = K_i t}$ *





* Now, consider a first order system :-





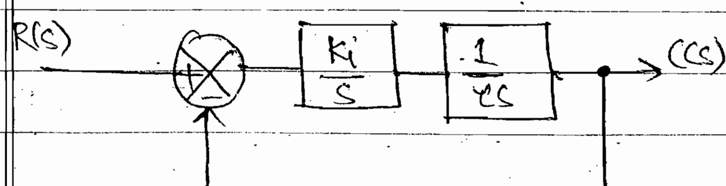
$$\frac{C(s)}{R(s)} = \frac{(1/\tau)}{(s+1/\tau)}.$$

Apply $R(s) = A/s$

$$\therefore C(s) = \frac{(A/\tau)}{s(s+1/\tau)} \Rightarrow c(t) = A\left[1-e^{-t/\tau}\right]$$

$$\boxed{e(t) = A e^{-t/\tau}} \ *$$

Now, applying Integral controller to this I^{st} order system:



$$\frac{C(s)}{R(s)} = \frac{(K_i/\tau)}{s^2+(K_i/\tau)}.$$, Here poles come on Imaginary axis, $\therefore$ system becomes Marginally stable when Integral controller is

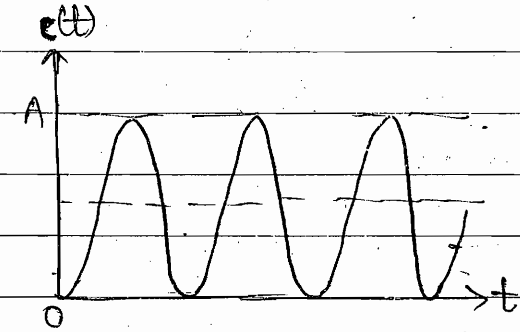
$\therefore$ For $r(t) = A \cdot u(t)$, $R(s) = A/s$,

$$\therefore \quad C(s) = \frac{(A \cdot K_i/\tau)}{s(s^2 + K_i/\tau)}. \qquad \therefore \boxed{c(t) = A\left[1 - \cos\sqrt{\frac{K_i}{\tau}}\,t\right]} \; *$$

$$\boxed{e(t) = A \cdot \cos\left(\sqrt{\frac{K_i}{\tau}}\right) \cdot t} \; *$$

$\therefore \boxed{t_s = \infty}$ *

Stability Yes.

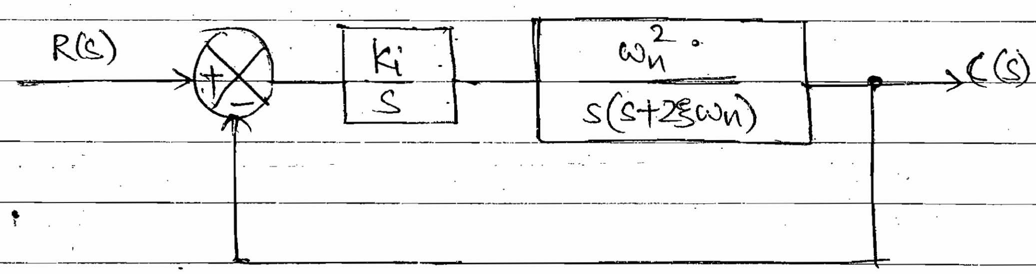


Also, $e_{av}(t) = \frac{1}{T_N}\int_0^{T_N} e(t) \cdot dt$

$\boxed{e_{av}(t) = 0}$ *

$\therefore$ Error will be reduced.

- Now, applying Integral controller to second order system :-



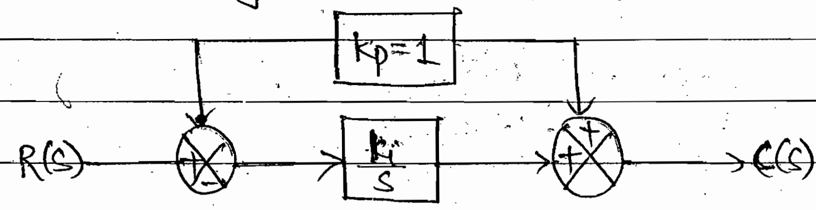
$$G(s) = \frac{K_i \cdot \omega_n^2}{s^2(s + 2\xi\omega_n)}.$$

$\therefore q(s) = s^3 + 2\xi\omega_n s^2 + K_i \cdot \omega_n^2 = 0 \Rightarrow$ Making system Unstable.

** Here, we will not apply Integral controller directly into second order system because Integral controller will convert a stable IInd order system into unstable IIIrd order system.

* As seen from $q(s)$, s^1 is missing which represents that atleast one pole will locate in the right-half side and system will be unstable. To overcome this, PI controller is used.

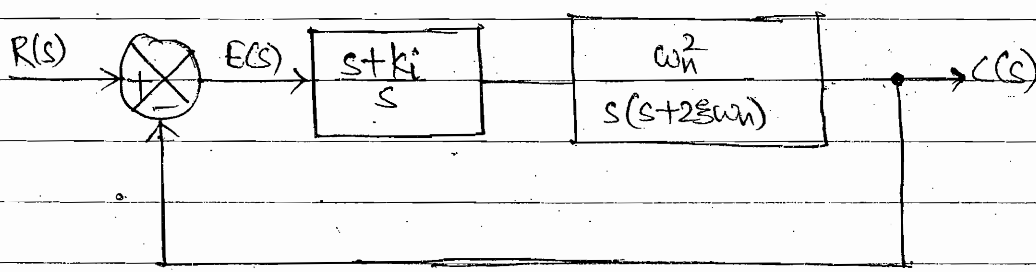
3> Proportional Integral (PI) Controller :-



$$\therefore \frac{C(s)}{R(s)} = k_p + \frac{k_i}{s} = \frac{sk_p + k_i}{s}$$

For $k_p = 1$, $\frac{C(S)}{R(S)} = \left(\frac{k_i + s}{s}\right)$.

- PI will add pole at origin as well as one zero at location other than zero.



$$\frac{C(s)}{R(s)} = \frac{(s+k_i)\cdot\omega_n^2}{s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + k_i\omega_n^2}$$

$$\therefore q(s) = s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + k_i\omega_n^2 = 0.$$

* For stable system, condition is :- $\boxed{2\xi\omega_n > k_i}$ *.

- Using Routh-Hurwitz Criteria :-

s^3	1	ω_n^2
s^2	$2\xi\omega_n$	$k_i\omega_n^2$
s^1	$\frac{(2\xi\omega_n - k_i)\cdot\omega_n^2}{2\xi\omega_n}$	
s^0	$k_i\cdot\omega_n^2$	

• For always stable system, $2\xi\omega_n - k_i > 0$

$$\therefore \boxed{2\xi\omega_n > k_i} \; *$$

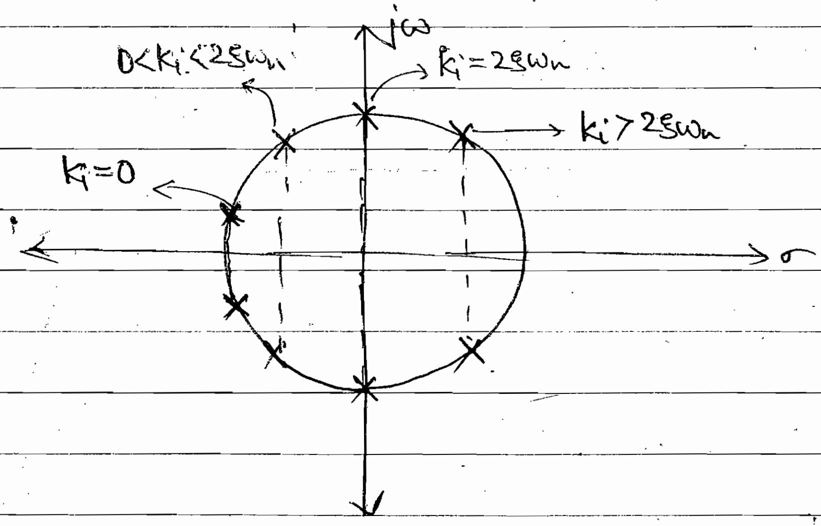
Put $k_i = 0$ now,

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$ [Second order system].

Now, if $0 < k_i < 2\xi\omega_n$, Poles will shift towards Right side.

For $k_i = 2\xi\omega_n$, Poles will lie on Imaginary axis now.

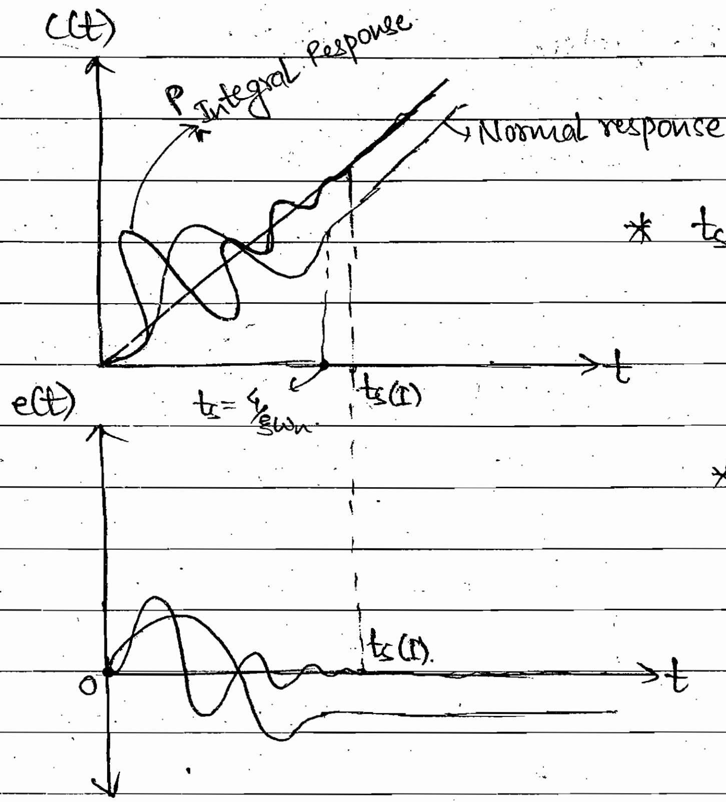
For $k_i > 2\xi\omega_n$, Poles will shift finally on Right-half.
↳ As $(2\xi\omega_n - k_i) < 0$, so both poles will go on Right-half of s-plane making system unstable.



Now, e_{ss} is : $K_a = \lim_{s \to 0} s^2 \cdot \frac{(s+k_i)\cdot\omega_n^2}{s^2(s+2\xi\omega_n)} \Rightarrow \boxed{K_a = \frac{k_i \cdot \omega_n}{2\xi}} \; *$

$$\therefore \boxed{e_{ss} = \frac{2\xi}{k_i\omega_n} \cdot A} \; * \text{ [For Parabolic Input]}$$

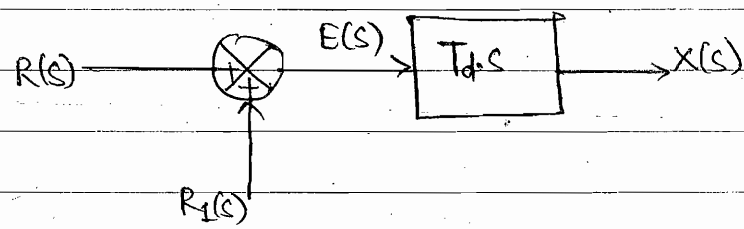
$$\boxed{e_{ss} = 0} \; * \text{ [For Ramp Input]}.$$



* t_{sI} increases, No. of cycles increases, stability decreases.

* Stability of system reduces as poles will shift towards Imaginary axis, as a result, $\% M_p$ will increase, t_s will also increase. But, it decreases e_{ss}.

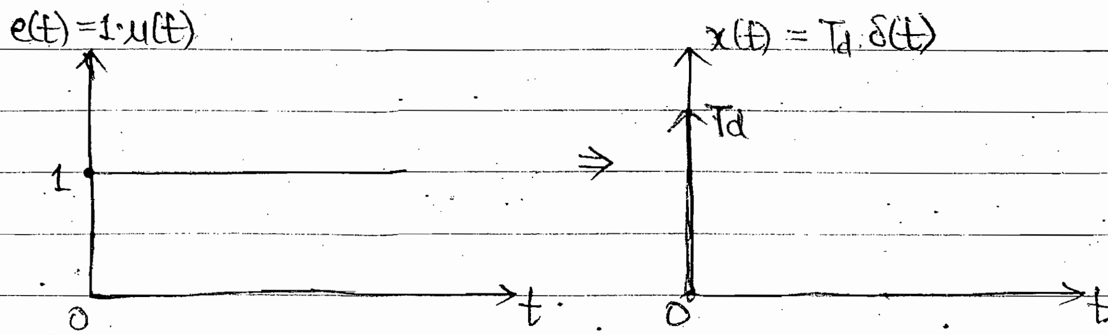
Derivative Controller :-



$$\frac{X(s)}{E(s)} = T_d \cdot s \qquad \text{where, } T_d = \text{Derivative Time Constant.}$$

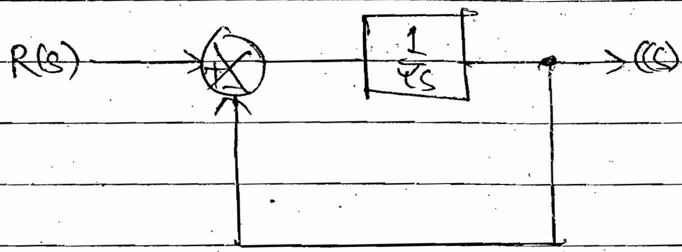
$$\therefore \; X(s) = T_d \cdot s \cdot E(s).$$

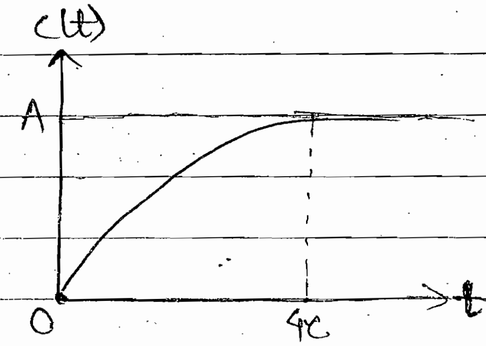
or $\boxed{x(t) = T_d \cdot \frac{d}{dt} e(t)}$ *



- Derivative controller will decrease t_r of system but it will increase e_{ss} as it decreases type of system by introducing zero at origin.
- Stability will also increase in this case.

* Consider a Ist order system :-





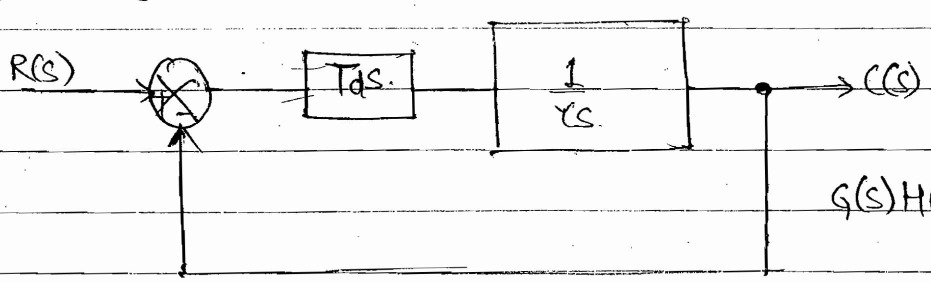
$$\frac{C(s)}{R(s)} = \frac{(1/\tau)}{(s+1/\tau)}.$$

For $r(t) = A.u(t)$, $R(s) = A/s$.

$$\therefore C(s) = \frac{(A/\tau)}{s(s+1/\tau)}$$

$$\boxed{c(t) = A\left[1 - e^{-t/\tau}\right]} \ *.$$

- If we apply Derivative Controller to Ist order system, then

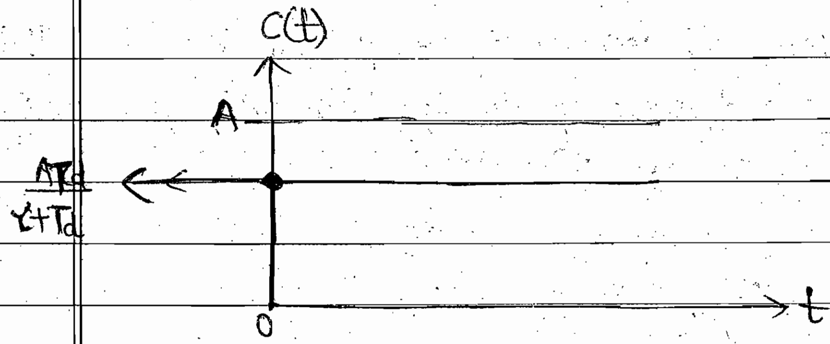


$$G(s)H(s) = \frac{T_d}{\tau}.$$

$$\therefore \frac{C(s)}{R(s)} = \frac{T_d}{\tau + T_d}$$

For $R(s) = A/s$, $\quad C(s) = \dfrac{A \cdot T_d}{s(\tau + T_d)}$.

$$\therefore \boxed{c(t) = \frac{A\,T_d}{\tau + T_d} \cdot u(t)} \ *$$

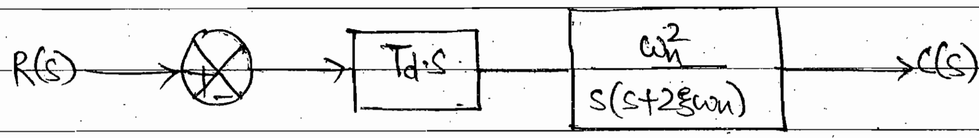


$\left(\dfrac{T_d}{\tau+T_d}\right) < 1$ always,

so, c(t) will remain less than desired input of A.

Here, $t_r = 0$ but e_{ss} has increased and become constant.

* Similarly, for II^nd order system :-

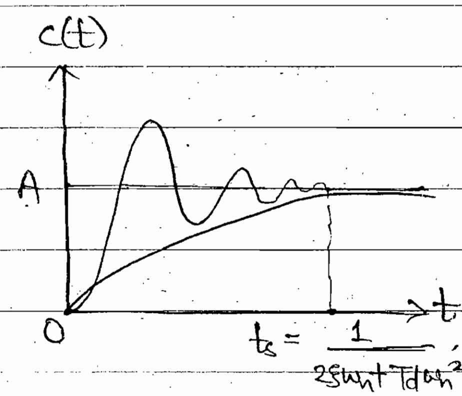


$$G(s)\cdot H(s) = \frac{T_d\cdot\omega_n^2}{(s+2\xi\omega_n)}$$

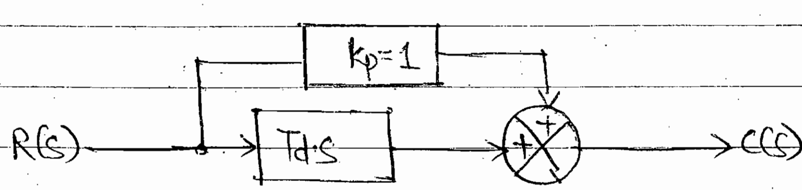
$$\therefore \frac{C(s)}{R(s)} = \frac{T_d\cdot\omega_n^2}{s+(2\xi\omega_n+T_d\cdot\omega_n^2)}$$

For $r(t) = A\cdot u(t)$, $R(s) = A/s$

$$\therefore C(s) = \frac{\frac{A}{s}\,T_d\cdot\omega_n^2}{s+(2\xi\omega_n+T_d\cdot\omega_n^2)}$$



5> Proportional Derivative Controller (PD) :-

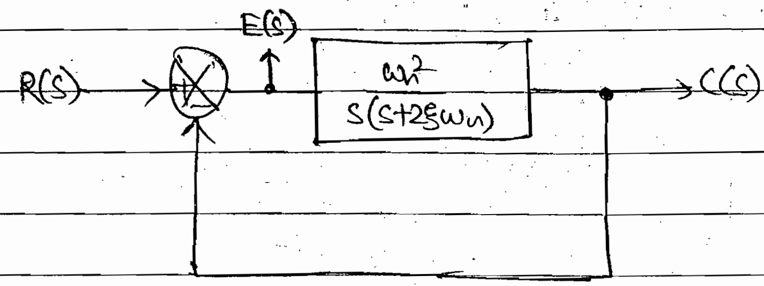


$$\frac{C(s)}{R(s)} = K_p + T_d\cdot s$$

$$\therefore \boxed{\frac{C(s)}{R(s)} = 1+T_d\cdot s} \ *$$

- This will locate zero other than origin.

Now, for second order system;



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

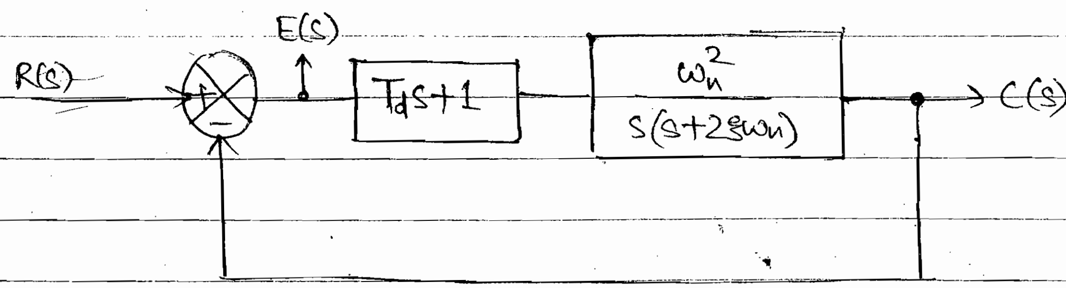
$$\therefore S_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \quad ; \; t_s = 4/\xi\omega_n$$

$$G(s)H(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}.$$

$$\therefore \quad K_v = \frac{\omega_n}{2\xi}, \quad \boxed{e_{ss} = \frac{2\xi}{\omega_n}.A} \; * \text{ [For Ramp Input].}$$

$$\boxed{e_{ss} = \infty} \; * \text{ [For Parabolic Input].}$$

* Now, if PD controller is applied, then :



$$G(s) = \frac{(T_d s+1)\omega_n^2}{s(s+2\xi\omega_n)}.$$

$$\text{or} \quad \frac{C(s)}{R(s)} = \frac{(T_d s+1)\cdot\omega_n^2}{s^2+2\xi\omega_n s+T_d s\cdot\omega_n^2+\omega_n^2}$$

$$\text{or} \quad \frac{C(s)}{R(s)} = \frac{(T_d s+1)\,\omega_n^2}{s^2+(2\xi\omega_n+T_d\omega_n^2)s+\omega_n^2}.$$

$\therefore \quad s^2 + (2\xi\omega_n + T_d \cdot \omega_n^2)s + \omega_n^2 = 0$

converting it into standard form;

$$s^2 + 2\xi'\omega_n' + (\omega_n')^2 = 0$$

$\therefore \quad \boxed{\omega_n' = \omega_n}$ *,

and $\quad T_d \cdot \omega_n^2 + 2\xi\omega_n = 2\xi'\omega_n'$

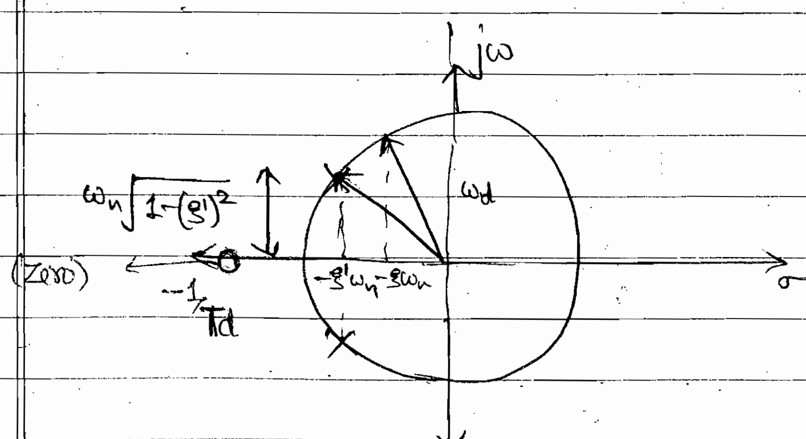
$\Rightarrow \quad T_d \cdot \omega_n^2 + 2\xi\omega_n = 2\xi'\omega_n$

$\Rightarrow \quad \boxed{\xi' = \xi + \frac{T_d \cdot \omega_n}{2}}$ *

$\therefore \quad \boxed{S_{1,2} = -\xi'\omega_n \pm j\omega_n\sqrt{1-(\xi')^2}}$ *

* As ~~ω_n~~ $\xi' > \xi \quad \Rightarrow \quad \xi'\omega_n' > \xi\omega_n$

$\therefore$ Real part of pole increases and Imaginary term decreases, so poles will shift left making system more stable than earlier.



As $\frac{C(s)}{R(s)} = \frac{(T_d s+1)\cdot \omega_n^2}{s^2+2\xi'\omega_n + \omega_n^2}$

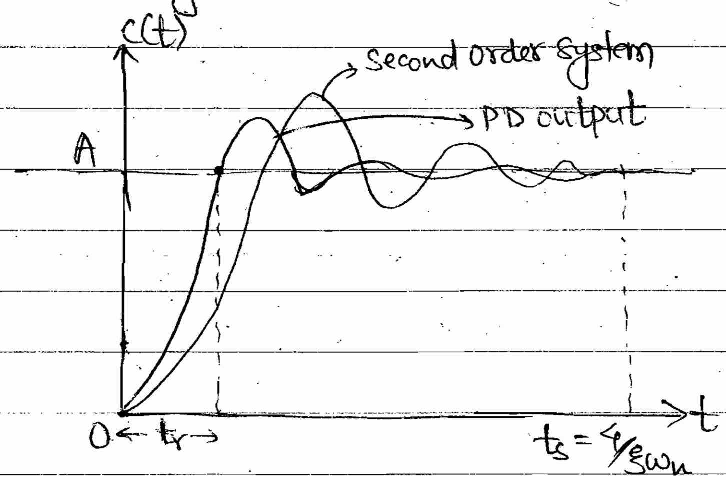
For $R(s) = A/s$.

$$\therefore C(s) = \left[\frac{\omega_n^2 \cdot (A/s)}{s^2+2\xi\omega_n' + \omega_n^2}\right] + \left\{ s\left[\frac{\omega_n^2\cdot(A/s)}{s^2+2\xi\omega_n'+\omega_n^2}\right]\right\} T_d.$$

$$\therefore c(t) = A\left[1 - \frac{e^{-\xi'\omega_n t}}{\sqrt{1-(\xi')^2}} \sin(\omega_d' t + \phi')\right] + T_d\left[\frac{d}{dt} A\left[1-\frac{e^{-\xi'\omega_n t}}{\sqrt{1-(\xi')^2}} \sin(\omega_d' t + \phi')\right]\right]$$

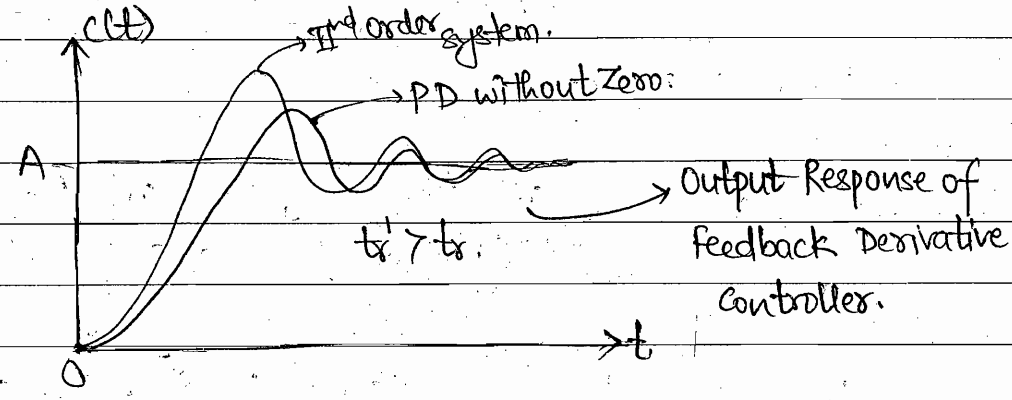
where, $\phi' = \tan^{-1}\left(\frac{\sqrt{1-(\xi')^2}}{\xi'}\right)$.

$\therefore$ PD will introduce a new derivative term in the output, so which will help to reduce transients or compress the transient region.



- The effect of PD controller is that:
  - * It will introduce one zero in transfer function and due to that zero, a derivative term will exist in the output response and that derivative term will compress the transient region. So, t_r will decrease.
  - * ξ will increase, hence $\% M_p$ will decrease.
  - * Poles shift away from the Imaginary axis, hence stability of system increases.

- If Zero were not included, then t_r would have increased.



- Formulae for t_r, M_p, t_s, t_p of PD controller will be different from t_r, M_p, t_s, t_p of Second order system.

* With application of PD Controller :-

- t_r will decrease, $\% M_p$ decreases and stability increases and t_s will decrease.

Now, e_{ss} will be :-

$$G(S)H(S) = \frac{(T_d s + K_p)\cdot \omega_n^2}{s(s+2\xi\omega_n)}$$

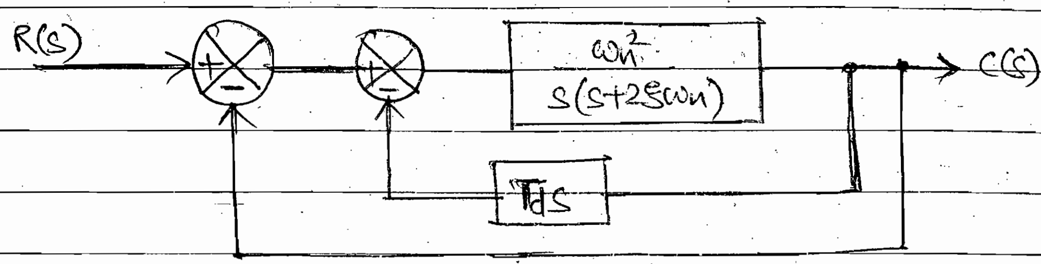
$$\therefore \quad K_v = \lim_{s\to 0} \cancel{s} \times \frac{(T_d s + K_p)\,\omega_n^2}{\cancel{s}(s+2\xi\omega_n)} \Rightarrow \boxed{K_v = \frac{K_p\cdot\omega_n}{2\xi}} \, *$$

$$\therefore \boxed{e_{ss} = \frac{2\xi\cdot A}{K_p\cdot\omega_n}} \, *$$

- T_d will not have effect on e_{ss}, as $T_d s$ will be 0 when limit $s\to 0$.
- So, e_{ss} will get affected only due to K_p.

* On e_{ss}, T_d will not effect because when we will apply limit of $s\to 0$, $T_d s$ term will always become Zero (0).
So, whatever the changes in e_{ss} are taking place, it is just due to K_p and if $K_p = 1$, then e_{ss} will remain same.

6) Feedback Derivative Controller :-



$$G(s) = \frac{\omega_n^2}{s[s+(2\xi\omega_n + T_d\omega_n^2)]}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+(2\xi\omega_n+T_d\omega_n^2)s+\omega_n^2}$$

$$\therefore q(s) = s^2+(2\xi\omega_n+T_d\cdot\omega_n^2)s+\omega_n^2 = 0.$$

$$\boxed{\omega_n' = \omega_n} * , \quad \boxed{\xi' = \xi + \frac{T_d\cdot\omega_n}{2}} *$$

$$S_{1,2} = -\xi'\omega_n \pm j\omega_n\sqrt{1-(\xi')^2}$$

$$\therefore C(s) = \frac{\omega_n^2 (A/s)}{s^2+2\xi'\omega_n + \omega_n^2}$$

$$\therefore c(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-(\xi')^2}}\sin(\omega_d' t+\phi')\right] ; \ \phi' = \tan^{-1}\left(\frac{\sqrt{1-(\xi')^2}}{\xi'}\right).$$

Now, all formulae will be applicable,

$$t_r' = \frac{\pi-\phi'}{\omega_n\sqrt{1-(\xi')^2}} \qquad (t_r' > t_r)$$

$$t_p' = \frac{\pi}{\omega_n\sqrt{1-(\xi')^2}} \qquad (t_p' > t_p)$$

$$\% M_p' = e^{-\pi\xi'/\sqrt{1-\xi'^2}} \times 100\% \quad (M_p' < M_p).$$

$$t_s' = 4/\xi'\omega_n \qquad (t_s' < t_s).$$

- With application of Feedback Derivative Controller :-
  - * ξ will increase and as a result, % M_p will decrease.
  - * t_r will increase, t_s will decrease and stability of system will increase.

* For e_{ss} :-

$$K_v = \lim_{s \to 0} s \times \frac{\omega_n^2}{s[s+(2\xi\omega_n + T_d\omega_n^2)]}$$

$$\Rightarrow \boxed{K_v = \frac{\omega_n}{2\xi + T_d\omega_n}} *$$

$$\therefore \boxed{e_{ss} = \frac{(2\xi + T_d\omega_n)A}{\omega_n}} * \text{ [For Ramp Input].}$$

** $\therefore$ e_{ss} will increase by (T_d ~~ω_n~~ A) factor with applying Feedback Derivative controller.

Here, e_{ss} will depend on T_d here.

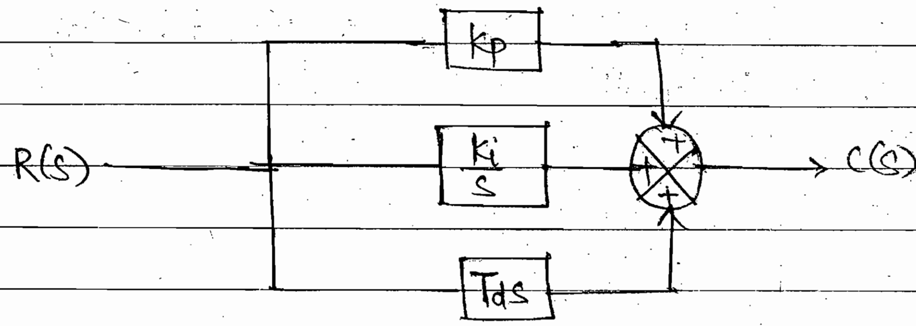
- It is used rarely. Example: Servomotors.

*→ Proportional Integral Derivative (PID) Controller :-

We know, for pure IInd order system,

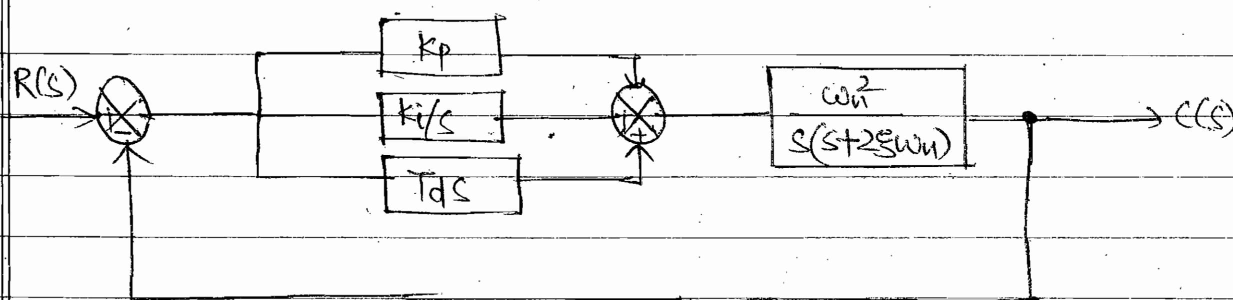
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad ; \quad e_{ss} = \frac{2\xi}{\omega_n} \text{ (for Ramp Input).}$$

Now, when PID controller is applied :-



$$\frac{C(s)}{R(s)} = T_d s + K_p + \frac{K_i}{s}$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{T_d s^2 + K_p s + K_i}{s}$$



$$G(s) = \frac{(T_d s^2 + K_p s + K_i)\,\omega_n^2}{s^2(s+2\xi\omega_n)}$$

and $$\frac{C(s)}{R(s)} = \frac{(T_d s^2 + K_p s + K_i)\,\omega_n^2}{s^3 + (2\xi\omega_n + T_d\omega_n^2)s^2 + K_p\omega_n^2 s + K_i\omega_n^2}$$

$\therefore$

s^3	1	$K_p\omega_n^2$
s^2	$2\xi\omega_n + T_d\omega_n^2$	$K_i\omega_n^2$
s^1	$\frac{(2\xi\omega_n + T_d\omega_n^2)K_p\omega_n^2 - K_i\omega_n^2}{(2\xi\omega_n + T_d\omega_n^2)}$	
s^0	$K_i\omega_n^2$	

$$(2\xi\omega_n + T_d\omega_n^2)K_p\cancel{\omega_n^2} > K_i\cancel{\omega_n^2}$$

$\therefore\ (2\xi\omega_n + T_d\omega_n^2)K_p > K_i$

** For more positive, T_d and K_p should be large and K_i should be small.

Numericals

Ques:- ① To design phase-lead compensator, which controller will be used :

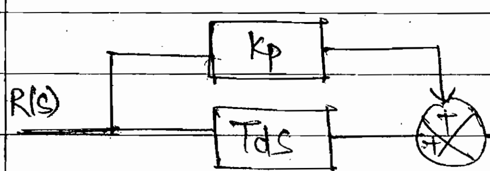
(a) P.D and Feedback Derivative controller

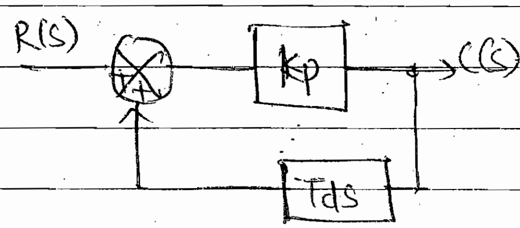
(b) Integral and Derivative controller.

(c) Proportional and Feedback Derivative controller.

(d) Derivative and PI controller

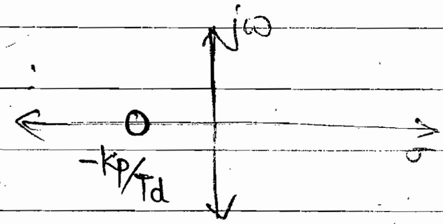
Ans:- (1)

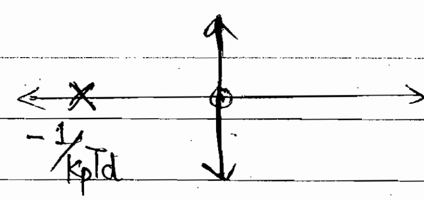




$$\frac{C(S)}{R(S)} = T_d s + K_p.$$

$$\frac{C(S)}{R(S)} = \frac{K_p}{K_p T_d s + 1}$$

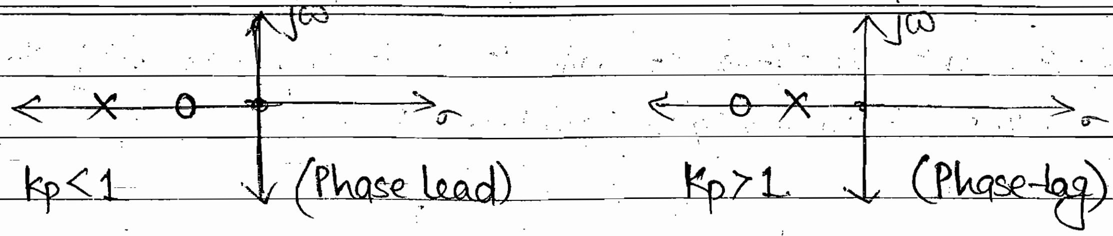




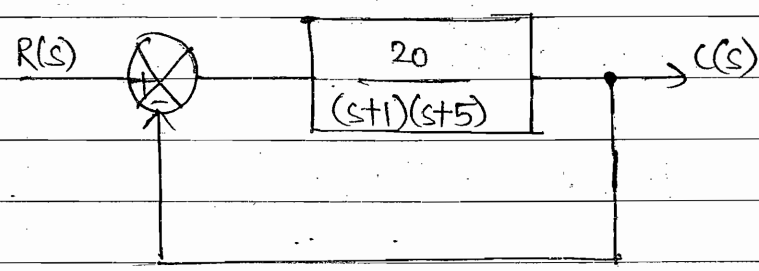
- To form Phase-lead compensator, both PD and Feedback Derivative controller will be cascaded.
- To make zero closer to imaginary axis,

$$\frac{K_p}{T_d} < \frac{1}{K_p T_d} \Rightarrow \boxed{K_p < 1} \ \text{* (For Phase-lead)}$$

$$\boxed{K_p > 1} \ \text{* (For phase-lag).}$$



Ques:-② For the given closed loop system, calculate its % M_p, ξ, ω_n, ω_d, First t_{po}, First t_{pu}, M_{pu1}, t_s and number of cycles upto t_s.



Ans:-② The closed Loop Transfer function can be obtained as:

$$\frac{C(s)}{R(s)} = \frac{20}{s^2+6s+25}$$

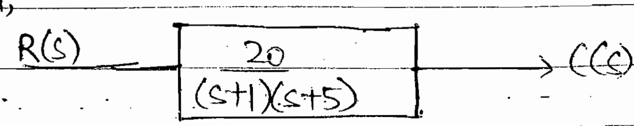
Here, characteristics Equation is : $s^2+6s+25=0$.

comparing with standard equation : $s^2+2\xi\omega_n s+25=0$.

$\boxed{\omega_n = 5 \text{ rad/sec}}$ * $\quad 2\xi \times 5 = 6 \Rightarrow \boxed{\xi = 0.6}$ *.

As $\omega_d = \omega_n\sqrt{1-\xi^2} \Rightarrow \boxed{\omega_d = 4 \text{ rad/sec}}$ *

- To understand this numerical better, first find its Open Loop Transfer Function,



$$\therefore \frac{C(s)}{R(s)} = \frac{20}{s^2+6s+5} = \frac{4 \times 5}{s^2+6s+5}$$

K=4 here.
(Gain).

- It means Amplifier of Gain $K=4$ is connected with Closed loop TF to make it Open Loop TF as:

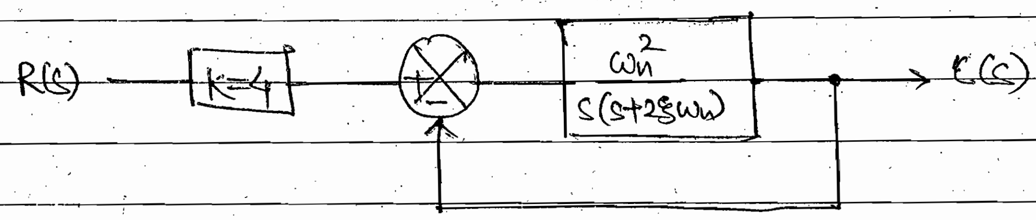
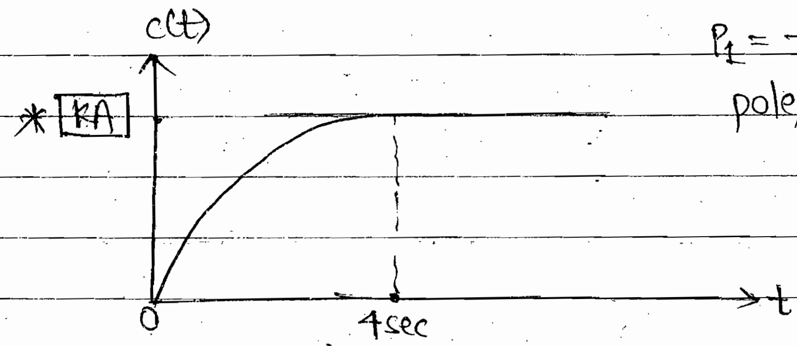


Fig: Open Loop Transfer Function.

Here, $q(s) = s^2+6s+5=0.$

$\omega_n = \sqrt{5}\ r/s$, $\xi = 3/\sqrt{5} = 1.3$

as $\xi > 1$, system is over-damped.



$P_1 = -1$ is dominant pole, so, t_s will be $\frac{4}{1} \Rightarrow t_s = 4$ sec.

- For DC Gain (K) $\neq 0$, Response will settle at [DC Gain X Input].

* Now, analyzing given closed loop system,

$$\therefore \frac{C(s)}{R(s)} = \frac{20}{s^2+6s+25} = \frac{0.8 \times 25}{s^2+6s+25}$$

$\boxed{K=0.8}$ *

$\therefore$ with increase in Feedback, DC Gain reduces.

$\therefore$ $\omega_n = 5\ r/s$, $\xi = 0.6$, $\xi\omega_n = 3$.

- Here, as $\xi < 1$, so Overdamped system changes to Under-damped system.

$\Rightarrow \omega_d = 4\ r/s.$

* Pole location, $s_{1,2} = -3 \pm j4.$

* Time constant, $\tau = \frac{1}{\xi \omega_n} \Rightarrow \tau = \frac{1}{3}$ seconds.

* t_s (for 2% error-band) $= \frac{4}{\xi \omega_n} = 1.33$ seconds.

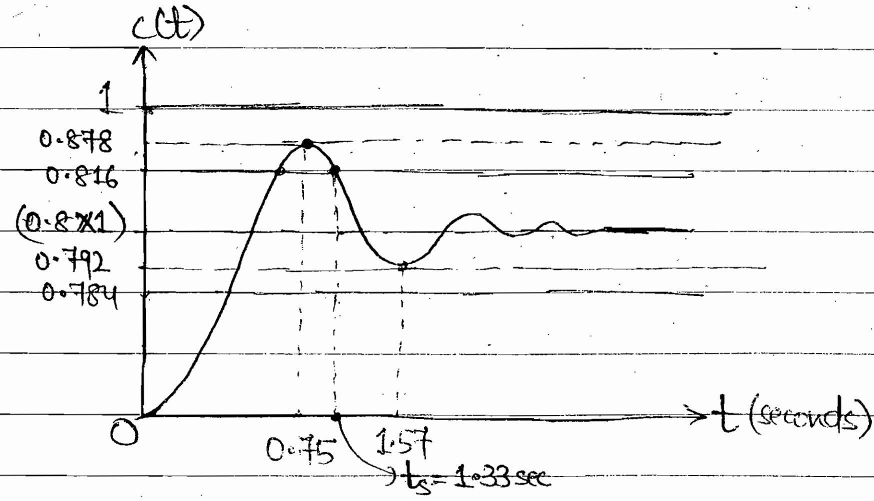
* $t_{po1} = \pi/\omega_d = \frac{3.14}{4} \Rightarrow t_{po1} = 0.785$ seconds.

* $t_{pu1} = \frac{2\pi}{\omega_d} = T_d = \frac{6.28}{4} = 1.57$ seconds. $= T_d$.

Note: *** As $T_d > t_s$, so output will settle before T_d without completing one-full cycle.

* $\% M_{po1} = e^{-\xi\pi/\sqrt{1-\xi^2}} = 9.8\ \%$

* $\% M_{pu1} = e^{-2\pi\xi/\sqrt{1-\xi^2}} = 0.89\%.$



Now, $\% M_p = \frac{c(t)|_{max} - c(t)|_{desired}}{c(t)|_{desired}} \times 100\ \%$

$\% M_p = \frac{c(t)|_{max} - 0.8}{0.8} = 9.8$

$$\therefore \boxed{c(t)|_{max} = 0.878} \ *$$

Similarly, $\% M_u = \dfrac{c(t)|_{desired} - c(t)|_{min}}{c(t)|_{desired}} \times 100\%$

$$\therefore 0.89\% = \frac{0.8 - c(t)|_{min}}{0.8} \times 100\%$$

$$\therefore \boxed{c(t)|_{min} = 0.792} \ *$$

* Number of cycles, $n = \dfrac{t_s}{T_d} = \dfrac{1.33}{1.5}$

$$\therefore \boxed{n = 0.8} \ *$$

* As $e_{ss} \propto \dfrac{1}{\text{Error-coefficients}}$ and Error-Coefficients = DC Gain

$$\therefore \boxed{e_{ss} \propto \frac{1}{K}} \ *$$

, So, as feedback increases, K decreases and e_{ss} increases.

* When feedback is increased in IInd order underdamped system, t_s will remain same. In under-damped, $\xi\omega_n$ remains constant, only ω_d changes.

Ques:-(3) The OLTF is $G(s)H(s) = \dfrac{K}{s(Ts+1)}$ [u.f.b]. Determine by what value K should be changed so that % overshoot decreases from 60% to 20%.

Ans:-(3) K = Variable here.

$$\therefore \frac{C(s)}{R(s)} = \frac{K}{Ts^2 + s + K} = \frac{(K/T)}{s^2 + s/T + K/T}$$

$\therefore$ Comparing with standard equation,

$$K' = K/T, \quad 2\xi\omega_n = 1/T, \quad K/T = \omega_n^2$$

$$\therefore \quad \omega_n = \sqrt{K/T}, \quad 2\times\xi\times\sqrt{K/T} = 1/T$$

$$\Rightarrow \quad \xi = \frac{1}{2\sqrt{KT}}$$

as K is only variable, $\boxed{\xi \propto \frac{1}{\sqrt{K}}}$ *

As $e^{-\pi\xi_1/\sqrt{1-\xi_1^2}} \times 100 = 60\% \Rightarrow e^{-\pi\xi_1/\sqrt{1-\xi_1^2}} = 0.6.$

$$\Rightarrow \quad \frac{-\pi\xi_1}{\sqrt{1-\xi_1^2}} = -0.5 \quad \Rightarrow \boxed{\xi_1 = 0.45} \ *$$

Similarly, $e^{-\pi\xi_2/\sqrt{1-\xi_2^2}} \times 100 = 20\% \Rightarrow \frac{-\pi\xi_2}{\sqrt{1-\xi_2^2}} = \ln(0.2) = -1.5$

$$\Rightarrow \boxed{\xi_2 = 0.16} \ *.$$

$$\left(\frac{0.45}{0.16}\right)^{-1} = \frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \frac{K_2}{K_1} = \left(\frac{0.16}{0.45}\right)^2 = 1/8$$

$\boxed{K_2 = \frac{1}{8}K_1}$ * $\Rightarrow$ K should be decreased by factor of 8.

Ques:- (4) The OLTF of a unity feedback system, $G(s)H(s) = \frac{10(s+5)}{s^2(s+2)(s+10)}$.

Determine steady state error if input $r(t) = 5t$.

Ans:- (4) $K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{s\times 10(s+5)}{s^2(s+2)(s+10)} \Rightarrow K_v = \infty$

$$\boxed{e_{ss} = 0} \ *$$

Ques:- 5 The open loop TF of a u.f.b system is: $\frac{10(s+5)}{s^2(s+2)(s+10)}$. If input is $2u(t)+5t+10t^2$. Determine steady state error.

Ans:- 5

As $e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$

Input is :- $2u(t) + 5t + 20\left(\frac{t^2}{2}\right)$

$K_a = \lim_{s\to 0} \cancel{s^2} \times \frac{10(s+5)}{\cancel{s^2}(s+2)(s+10)} = 5/2$.

$\therefore \quad e_{ss} = \frac{20}{5} \times 2$

$\boxed{e_{ss} = 8}$ *.

Ques:- 6 The open loop TF of a u.f.b is: $G(s)H(s) = \frac{K(s+2)}{s^2(s+5)(s+20)}$. Input given is: $1u(t) + 2t + 5t^2$. Determine value of K to limit e_{ss} at 8 ?

Ans:- 6

$K_a = \lim_{s\to 0} \cancel{s^2} \frac{K(s+2)}{\cancel{s^2}(s+5)(s+20)} = \frac{2K}{100} = K/50$

$\therefore \quad e_{ss} = \frac{10}{K} \times 50$

$8 = \frac{500}{K} \Rightarrow K = \frac{5 \times \cancel{100}^{25}}{\cancel{8}2}$

$\boxed{K = 62.5}$ *.

Ques:- 7 The forward gain of a system is: $\frac{10(s+5)}{s(s+2)(s+10)}$; $H(s) = 1/s$.

Input to this system $r(t) = 5u(t)$ ~~+10t~~. Determine e_{ss}.

Ans:- 7

$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)-G(s)} = \frac{\frac{10(s+5)}{s(s+2)(s+10)}}{1+\frac{10(s+5)}{s^2(s+2)(s+10)}+\frac{10(s+5)}{s(s+2)(s+10)}}$

$$G'(s) = \frac{10\,s(s+5)}{s^2(s+2)(s+10)+10(s+5)-10s(s+5)}$$

Type = 0

$$\therefore\ K_p = \lim_{s\to 0} \frac{10\,s(s+5)}{s^2(s+2)(s+10)+10(s+5)-10s(s+5)} = 0.$$

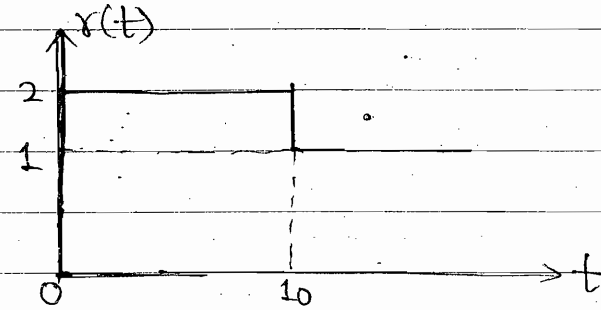
$\boxed{~~e_{ss} = \infty~~}$ *

$$e_{ss} = \frac{A}{1+K_p} = \frac{5}{1+0}.$$

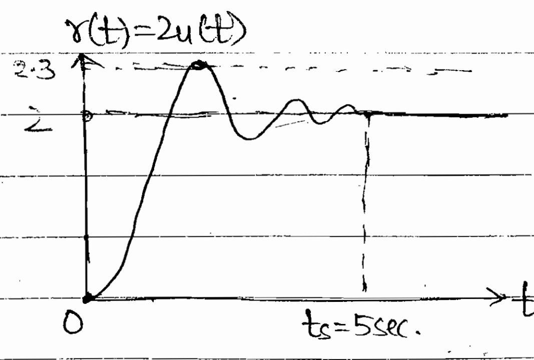
$\boxed{e_{ss} = 5}$ *

Ques:- (8) A second order under-damped system having $\% M_p = 15\%$, $t_s = 5$ sec, if input to this time is : $r(t) = 2u(t) - 1u(t-10)$. Obtain its output response. Also plot its output graph.

Ans:- (8)



$\therefore$ Response when $r(t) = 2u(t)$.



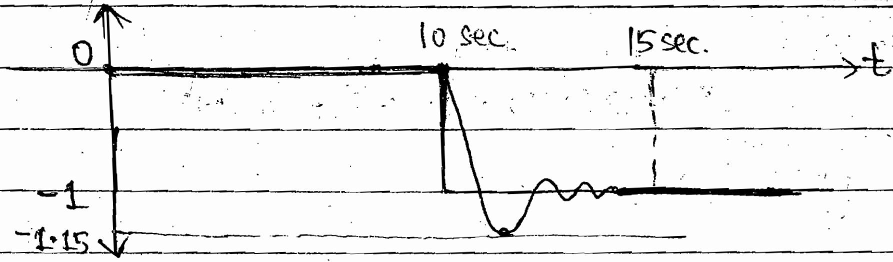
Now, $\frac{c(t)|_{max} - c(t)|_{des}}{c(t)|_{des}} \times 100 = 15\%$.

$\Rightarrow\ c(t)|_{max} = 2.3$

$$\therefore\ c(t) = 2\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)\right]$$

Now, when second input $r(t) = -1u(t-10)$ is applied, then

$r(t) = -u(t-10)$

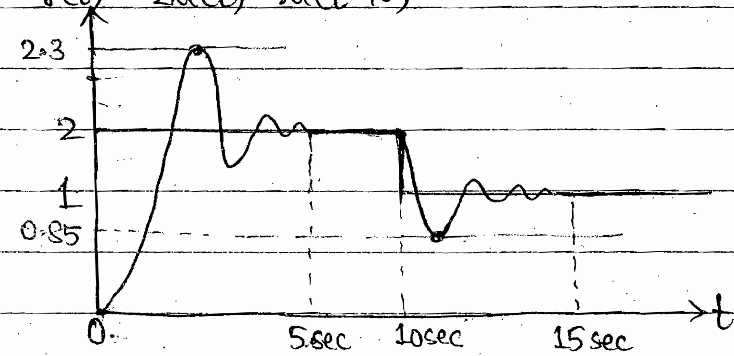


$\therefore$ Total output,

$$c(t) = 2\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi)\right] - (1)\left[1 - \frac{e^{-\xi\omega_n(t-10)}}{\sqrt{1-\xi^2}}\sin(\omega_d(t-10)+\phi)\right].$$

$\therefore$ Final output will be:

$r(t) = 2u(t) - u(t-10)$



As $\quad M_p\% = 15\% \Rightarrow e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.15$

$$\Rightarrow \boxed{\xi = 0.5} \ *$$

As $\quad t_s = \frac{4}{\xi\omega_n} \Rightarrow \omega_n = \frac{40}{0.5 \times 5} \Rightarrow \boxed{\omega_n = 1.6 \text{ rad/sec}} \ *$

$$\therefore \omega_d = \omega_n\sqrt{1-\xi^2} = 1.6\sqrt{1-(0.5)^2}$$

$$\boxed{\omega_d = 1.38 \text{ r/sec}} \ *$$

As $\quad \phi = \cos^{-1}(\xi) = \cos^{-1}(0.5), \quad \phi = 60^\circ$

$$\therefore c(t) = 2\left[1 - \frac{e^{-\frac{4}{5}t}}{\sqrt{0.75}}\sin\left(1.38t + \frac{\pi}{3}\right)\right]u(t) - 1\left[1 - \frac{e^{-\frac{4}{5}(t-10)}}{\sqrt{0.75}}\sin\left(1.38(t-10)+\frac{\pi}{3}\right)\right]u(t-10).$$

At t_s, $c(5) = 2\left[1 - \frac{e^{-4}}{\sqrt{0.75}} \sin\left(1.38\times 5+\frac{\pi}{3}\right)\right] + 0.$

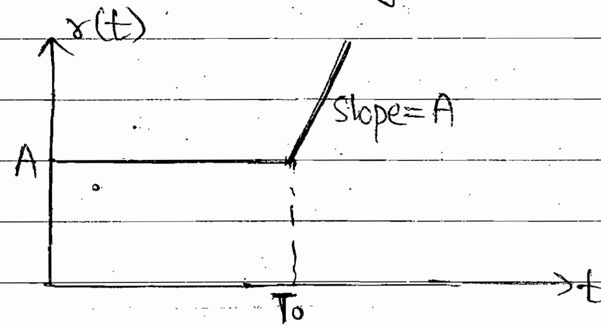
$\boxed{c(5) = 2}$ * (For 2% Error-band).

At $t = 15$, $c(15) = 2\left[1 - \frac{e^{-\frac{4}{3}\times 15}}{\sqrt{0.75}} \sin\left(1.38\times 15+\frac{\pi}{3}\right)\right]u(t) - 1\left[1 - \frac{e^{-\frac{4}{3}\times 5}}{\sqrt{0.75}} \sin\left(1.38\times 5+\frac{\pi}{3}\right)u(t-10)\right]$

$\Rightarrow \quad c(15) = 2 - 1$

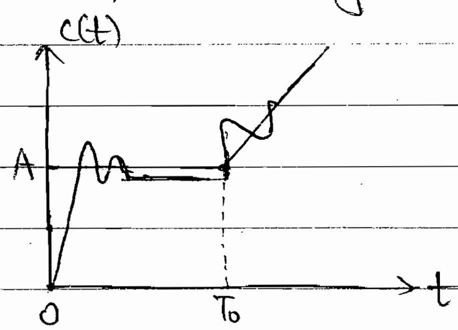
$\therefore \quad \boxed{c(15) = \mathbf{1}}$ * [For 2% error-band].

Ques:- (9) The OLTF of a u.f.b second order underdamped system is: $G(s)H(s) = \frac{K}{s(Ts+1)}$. If input to this system is:

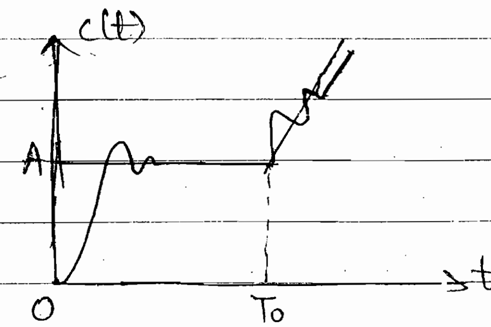


Which of the following will be its output response?

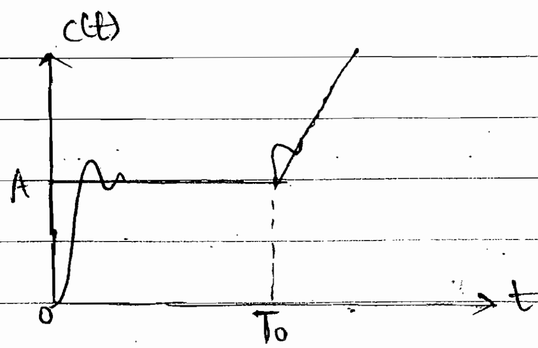
(a)

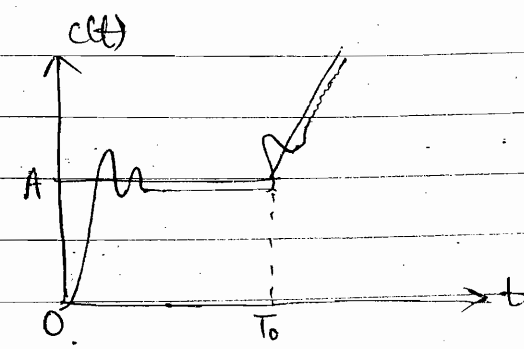


(b)



(c)





Ans:- (9) output will be :-

$$c(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)\right]u(t) + A\left[(t-T_0) - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi')\right]u(t-T_0)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) \qquad \phi' = \tan^{-1}\left(\frac{2\xi\sqrt{1-\xi^2}}{2\xi^2-1}\right).$$

Ans (b)

As type is 1, For unit step, $e_{ss} = 0$

For Ramp, e_{ss} = Finite.

Ques:- (10) For Closed loop TF of a unity feedback system, $\frac{C(s)}{R(s)} = \frac{K(s+\alpha)}{s^2+\alpha s+\beta}$

determine e_{ss} for unit step input.

Ans:- (10) $$OLTF = \frac{K(s+\alpha)}{s^2+\alpha s+\beta-Ks-\alpha K} = \frac{K(s+\alpha)}{s^2+s(\alpha-K)+(\beta-\alpha K)}$$

$$\therefore \quad K_p = \lim_{s\to 0} \frac{K(s+\alpha)}{s^2+s(\alpha-K)+(\beta-\alpha K)}$$

$$\Rightarrow \quad K_p = \frac{K\alpha}{(\beta-\alpha K)}$$

$$\therefore \quad e_{ss} = \frac{1}{1+K_p} = \frac{(\beta-\alpha K)}{\beta-\cancel{\alpha K}+\cancel{K\alpha}}$$

$$\therefore \quad \boxed{e_{ss} = \frac{\beta-K\alpha}{\beta}} \; *$$

Ques:- (11) The error response of a servomotor for step-input is:

$e(t) = 1.67\, e^{-8t} \sin\{6t+37°\}$. Determine

ξ, ω_n, ω_d, t_{po1}, $\%M_{po1}$ and t_s for 5 % error-band.

Ans:- (11) For $r(t) = Au(t)$.

$$\therefore c(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)\right]u(t).$$

and $e(t) = r(t) - c(t) = Au(t) - c(t)$.

$$\therefore\ e(t) = \frac{A}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + \phi) \quad \text{—— (1)}.$$

Comparing given $e(t)$ with equation (1),

$$\therefore\ 1.67 = \frac{A}{\sqrt{1-\xi^2}}, \quad \xi\omega_n = 8.$$

Also, $\omega_d = 6 \Rightarrow \omega_n\sqrt{1-\xi^2} = 6.$

$$\omega_n = 6 \times 1.67 \Rightarrow \boxed{\omega_n \simeq 10 \text{ rad/sec}} *$$

$$\therefore\ \boxed{\xi \simeq 0.8} *. \quad \boxed{\omega_d = 6 \text{ r/sec}} *.$$

Now, $t_{po1} = \frac{\pi}{\omega_d} = \frac{3.14}{6} \Rightarrow \boxed{t_{po1} = 0.52 \text{ sec}} *$

$$t_{pu1} = \frac{2\pi}{\omega_d} = 1.04 \text{ seconds} = T_d.$$

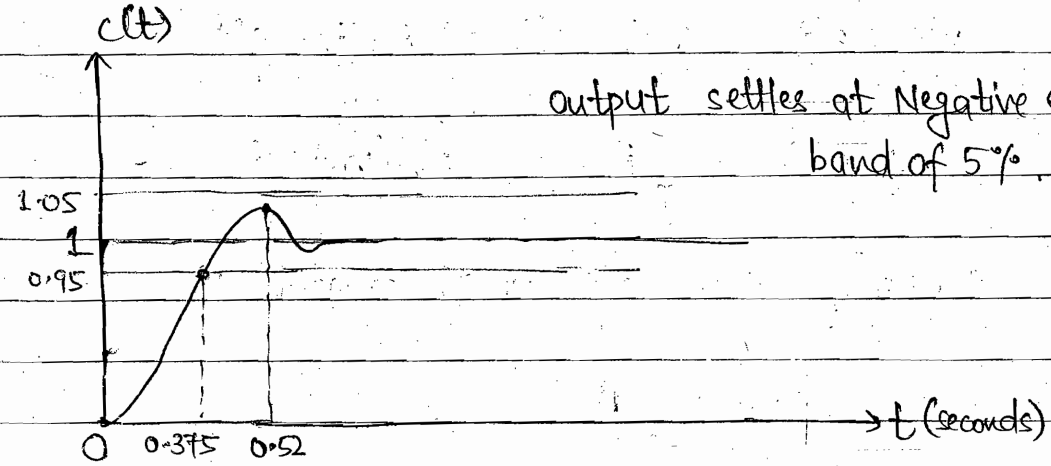
$$t_s = 3\tau = \frac{3}{\xi\omega_n} \Rightarrow t_s = \frac{3}{8}$$

$$\boxed{t_s = 0.375 \text{ sec}} *$$

$$\% M_{po1} = e^{-\xi\pi/\sqrt{1-\xi^2}} = e^{-0.8 \times 3.14 \times 1.67} \quad \boxed{\% M_{po1} = 1.5\ \%} *.$$

$$\therefore\ \frac{A}{\sqrt{1-\xi^2}} = 1.67 \Rightarrow \boxed{A \simeq 1} *$$

As $t_s < T_d$, so output will settle before half-cycle is completed.



output settles at Negative error band of 5%.

Ques:-12, The OLTF of u.f.b is: $G(s)H(s) = \frac{K(s+2)}{s^3+\beta s^2+4s+1}$. Determine the value of K and β so that ξ will be maintained at 0.2 and ω_n at 3 rad/sec.

Ans:-(12) Now, $\frac{C(s)}{R(s)} = \frac{K(s+2)}{s^3+\beta s^2+s(K+4)+(1+2K)} = \frac{K(s+2)}{s^3+\beta s^2+s(K+4)+(1+2K)}$

$\therefore \quad q(s) = 1+G(s)H(s) = 0$

$\Rightarrow \quad s^3+\beta s^2+s(4+K)+(1+2K) = 0.$

- This third order system has been made from one I^st order and one II^nd order under-damped system.

$\therefore \quad (s+\xi\omega_n+j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n-j\omega_n\sqrt{1-\xi^2})(s+p) = 0.$

$\Rightarrow \quad (s^2+2\xi\omega_n s+\omega_n^2)(s+p) = 0$

$\Rightarrow \quad s^3+ps^2+2\xi\omega_n s^2+2\xi\omega_n ps+\omega_n^2 s+\omega_n^2 p = 0.$

$\Rightarrow \quad s^3+s^2(p+2\xi\omega_n)+s(\omega_n^2+2\xi\omega_n p)+\omega_n^2 p = 0.$

$\Rightarrow \quad p+2\xi\omega_n = \beta \quad ; \quad 4+K = \omega_n^2+2\xi\omega_n p \quad , \quad 1+2K = \omega_n^2 p.$

or $\quad p+1.2 = \beta. \quad ; \quad 12p+9 = 4+K \quad ; \quad 1+2K = 9p.$

$\therefore \quad p = 1.66 \; ; \quad K = 7 \; ; \quad \beta = 2.85.$

- As p is positive here (we consider p as left-half pole), so system, here, is stable.
  If, by chance, p is negative, then system will be unstable.

Ques:- (13) The OLTF of a u-f-b is: $G(s)H(s) = \dfrac{20}{s(s+2)}$. If input $r(t) = a_0 u(t) + a_1(t) + a_0 \dfrac{t^2}{2}$, Determine Dynamic error as well as e_{ss}.

Ans:- (13) Type = 1.

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} = \frac{2s+s^2}{20+2s+s^2}.$$

Using Long Division; $20+2s+s^2 \,\big)\, 2s+s^2 \,\big(\, 0 + s/10 + \frac{s^2}{25}$

$$\begin{array}{l} \cancel{2s} + s^2 \\ \cancel{2s} + s^2/5 + s^3/10 \\ (-)\quad(-)\quad(-) \\ \hline \frac{4}{5}s^2 - \frac{s^3}{10} \\ \frac{4}{5}s^2 + \frac{2}{25}s^3 + \frac{s^4}{25} \\ (-)\quad(-) \\ \hline \end{array}$$

(Neglect Remainder).

$$\therefore \frac{E(s)}{R(s)} = \frac{s}{10} + \frac{s^2}{25} = C_0 + C_1 s + C_2 s^2$$

$$\text{or } E(s) = \frac{sR(s)}{10} + \frac{s^2 R(s)}{25}. \quad \text{--- (1)}$$

$\therefore \quad C_0 = 0 \quad ; \quad K_p = \infty$

$C_1 = 1/10 \quad , \quad C_2 = 1/25.$

$K_v = 10 \quad , \quad K_a = 25.$

$$\therefore \text{From eq-(1)}, \quad e(t) = \frac{\frac{d}{dt}(r(t))}{10} + \frac{\frac{d^2}{dt^2} r(t)}{25}.$$

$$\boxed{e(t) = \frac{(a_1 + a_2 t)}{10} + \left(\frac{a_2}{25}\right).} \quad *$$

$$e_{ss} = \lim_{t \to \infty} e(t) \quad \Rightarrow \quad \boxed{e_{ss} = \infty} \quad *.$$

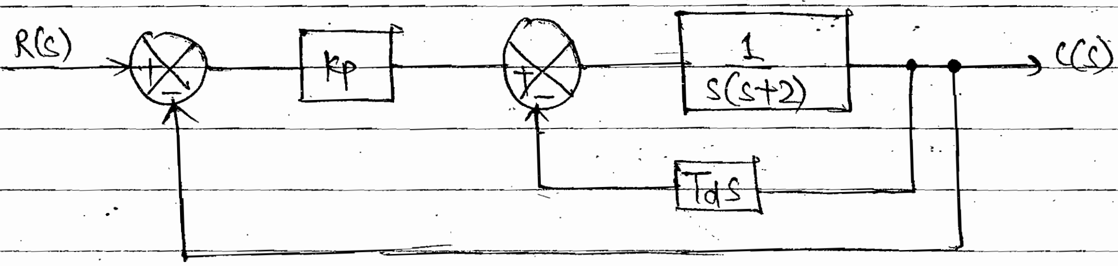
* If, initially, Parabolic input is not applied, then $a_2 = 0$.

$$\therefore e_{ss} = \lim_{t \to \infty} \left(a_1/10 \right) \Rightarrow \boxed{e_{ss} = \frac{a_1}{10}} *$$

* e_{ss} can also be found by :

$$K_v = \lim_{s \to 0} \frac{\cancel{s} \times 20}{\cancel{s}(s+2)} = 10.$$

$$\boxed{e_{ss} = \cancel{a_1}10^{-1}} * \quad \text{or} \quad \boxed{e_{ss} = \frac{a_1}{10}} *$$

Ques:-(14) 

For the given block, determine the values of kp and Td so that ξ will be maintained at 0.7 and e_{ss} at 0.2

Ans:-(14) $\Rightarrow$ Inside block $\Rightarrow$ $$\frac{1/s(s+2)}{1 + \frac{T_d \cdot s}{s(s+2)}}$$

$$\Rightarrow \quad = \frac{1}{s^2 + 2s + T_d s} = \frac{1}{s^2 + s(T_d + 2)}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{k_p}{s^2 + s(2+T_d) + k_p}$$

$$\therefore q(s) = s^2 + s(2+T_d) + k_p = 0.$$

$$\omega_n = \sqrt{k_p} \quad \text{and} \quad 2 + T_d = 2(0.7)\sqrt{k_p}$$

$$\therefore \quad 2 + T_d = 1.4\sqrt{k_p}.$$

Also, $K_v = \lim_{s\to 0} s\left(\dfrac{K_p}{s^2+s(2+T_d)}\right)$

$$\Rightarrow K_v = \lim_{s\to 0} \cancel{s} \times \frac{K_p}{\cancel{s}(s+2+T_d)} = \frac{K_p}{2+T_d}.$$

$$\therefore e_{ss} = \frac{2+T_d}{K_p} \Rightarrow 0.2 = \frac{2+T_d}{K_p}$$

$$\Rightarrow 0.2K_p = 2+T_d.$$

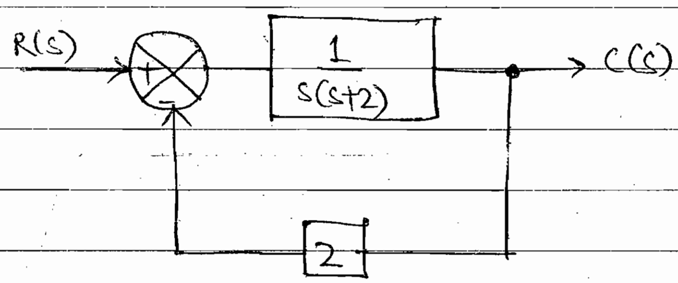
$$\therefore 0.2K_p = 1.4\sqrt{K_p} \Rightarrow \cancel{0.2}\sqrt{K_p} = \cancel{1.4}\ 7$$

$$\therefore \boxed{K_p = 49} *$$

and $2+T_d = 0.2K_p \Rightarrow 2+T_d = 0.2\times 49$

$$T_d = 9.8-2 \quad \therefore \boxed{T_d = 7.8} * \text{(seconds)}$$

Ques:- (15)



For the given block, calculate e_{ss} for unit step and unit Ramp input.

Also calculate e_{ss} for its unity feedback system for unit step input.

Ans:- (15) $G(s)H(s) = \dfrac{2}{s(s+2)}$; Type = 1

So, $K_v = \lim_{s\to 0} \cancel{s} \times \dfrac{2}{\cancel{s}(s+2)} \Rightarrow K_v = 1.$

$$\therefore \boxed{e_{ss} = 1} * \text{(For Ramp Input)}$$

For step Input, $\boxed{e_{ss} = 0}$ *

Converting it into unity feedback system;

$$\frac{C(s)}{R(s)} = \frac{1/s(s+2)}{1+2/s(s+2)} = \frac{1}{s^2+2s+2}$$

$$\therefore \quad OLTF = \frac{1}{s^2+2s+1}.$$

$$\therefore \quad K_p = \lim_{s \to 0} \frac{1}{s^2+2s+1} = 1/1.$$

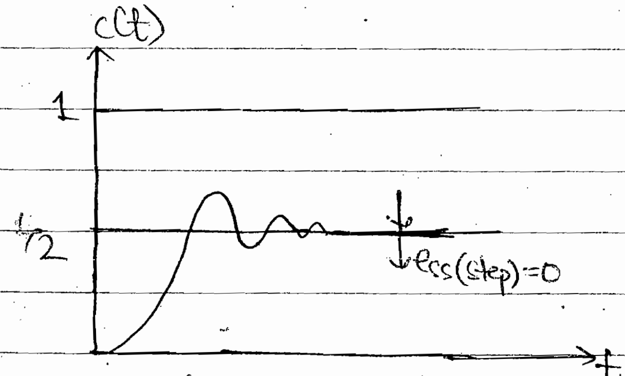
$$\therefore \quad \boxed{e_{ss} = 0.5} \ * \text{ (For unit step).}$$

$$\boxed{e_{ss} = \infty} \ * \text{ (For unit Ramp).}$$

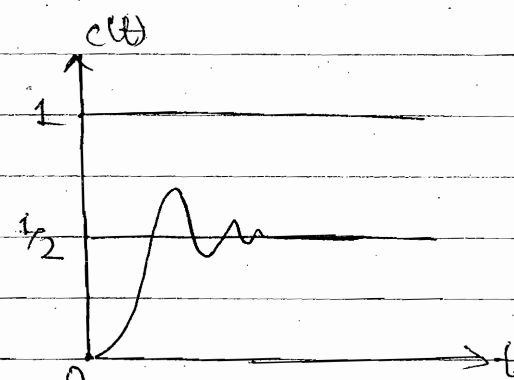
Now, $\dfrac{C(s)}{R(s)} = \dfrac{1}{s^2+2s+2}$

Comparing with Standard IInd order system; $K = 1/2$.

- And For given block, $\dfrac{C(s)}{R(s)} = \dfrac{1}{s^2+2s+2}$, here also DC gain $(K) = 1/2$.



(For u.f.b system)



(For Given block)

Chapter:-4

SENSITIVITY

* Sensitivity is defined with respect to two parameters.
  * Sensitivity in terms of input and for better performance of a system, it should be as large as possible;
  and second with respect to variation of parameters and it should be as small as possible.
* If Sensitivity wrt input is Large, then that system will detect smallest input possible and if sensitivity wrt variation in parameter is small, then effect on output due to variation of parameter will be small.
  * It is represented by: S_K^T.

$$\therefore \boxed{S_K^T = \frac{(dT/T)}{(dK/K)} = \frac{K}{T} \cdot \frac{dT}{dK}} \quad *$$

T = Transfer Function
K = Any parameter.

$\frac{dT}{\cancel{dK}\,T}$ = Limiting error in transfer function.

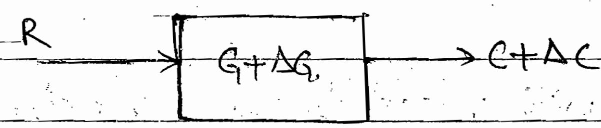
$\frac{dK}{K}$ = Limiting error of any parameter.

$$\boxed{\text{Limiting Error } (\epsilon) = \frac{\text{Measured value} - \text{True value}}{\text{True Value}} \times 100\,\%} \quad *$$

- For transfer Function, $\epsilon_T = \frac{T_M - T}{T} = \frac{dT}{T}$.
- For Parameter, $\epsilon_K = \frac{K_M - K}{K} = \frac{dK}{K}$.

$$\therefore \boxed{S_K^T = \frac{\epsilon_T}{\epsilon_K} = \frac{K}{T} \cdot \frac{dT}{dK}} \quad *$$

* Sensitivity of Open Loop System due to variation in Forward Gain :-



Earlier, $\frac{C}{R} = G = T$

Now, $C + \Delta C = (G + \Delta G)R = GR + (\Delta \cdot G) \cdot R$

$$C + \Delta C = C + R(\Delta \cdot G)$$

$$\therefore \Delta C = R(\Delta \cdot G)$$

or $$\left(\frac{\Delta C}{R}\right) = \Delta G = \Delta T$$

$$\therefore \left(\frac{\Delta T}{T}\right) = 1 \cdot \left(\frac{\Delta G}{G}\right)$$

$$\therefore S_G^T = \frac{\Delta T/T}{\Delta G/G}$$

$$\therefore \boxed{S_G^T = 1} \; *$$

As, $\Delta T = \frac{\Delta C}{R}$ and $T = C/R$

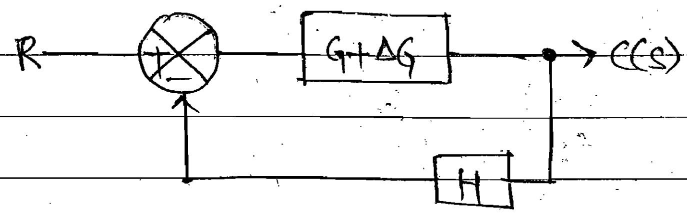
$$\therefore \boxed{\frac{\Delta T}{T} = \frac{\Delta C}{C}} \; *$$ (Valid only if Input R(s) is constant)

- Same can be obtained by using formula,

$$S_G^T = \frac{G}{T} \cdot \frac{dT}{dG} \Rightarrow \boxed{S_G^T = 1} \; * \quad (T = G \text{ here})$$

* In case of Open Loop System, S_G^T (Forward Gain) = 1, it means both transfer function and Forward Gain will change with same percentage and hence, output will also change by same percentage.

* Sensitivity of Closed Loop system due to variation of parameter- Forward Gain :-



$$\frac{C}{R} = T = \frac{G}{1+GH}$$

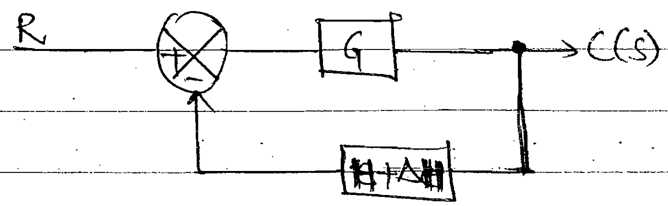
$$\frac{dT}{dG} = \frac{(1+GH) - GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

$$\therefore \quad S_G^T = \frac{G}{T} \cdot \frac{dT}{dG} = \frac{G}{T} \cdot \times \frac{1}{(GH+1)^2} = \frac{\cancel{G}}{\cancel{G}}\cancel{(1+GH)} \times \frac{1}{(GH+1)^{\cancel{2}}}$$

$$\therefore \quad \boxed{S_G^T = \frac{1}{1+GH}} \; *$$

- Sensitivity of closed loop system due to variation in Forward Gain is $(1+GH)$ times less than the sensitivity of open Loop system due to variations in Forward Gain, hence Performance of Closed loop systems due to variation in Parameter is better than that of open Loop systems.

* Sensitivity of Closed Loop System due to variation in Feedback Gain :-



Now, $\quad T = \frac{C}{R} = \frac{G}{1+GH}$

and $\dfrac{dT}{dH} = \dfrac{(1+GH)(0) - G^2}{(1+GH)^2} = \dfrac{-G^2}{(1+GH)^2}$

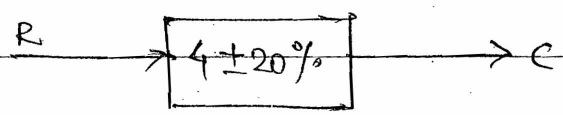
$\therefore \; S^T_H = \dfrac{H}{G} \times \cancel{(1+GH)} \times \dfrac{(-G^2)}{(1+GH)^{\cancel{2}}}$

$\Rightarrow \; \boxed{S^T_H = \dfrac{-GH}{(1+GH)}}$ *

- sensitivity of closed loop system due to variations in Feedback Gain is (−GH) times greater than sensitivity of closed loop system due to variation of Forward Gain. So, In Feedback Gain, proper shielding is required.

*• Negative sign in S^T_H represents that effect will be opposite in nature. If T increases, H will decrease and vice-versa.

Ques:- (1)



For the given block, calculate % change in output of open loop system and closed loop system with unity feedback condition.

Ans:- (1) $\dfrac{dG}{G} \times 100 = 20\%$

$\therefore \; dG/G = 0.2 \;\Rightarrow\; dG = \pm 0.2G$

$\therefore \; dG = \pm(0.2 \times 4) \;\Rightarrow\; dG = \pm 0.8$

So, Gain will change between 3.2 to 4.8.

Now, $S^T_G = \dfrac{G}{T} \cdot \dfrac{dT}{dG} = 1$

$\Rightarrow \; dT/T = 1 \times \dfrac{dG}{G} \;\Rightarrow\; \dfrac{dC}{C} = 1 \times \dfrac{dG}{G}$

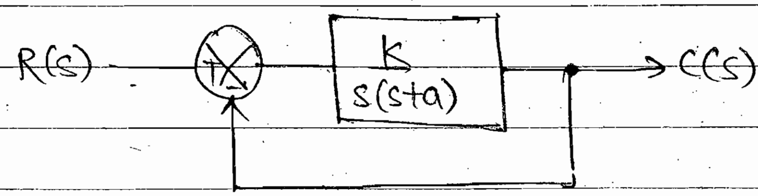
or $\boxed{\frac{dC}{C} = 20\%}$ *.

• For closed loop system with u.f.b.:

$$S_G^T = \frac{1}{1+GH} = \frac{1}{1+4\times 1} = 1/5.$$

$$\therefore \frac{dC}{C} = \frac{1}{5}\times 20\% \Rightarrow \boxed{\frac{dC}{C} = 4\%} *.$$

Ques: (2) For the given block, calculate sensitivity of e_{ss} due to variation of K and a.



Ans: (2) $G(s)H(s) = \frac{K}{s(s+a)}$.

$$\therefore K_v = \lim_{s\to 0} s \times \frac{K}{s(s+a)} \Rightarrow K_v = \frac{K}{a}.$$

$$\therefore e_{ss} = (a/K).$$

Now, $S_K^{e_{ss}} = \frac{K}{(a)}\times K \times (-a/K^2)$

$\boxed{S_K^{e_{ss}} = -1}$ *. $\boxed{\frac{de_{ss}}{e_{ss}} = -\frac{dK}{K}}$ *

Also, $S_a^{e_{ss}} = \frac{a}{a}\times K \times 1/K$ $\boxed{S_a^{e_{ss}} = 1}$ *.

or $\boxed{\frac{de_{ss}}{e_{ss}} = \frac{da}{a}}$ * e_{ss} and a will change proportionally.

Chapter:-5

STABILITY AND ROUTH-HURWITZ STABILITY CRITERIA

* → Stability is defined in terms of two parameters i.e
(i) Absolute Stability (ii) Relative Stability.

(i) **Absolute stability:-**

- It is defined in terms of Location of pole and for closed loop system to be stable, all poles should lie in the left half plane.
Routh-Hurwitz Criteria, Root Locus, Nyquist Plot define Absolute Stability.

(ii) **Relative stability:-**

- It is defined in terms of Damping Ratio, Gain Margin and Phase Margin. If open loop system is Non-minimum phase system, then for closed loop system to be stable, Gain Margin and Phase Margin should be Negative.

* If open loop system is Minimum Phase system, then for closed loop system to be stable, then Gain Margin and Phase Margin should be Positive.
* In IInd order system, Relative stability is defined by ξ. For IInd order system to be stable, ξ will be positive.

- In R-H criterion for stability:-
  * R-H table gives information about poles lying in Right-half of s-plane. It will not give any information about poles lying on left-half of s-plane.

*> Non-Minimum Phase System :-

- It is a system which contains atleast one pole or zero on right-half of s-plane.

*> Minimum Phase System :-

- It is a system which does not contain any pole or zero in the right-half of s-plane.

*> Properties of Routh Table :-

(i) R-H criterion is applicable only for Closed loop systems because in ~~in~~formation for Routh table, we use characteristics equation and characteristics equation is defined for Closed loop systems.

(ii) For Closed loop system to be Stable, all elements in first column of Routh-table must be of same sign; either positive or Negative. No. of times change in sign represents the number of poles lying on Right-half of s-plane, thus system will be unstable.

(iii) In characteristics equation, if any power of s is missing, that represents the presence of atleast one pole on Right-half and Closed loop system will be unstable.

(iv) If characteristics equation contains either even power of s or odd power of s, then there will be possibility that poles can locate on Imaginary axis.

(v) In characteristics equation, the co-efficient of s should be Real, it should not be either Imaginary, Complex or Sinusoidal.

Example:- $q(s) = 1+G(s)H(s) = a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0.$

Solution:-

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	
s^4	$(a_5a_4 - a_6a_3)/a_5$	$(a_5a_2 - a_6a_1)/a_5$	0	
s^3	$(b_1a_3 - b_2a_5)/b_1$	$(b_1a_1 - a_5a_2)/b_1$	0	
s^2	$(c_1b_2 - b_1c_2)/c_1$	a_0		
s^1	$(d_1c_2 - c_1a_0)/d_1$	0		
s^0	a_0			

where, $b_1 = \dfrac{a_5a_4 - a_6a_3}{a_5}$; $b_2 = \dfrac{a_5a_2 - a_6a_1}{a_5}$

$$c_1 = \frac{b_1a_3 - b_2a_5}{b_1} \qquad c_2 = \frac{(b_1a_1 - a_5a_2)}{b_1}$$

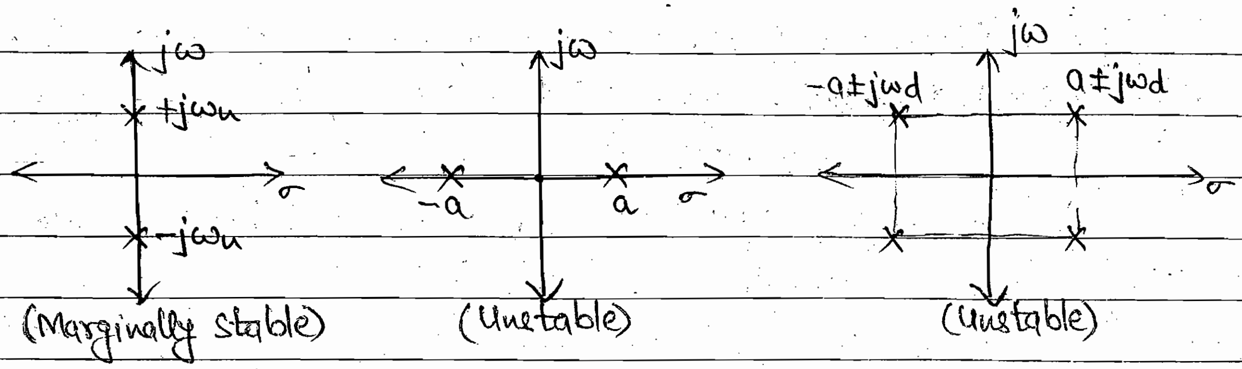
$$d_1 = \frac{(c_1b_2 - b_1c_2)}{c_1} \qquad e_1 = \frac{(d_1c_2 - c_1a_0)}{d_1}$$

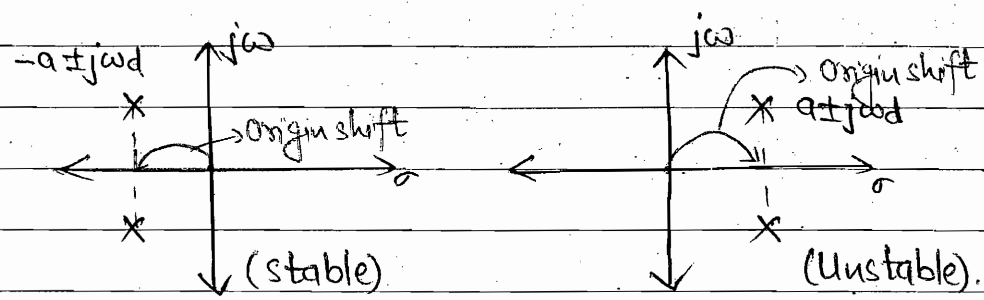
*→ **Case:-I :- Any element of First Column becomes Zero(0) :-**

- If any element of first column becomes 0, in that case, all elements of Next row will become ∞ and R-H table will terminate at that row. To extend R-H table, we will replace 0 with some variable, after that we will complete our Routh table and after completion of Routh table, again we will replace that variable with 0 in entire Routh table.

*→ **Case:-II :- All elements of any odd row becomes Zero(0) :-**

- If all elements of any odd row becomes 0, then Auxillary characteristics equation of Even row just above that odd row will be purely divisible with main characteristics equation and main characteristics equation will contain all poles of Auxillary characteristics equation and poles of Auxillary equation will locate at Image Location about either x or y-axis.





- In normal case, Routh-table gives only information about poles but for these cases, Routh-table gives the location of poles.

  * The Auxillary characteristics equation will give exact location of Image pole.

* If more than one time, odd row is becoming Zero(0), then that represents the existence of more than one pair of pole at common Image location and system will be unstable.

Numericals

Ques:- (1) The characteristics equation of a system is : $s^5+s^4+2s^3+2s^2+3s+5=0$
Determine total number of poles lying in the right-half of s-plane.

Ans:- (1)

s^5	1	2	3
s^4	1	2	5
s^3	$0 = d$ -	-2	
s^2	$(2d+2)/d = \infty$	5	
s^1	$\left[\left(\frac{-4d-4}{d}\right)-5d\right]/\left(\frac{2d+2}{d}\right)$		
s^0	5		

$$\frac{-2-5d^2}{(2d+2)} = -2 \quad (\text{at } d=0)$$

∴ Two times, sign changes, so, Right-pole = 2.

- In this case, 2 times sign changes. It represents out of total 5 poles,

  RHP = 2

  LHP = 3. and system will be Unstable.

Ques:- (2) The characteristics equation of a system is:

$$s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0.$$

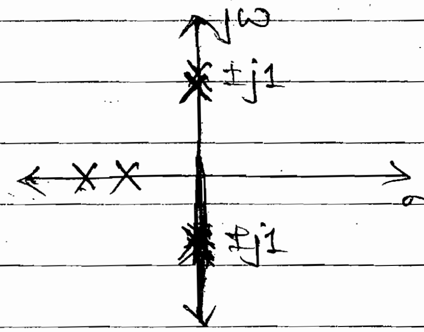
Comment on the stability of system. Also, determine total poles lying on $j\omega$ axis and calculate undamped Natural frequency.

Ans:- (2)

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	$0 \to 8$	$0 \to 8$		
s^2	2	2		
s^1	$0 \to 4$			
s^0	2			

Symmetric Poles

$$\frac{\frac{5}{2} \times 6 - 16}{5/2} \Rightarrow \frac{30 - 32}{5} = -\frac{2}{5}$$



$\therefore$ $A(s) = 2s^4 + 4s^2 + 2 = 0$

$$\frac{dA(s)}{ds} = 8s^3 + 8s$$

$\therefore$ Finding solutions of $A(s)$.

$\therefore$ $2(s^2+1)(s^2+1) = 0 \Rightarrow s = \pm j1, \pm j1$

$\therefore$ Imaginary Poles = 4

LHP = 2

RHS = 0.

- Conventionally, To find remaining poles, divide $q(s)$ with $A(s)$.

$$\therefore\quad 2s^4+4s^2+2\ \overline{\big)\ s^6+3s^5+4s^4+6s^3+5s^2+3s+2\ \big(}\ \frac{s^2}{2}+\frac{3}{2}s+1$$

$$\underset{(-)}{\cancel{s^6}}\quad \underset{(-)}{+2s^4}\quad \underset{(-)}{+s^2}$$

$$\cancel{3s^5}+2s^4+\cancel{6s^3}+4s^2+3s+2$$

$$\underset{(-)}{\cancel{3s^5}}\quad \underset{(-)}{\cancel{6s^3}}\quad \underset{(-)}{+3s}$$

$$+2s^4+4s^2+2$$

$$2s^4+4s^2+2$$

$$\times\times\times.$$

$$\therefore\quad 2s^4+4s^2+2=0 \qquad \text{and} \qquad s^2+3s+2=0.$$

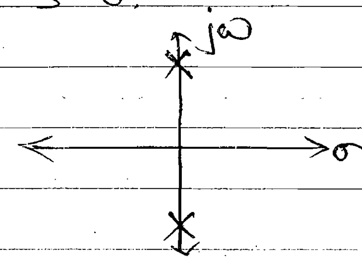
$$s=\pm j1,\ \pm j1 \qquad\qquad (s+1)(s+2)=0$$

$$s=-1,\ -2.$$

$\therefore$ Two poles lie on left-half of s-plane.

$$\therefore\quad s^2+2\xi\omega_n s+\omega_n^2=0\ ,\ \text{for } \xi=0.$$

$$\boxed{s^2+\omega_n^2=0}\ *$$

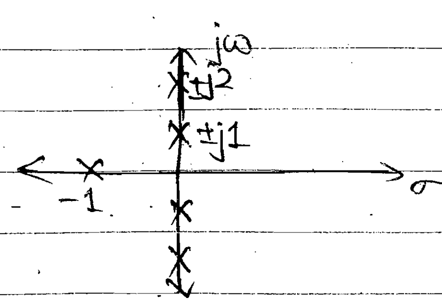


$$\boxed{\omega_n=1\ \text{rad/sec}}\ *$$

System will be Unstable. Ans

** For Example:-

(1)



$$\therefore\quad (s+1)(s^2+4)(s^2+1)=0.$$

$\therefore q(s) = s^5 + s^4 + 5s^3 + 5s^2 + 4s + 4 = 0.$

* Here the 5^{th} order system is made up of 2 2^{nd} order systems having ω_n of 1 rad/sec and 2 rad/sec.

Using Routh-Hurwitz Criterion,

s^5	1	5	4
s^4	1	5	4
s^3	$0 \to 4$	$0 \to 10$	
s^2	2.5	4	
s^1	3.6		
s^0	4		

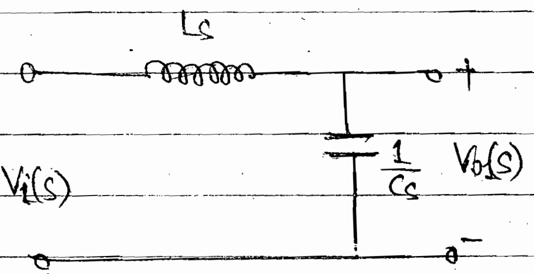
$A(s) = s^4 + 5s^2 + 4 = 0$

$\frac{dA(s)}{dt} = 4s^3 + 10s$

** $s^1 \neq 0$ here because $A_1(s) = 2.5s^2 + 4 = 0$ is not divisible with $q(s)$.

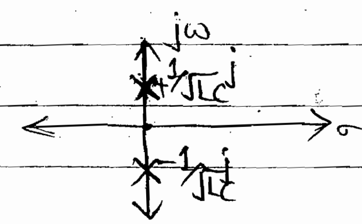
* If $q(s)$ is divisible by $A_1(s)$, then odd row will become 0.

** In 2^{nd} order systems, Marginally stable conditions come for L-C circuit.

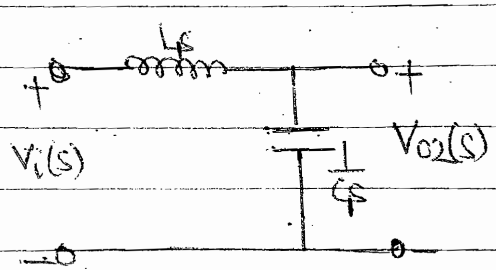


$\therefore \frac{V_{o1}(s)}{V_i(s)} = \frac{1/Cs}{Ls + 1/Cs}$

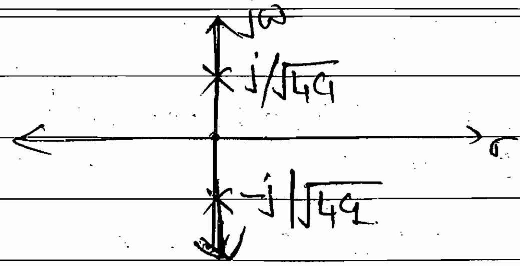
or $\frac{V_{o1}(s)}{V_i(s)} = \frac{1/LC}{s^2 + 1/LC}$



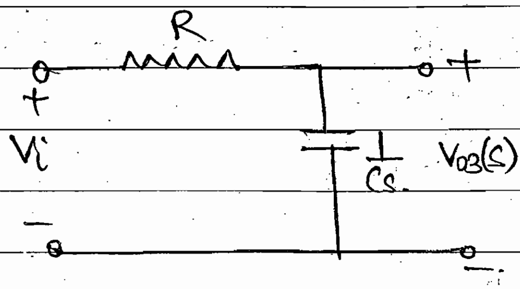
Similarly



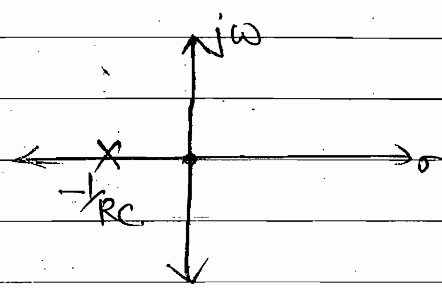
$\Rightarrow \frac{V_{o2}(s)}{V_i(s)} = \frac{1/L_1C_1}{s^2 + 1/L_1C_1}$



Again,

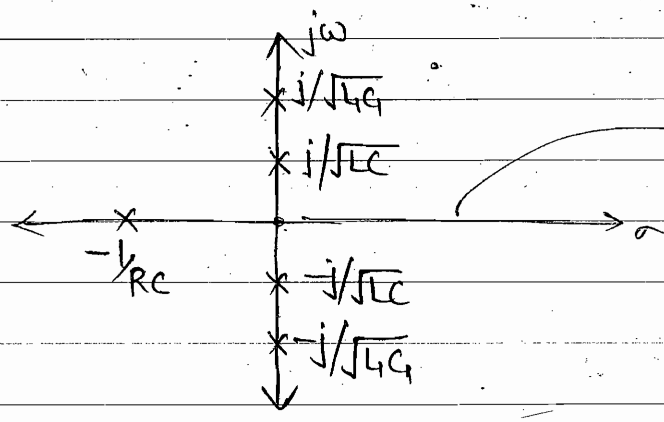


$$\frac{V_{03}(s)}{V_i(s)} = \frac{1/RC}{s + 1/RC}$$



* Cascading all the three networks,

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{(LCs^2+1)(L_1C_1s^2+1)(RCs+1)}$$



It looks same as our taken example. [Take $L=C=1$, $L_1=C_1=\frac{1}{2}$, $R=C=1$].

Ques: 3 The ORTF of a system is : $G(s)H(s) = \frac{K}{s(Ts+1)}$

Determine value of K and T so that all poles lie on left half side of line $s = -a$.

Ans:- 3 $q(s) = 1 + \frac{k}{s(Ts+1)} = Ts^2 + s + k = 0.$

or $s^2 + \frac{s}{T} + k/T = 0.$

$s + a = 0$

$z = s + a.$

$\boxed{s = z - a}$

s^2	1	k/T
s^1	$1/T$	
s^0	k/T	

Replace 's' with 's-a' :

$$q(s-a) = (s-a)^2 + \frac{(s-a)}{T} + k/T = 0.$$

$$\Rightarrow T[s^2 + a^2 - 2as] + (s-a) + k = 0$$

or $q(s-a) = Ts^2 + (1 - 2aT)s + (k - a + a^2T) = 0$

s^2	T	$k - a + a^2T$
s^1	$(1-2aT)$	0.
s^0	$k - a + a^2T$	

$\therefore 1 - 2aT > 0 \Rightarrow T < 1/2a$

$\therefore$ For system to be stable; $\boxed{0 < T < 1/2a}$ *

Also, $k - a + a^2T > 0 \Rightarrow k > a - a^2T$

$\therefore \boxed{k > a - a^2T}$ *

- For Marginally stable, $\boxed{T = 1/2a}$ *

* Checking boundary conditions:-

For $T = 0$,

s^2	0	$(k-a)$
s^1	1	
s^0	$(k-a)$.	

$\therefore \boxed{K > a}$ *

For $T = 1/2a$,

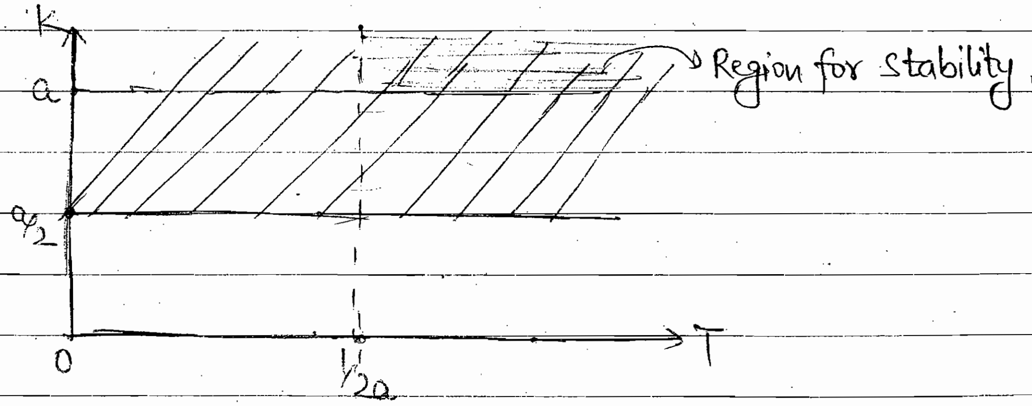
s^2	$1/2a$	$k - a/2$
s^1	0	
s^0	$k - a/2$	

$\therefore \boxed{K > a/2}$ *

$\therefore \quad Ts^2 + s + k = 0 \Rightarrow \frac{1}{2a}s^2 + s + k = 0.$

$\Rightarrow \quad s^2 + 2as + 2ak = 0.$

$\therefore \boxed{\xi = \sqrt{\frac{a}{2k}}}$ *



* For overall stability ; $0 < T < 1/2a$, $K > 0$.

Ques:- (4) The $q(s) = 1 + G(s)H(s) = s^4 + 5s^3 + 2s^2 + 5s + k = 0$. Determine the range of k for system to be stable.

Ans:- (4)

s^4	1	2	K
s^3	5	5	
s^2	1	K	
s^1	$(5-5K)/1$		
s^0	K		

• For stable system,

$$\boxed{0 < K < 1}\ *$$

• At $K=0$;

s^4	1	2	0
s^3	5	5	
s^2	1	0	
s^1	5		
s^0	0		

∴ For $K=0$ also, system is stable.

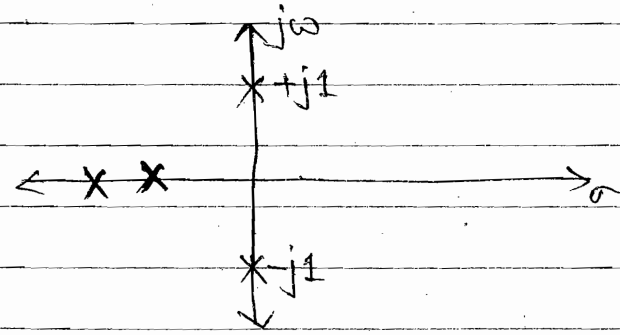
• At $K=1$;

s^4	1	2	1
s^3	5	5	
s^2	1	1	
s^1	0		
s^0	1		

→ Poles are at Image location

For $K=1$, System is Marginally stable.

$$A(s) = s^2+1=0 \Rightarrow \boxed{s=\pm j1}\ *$$



Ques:- (5) The OLTF of a system is: $G(s)H(s) = \dfrac{K(s+1)}{s^3+bs^2+3s+1}$

Determine the value of K and b, so that system oscillates at the frequency of 2 rad/sec.

Ans:- (5)

$$q(s) = 1+G(s)H(s) = s^3+bs^2+3s+1+Ks+K=0.$$

s^3	1	$(K+3)$
s^2	b	$(K+1)$
s^1	$\frac{b(K+3)-K-1}{b}$	
s^0	$K+1$	

$\therefore$ For poles at Image location, $s^1=0$.

$$\therefore \quad b(K+3) = K+1$$

$$\therefore \quad \boxed{b = \frac{K+1}{K+3}} *$$

Now, $A(s) = bs^2+(K+1)=0.$

$$\text{or} \quad s^2 = \frac{-(K+1)}{b} = \frac{-\cancel{(K+1)}}{\cancel{(K+1)}} \times (K+3).$$

$$\text{or} \quad s = \pm j\sqrt{(K+3)}.$$

As $\omega_n = 2$ rad/sec $\Rightarrow 2 = \sqrt{K+3}$.

$$\text{or} \quad K+3=4 \quad \Rightarrow \quad \boxed{K=1} * \quad \boxed{b=\tfrac{1}{2}} *$$

Ques:-(6) The OLTF of a system is: $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$. Determine the value of K so that settling time of II^{nd} order underdamped system will be 12 seconds.

Ans:-(6) $q(s) = 1+G(s)H(s) = s[s^2+4s+3]+K=0$

$$\text{or} \quad q(s) = s^3+4s^2+3s+K=0$$

s^3	1	3
s^2	4	K
s^1	$\frac{12-K}{4}$	
s^0	K	

As $t_s = \frac{4}{\xi\omega_n} = 12 \Rightarrow \xi\omega_n = \left(\frac{1}{3}\right)$.

$\therefore (s^2 + 2\xi\omega_n s + \omega_n^2)(s+p) = 0$.

$\Rightarrow s^3 + (2\xi\omega_n s^2) + \omega_n^2 s + ps^2 + 2\xi\omega_n ps + \omega_n^2 p = 0$

$\Rightarrow s^3 + (2\xi\omega_n + p)s^2 + s(\omega_n^2 + 2\xi\omega_n p) + \omega_n^2 p = 0. \quad \text{—(1)}$

Comparing eq-(1) with $q(s)$;

$2\xi\omega_n + p = 4, \qquad \omega_n^2 + 2\xi\omega_n p = 3 \quad ; \quad \omega_n^2 p = K$

$\Rightarrow \frac{2}{3} + p = 4, \qquad \omega_n^2 + \frac{2p}{3} = 3 \quad ; \quad \omega_n^2 p = K$

By solving this, $p = 4 - 2/3 = 10/3$.

$\omega_n^2 = 3 - 2\!\!/_9 = 7/9 \Rightarrow \omega_n = \frac{\sqrt{7}}{3}$.

$\therefore K = 7/9 \times 10/3 = 70/27$

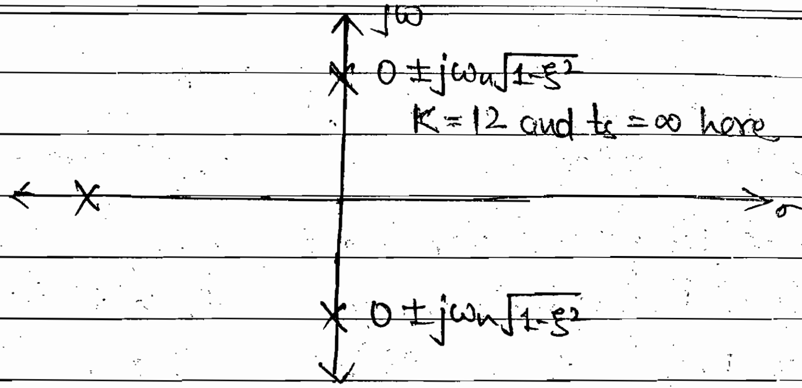
$\therefore \boxed{K = \frac{70}{27}}$ *

And $\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{2/3}$.

* From Routh-table, $s^1 = 0 \Rightarrow K = 12$.

and $s = \pm j\sqrt{3} \quad \therefore \omega_n = \sqrt{3}$ rad/sec.

As $t_s = \frac{4}{\xi\omega_n} \Rightarrow \xi\omega_n = \frac{1}{3} \Rightarrow \xi = \frac{1}{3\sqrt{3}}$



- For $t_s \neq 0$, Real part must exist. so, origin must be shifted to $-\frac{1}{3}$.

$\therefore \quad q(s-\frac{1}{3}) = (s-\frac{1}{3})^2 + 4(s-\frac{1}{3})^2 + 3(s-\frac{1}{3}) + K = 0.$

so, $\quad K = \frac{70}{27}$ will be got after solving above equation.

and $s = \pm j\sqrt{2/3}$, this $\sqrt{2/3}$ must be ω_d.

Chapter :- 6

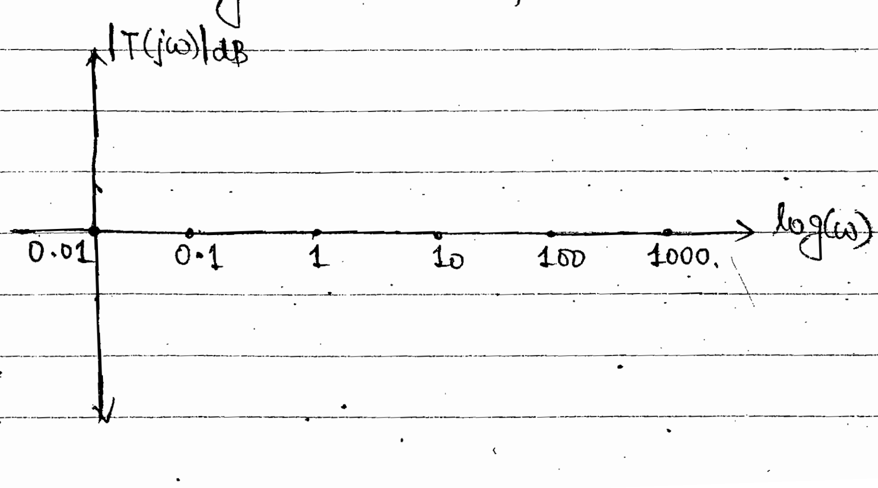
BODE PLOT

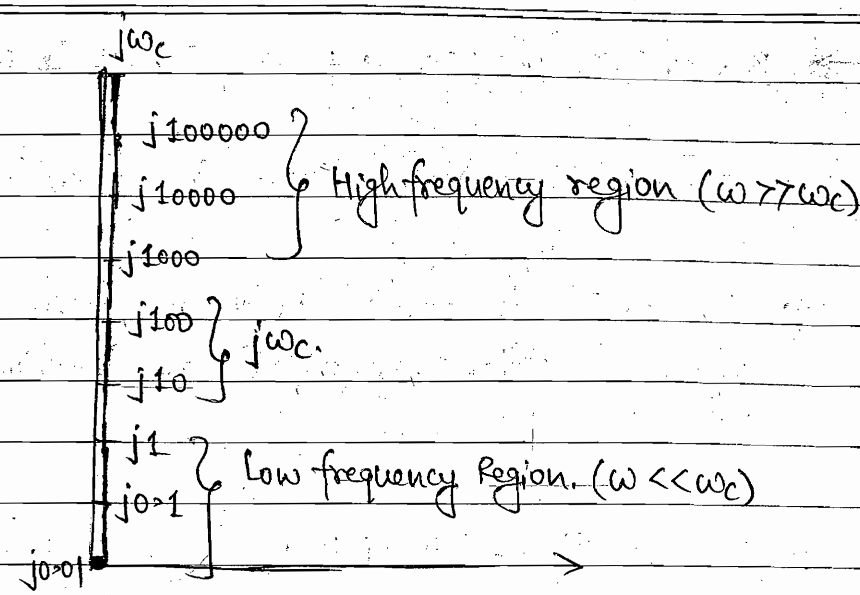
** • In case of Bode Plot, variable is Positive frequency line of s-plane. For each frequency, we will calculate Magnitude as well as Phase Angle and then we will connect Locus of all points and that plot will be known as Bode Plot.

* In Bode Plot, Magnitude plot is Logarithmic plot and it is in dB and Phase angle plot is in degrees.

** As Magnitude plot is logarithmic plot, so its abscissa is in terms of $\log(\omega)$ and as we know that $\log(0)$ is invalid quantity, so it is not possible to calculate Magnitude at $\omega = 0$. Hence, origin of Bode plot will never be 0, it will be 0.1, 0.001 ---- however, Phase angle plot is in terms of ω and $\omega = 0$ is a valid quantity but in Bode Plot, for each frequency, we calculate Magnitude as well as Phase Angle. If Magnitude is undefined at $\omega = 0$, then phase angle will also be undefined at $\omega = 0$.

** Bode Plot defines Relative Stability i.e stability will be defined in terms of Gain Margin and Phase Margin. As we know that Bode Plot is valid only for Minimum phase system and for closed loop system to be stable, both Gain Margin and Phase Margin should be positive.



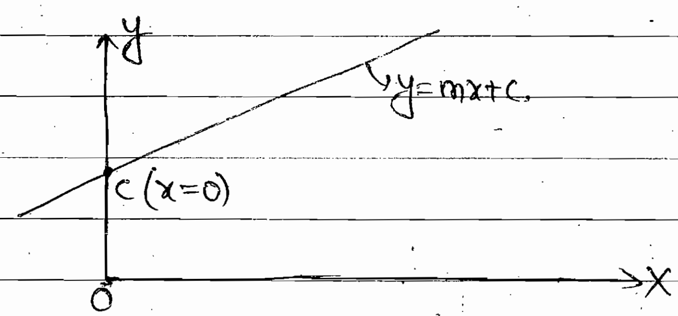


Now, $\left(\frac{P_o}{P_i}\right)_{dB} = 10\log_{10}\left(\frac{V_o}{V_i}\right)^2 = 20\log_{10}\left(\frac{V_o}{V_i}\right) = 20\log_{10}\left(\frac{I_o}{I_i}\right) = 20\log_{10}[T(j\omega)]$.

$\left(\frac{P_o}{P_i}\right)_{octave} = 3\log_2\left[\frac{P_o}{P_i}\right] = 3\log_2\left[\frac{P_o}{P_i}\right]^2 = 6\log_2\left(\frac{V_o}{V_i}\right) = 6\log_2[T(j\omega)]$.

- In case of Bode Plot, we use equation of Line: $y = mx + c$, where m = slope and c = y-intercept or intercept at ($\omega = 1$).

* This intercept itself represents DC gain of the system.



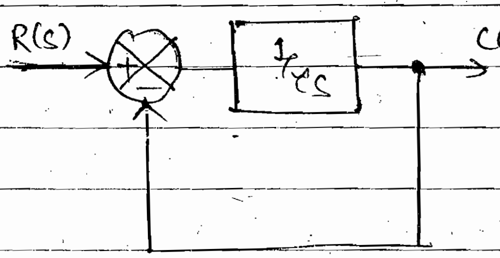
$$\therefore \boxed{|T(j\omega)|_{dB} = m \cdot \log(\omega) + c} \; *$$

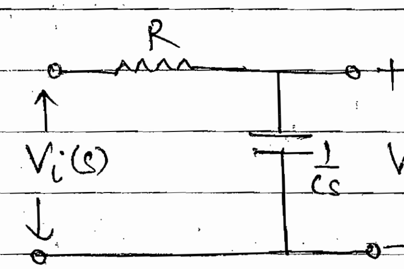
At $\omega = 1$, $\boxed{K = |T(j1)| = c = \text{DC Gain}}$ *.

Bandwidth of open Loop and closed Loop Systems :-

* Bandwidth is not defined for system whose pole lies on origin.
* Bandwidth is defined only for system whose poles lie on any point other than origin and location of Dominant pole itself is Bandwidth.

* Bandwidth of Closed Loop System > Bandwidth of Open Loop System.





$$\frac{C(s)}{R(s)} = \frac{(1/\tau)}{(s+1/\tau)} = \frac{1}{\tau s+1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{(1/RC)}{(s+1/RC)} = \frac{1}{RCs+1}$$

Put $\boxed{s=j\omega}$ *

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{RC(j\omega)+1}$$

or $$\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

Now, * For $\omega << \omega_c$, i.e $\omega \to 0$ $\quad \therefore \frac{V_o}{V_i} = 1 \Rightarrow V_o = V_i$

or $\boxed{P_o = P_i}$ *. i-e Power dissipation across Resistor $(P_R) = 0$.

* For $\omega >> \omega_c$, i.e $\omega \to \infty$. $\quad \frac{V_o}{V_i} = 0 \Rightarrow V_c = 0 \Rightarrow \boxed{P_c = 0}$ *.

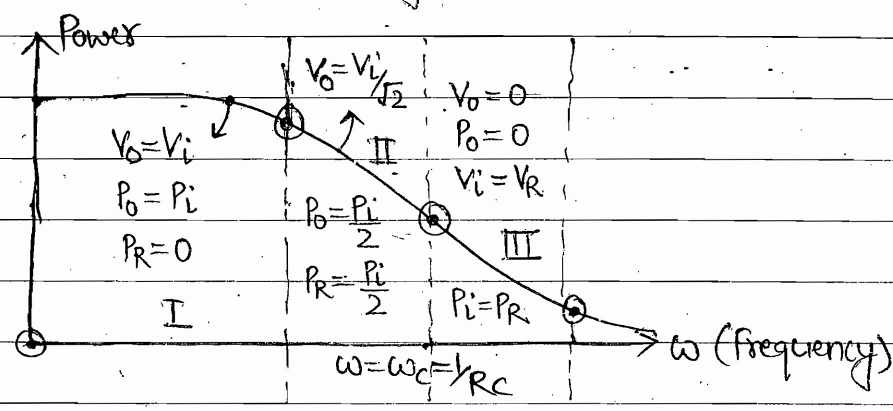
$\therefore$ $\boxed{P_i = P_R}$ * All power is dissipated by Resistor.

For $\omega = 1/RC$, $\frac{V_o}{V_i} = \frac{1}{\sqrt{2}} \Rightarrow \boxed{V_i = \sqrt{2}\,V_o}$ *

$\boxed{P_o = \frac{1}{2} P_i}$ * $\boxed{P_R = \frac{1}{2} P_i}$ *

So, half power is stored by capacitor and half power is being dissipated by Resistor.

* Graph of Power vs Frequency :-



* • In Region II, in comparison to region 1, P_o is decreasing and P_R is increasing.
* • Again, in Region III, P_o further decreases to 0 and $P_i = P_R$ here.

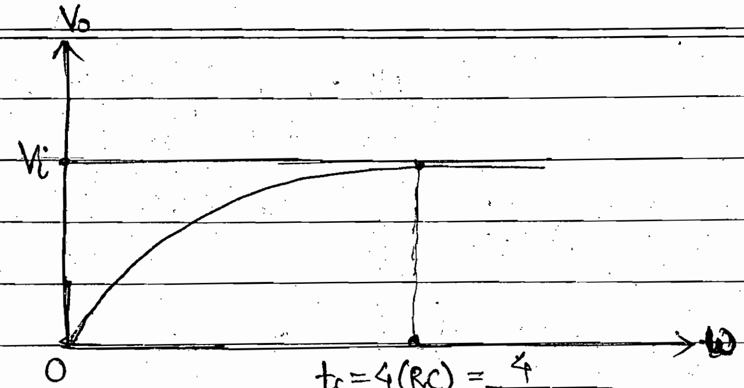
* At the most, we can select frequencies upto 0 to ω_c because after ω_c, power reduces by more than $\frac{1}{2}$.

$\therefore$ Bandwidth $= \omega_H - \omega_L \Rightarrow \Delta\omega = \omega_c - 0$

$\therefore$ $\boxed{\text{Bandwidth } (\Delta\omega) = \omega_c = 1/RC = \text{location of Pole}}$ *.

Now, as $\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\sqrt{2}}$, converting it into dB,

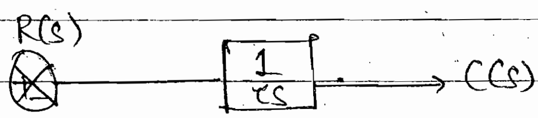
$\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right|_{dB} = 20\log_{10}\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \boxed{\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right|_{dB} = 3\text{ dB}}$ *



$t_s = 4(RC) = \dfrac{4}{\text{Bandwidth}}$

- Higher the bandwidth, faster will be the response of the system.

* Bandwidth of system with pole at origin :-

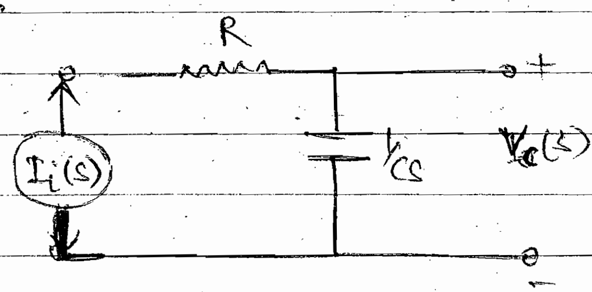


$$\therefore \frac{C(s)}{R(s)} = \frac{(1/\tau)}{s} \quad \text{—— (1)}$$

For step input : $R(s) = A/s$

$$\therefore c(t) = \frac{A}{\tau} \cdot t.$$

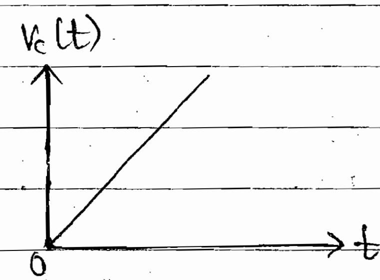
Now, this is equivalent to :-



$$\frac{V_c(s)}{I_i(s)} = \frac{(1/C)}{s}$$

For Step current, $I_i(s) = I_0/s$.

$$\therefore V_c(t) = \frac{I_0}{C} \cdot t$$

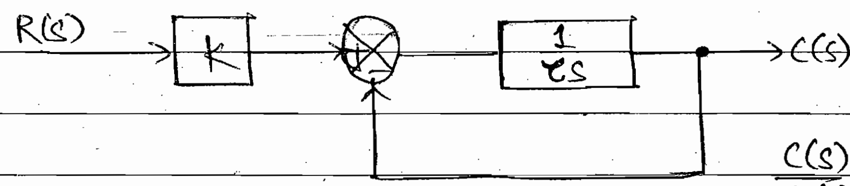


- Here, P_R and P_C are independent of Frequency, so as they are constant, so bandwidth is not defined.

From eq-(1), As $\tau = 1/\omega_c$

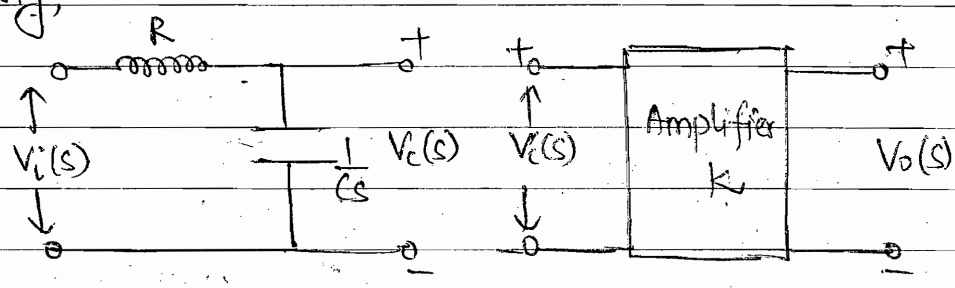
$$\therefore \frac{C(s)}{R(s)} = \frac{1}{(s/\omega_c + 1)}, \quad \omega_c \rightarrow \text{Corner frequency of Pole.}$$

* Making closed loop system an Open Loop :-



$$\frac{C(s)}{R(s)} = \frac{(k/\tau)}{(s+1/\tau)} = \frac{k}{\tau s+1}$$

Similarly,



$$\frac{V_o(s)}{V_c(s)} = K$$

$\therefore$ Cascading above two systems,

$$\frac{V_c(s)}{V_i(s)} = \frac{1/RC}{(s+1/RC)}$$

$$\Rightarrow \frac{\cancel{V_c(s)}}{V_i(s)} \times \frac{V_o(s)}{\cancel{V_c(s)}} = \frac{K/RC}{s+1/RC}$$

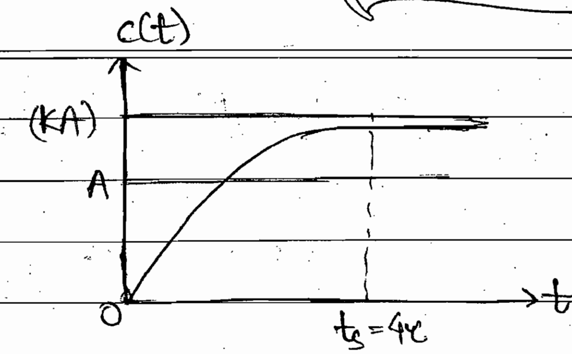
$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{(K/\tau)}{(s+1/\tau)}} *$$

$\Rightarrow$ Applying Step Input : $r(t) = A \cdot u(t) = A/s$.

$$C(s) = \frac{(K/\tau) \cdot A}{s(s+1/\tau)} = \frac{K \cdot A}{s} - \frac{(K \cdot A)}{(s+1/\tau)}.$$

$$\boxed{c(t) = K \cdot A\left[1 - e^{-t/\tau}\right]} *.$$

$\therefore$ At $c(t_s = 4\tau) = K \cdot A.$

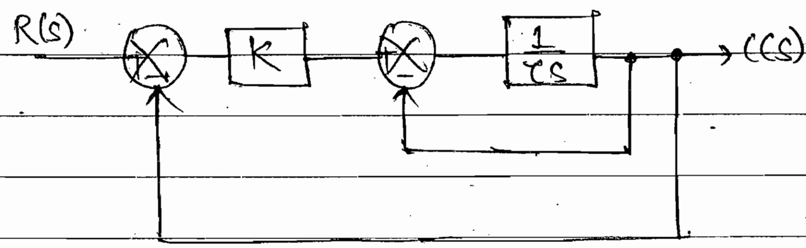


Here, DC Gain $= K$

Time Constant $= \tau$

Bandwidth $= 1/\tau$.

Now, Making this open loop a closed loop system by providing unity feedback:



$$\therefore \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1 + K} = \frac{(K/\tau)}{s + (1+K/\tau)}$$

So, Bandwidth $= \left(\frac{1+K}{\tau}\right)'' = (1 + K/\tau)$. (> open loop system's).

* For step input, $R(s) = A/s$.

$$\therefore C(s) = \frac{\left(\frac{K}{K+1}\right)A}{s} - \frac{\left(\frac{K}{K+1}\right)A}{s + (1+K/\tau)}.$$

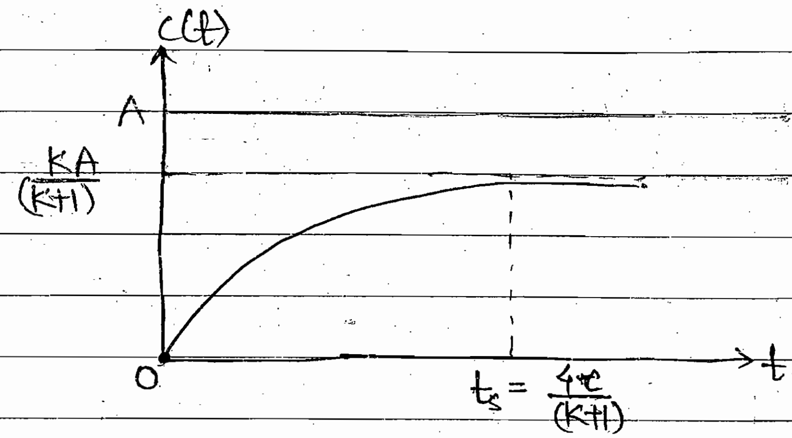
$$\therefore c(t) = \frac{K \cdot A}{(K+1)}\left[1 - e^{-\frac{(K+1)}{\tau}t}\right].$$

$\therefore$ At $t = t_s = 4\tau$, $c(4\tau) = \frac{K \cdot A}{(K+1)}$.

Here; DC Gain $= \frac{K}{K+1}$ (↓es)

Time Constant $= \frac{\tau}{K+1}$ (↓es)

Bandwidth $= \frac{(K+1)}{\tau}$. (↑es).



* So, conclusions are :-

(i) Bandwidth of closed loop > Bandwidth of open loop system.

(ii) with increasing the feedback, DC gain decreases, t_s decreases, Bandwidth increases.

(iii) Closed loop systems have more speed or faster response than Open loop systems.

*→ <u>How to Plot Bode Plot</u> :-

1> $T(s) = K.$

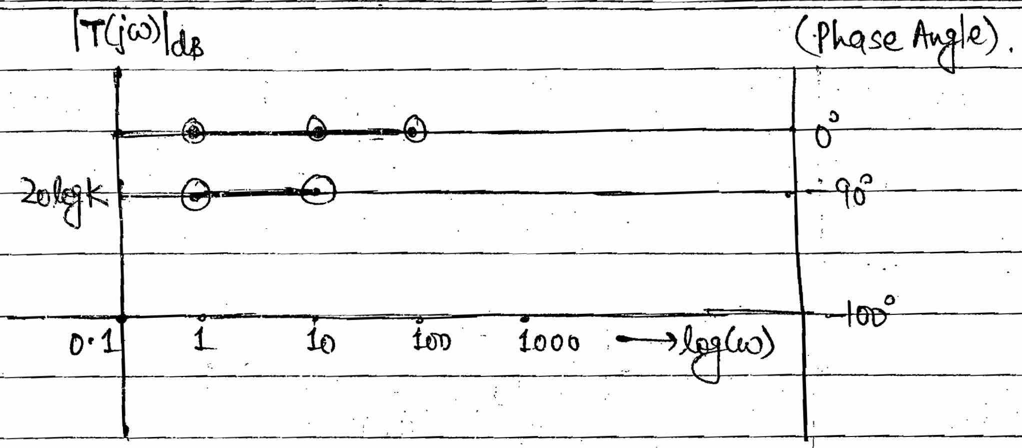
$\Rightarrow T(j\omega) = K + j0 \Rightarrow |T(j\omega)| = \sqrt{K^2} = K.$

or $|T(j\omega)|_{dB} = 20 \log K.$

$\therefore \boxed{|T(j\omega)|_{dB} = 20 \log (K)}$ * $\boxed{\angle T(j\omega) = 0^\circ}$ *

Now, $|T(j0.1)| = 20 \log (K).$, $\angle T(j0.1) = 0^\circ$

$|T(j1)| = 20 \log (K)$ $\angle T(j0.01) = 0^\circ.$



2) $T(s) = 1/s$.

$\Rightarrow T(j\omega) = \frac{1}{j\omega}$

$|T(j\omega)| = \frac{1}{\omega}$, $\angle T(j\omega) = -90^\circ$.

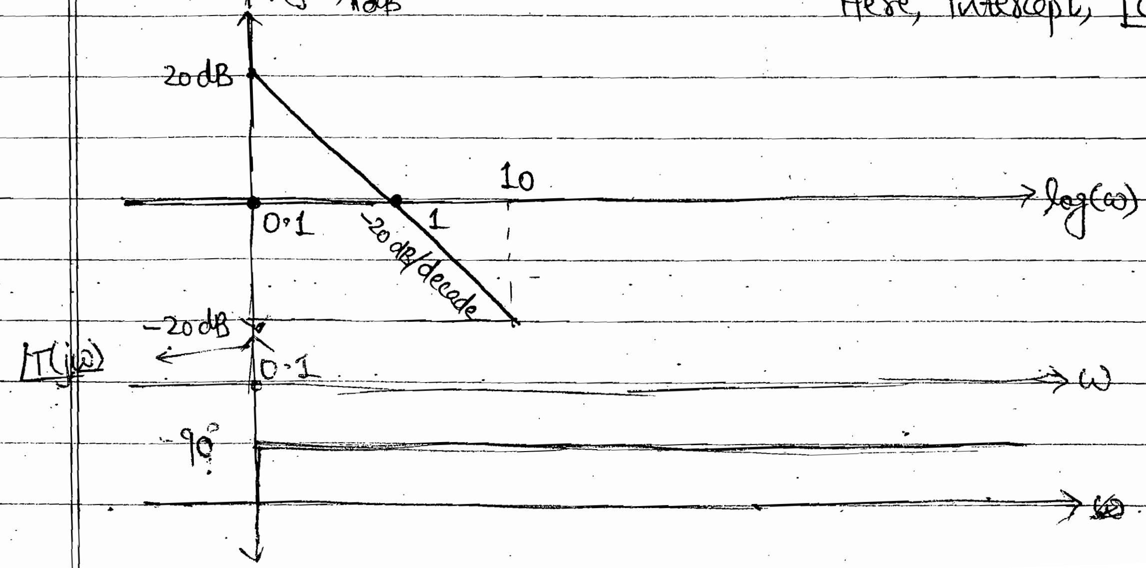
$\therefore$ $\boxed{|T(j\omega)|_{dB} = -20\log(\omega)}$ * $\boxed{\angle T(j\omega) = -90^\circ}$ *

$\therefore |T(j0{\cdot}1)| = -20\log(0{\cdot}1) = 20\text{ dB}$. ; $\angle T(j0{\cdot}1) = -90^\circ$

$|T(j1)| = 0\text{ dB}$; $\angle T(j1) = -90^\circ$

$|T(j10)| = -20\text{ dB}$. ; $\angle T(j10) = -90^\circ$.

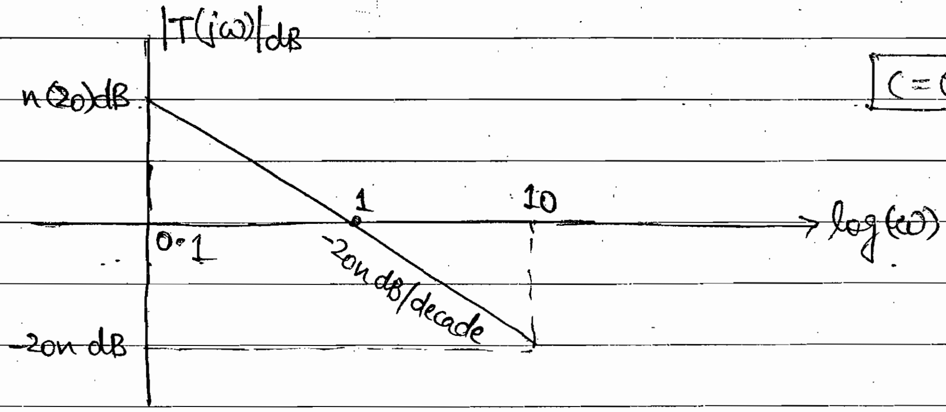
Here, intercept, $\boxed{C=0}$ *



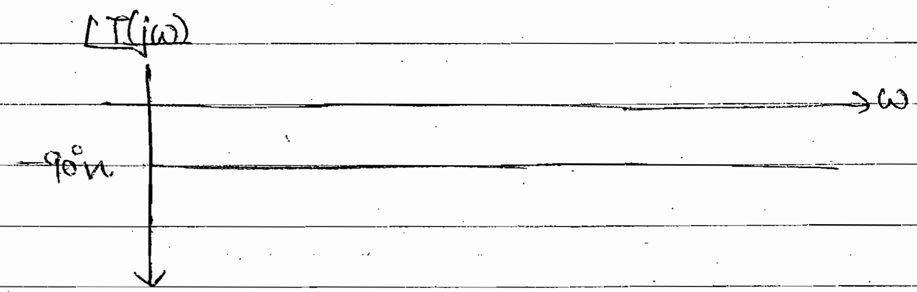
3) $T(s) = \frac{1}{s^n}$

$\Rightarrow T(j\omega) = \frac{1}{(j\omega)^n} \Rightarrow |T(j\omega)| = \frac{1}{\omega^n}$

$\therefore \boxed{|T(j\omega)|_{dB} = (-n20)\log(\omega)}$ * $\boxed{\angle T(j\omega) = -90^\circ n}$ *



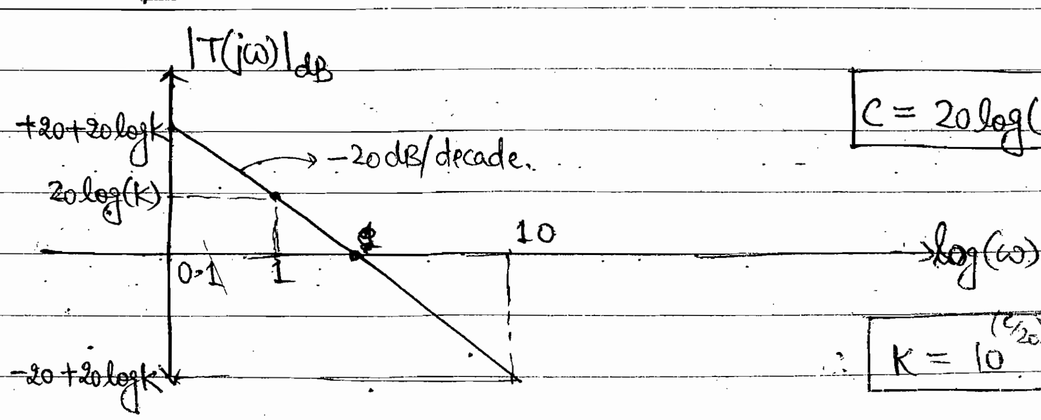
$\boxed{C=0}$ *.



4) $T(s) = \frac{K}{s}$

$\Rightarrow T(j\omega) = \frac{K}{j\omega} \Rightarrow |T(j\omega)| = \frac{K}{\omega}$

$\therefore \boxed{|T(j\omega)|_{dB} = 20\log(K) - 20\log(\omega)}$ * $\boxed{\angle T(j\omega) = -90^\circ}$ *



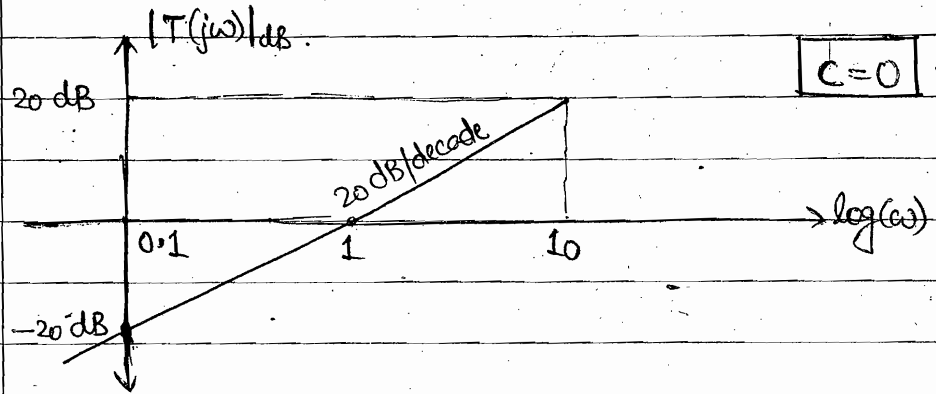
$\boxed{C = 20\log(K)}$ *.

$\therefore \boxed{K = 10^{(C/20)}}$ *

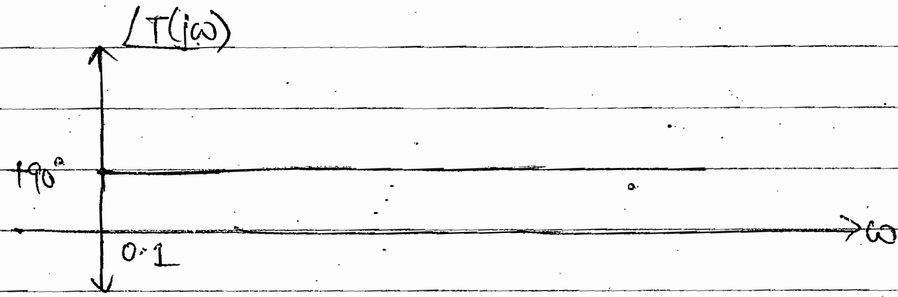
5) $T(s) = s$.

$\Rightarrow T(j\omega) = j\omega$.

$\boxed{|T(j\omega)|_{dB} = 20\log(\omega)}$ * $\boxed{\angle T(j\omega) = 90^\circ}$ *.



$\boxed{C = 0}$ *.



6) $T(s) = s^n$

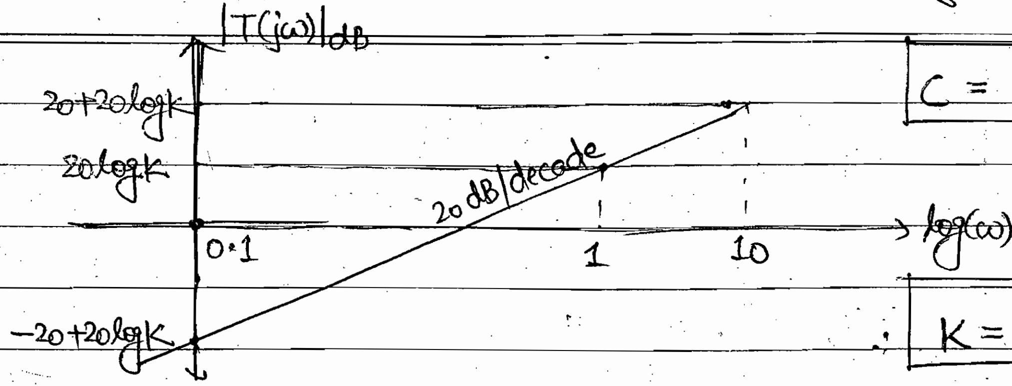
$\Rightarrow T(j\omega) = (j\omega)^n \Rightarrow |T(j\omega)| = \omega^n$.

$\boxed{|T(j\omega)|_{dB} = (20n)\log(\omega)}$ * $\boxed{\angle T(j\omega) = 90^\circ n}$ *.

7) $T(s) = Ks$.

$\Rightarrow T(j\omega) = K(j\omega) \Rightarrow |T(j\omega)| = K\omega$.

$\therefore \boxed{|T(j\omega)|_{dB} = 20\log(K) + 20\log(\omega)}$ * $\boxed{\angle T(j\omega) = 90^\circ}$ *



$$\boxed{C = 20\log(K)} \ *$$

$$\therefore \boxed{K = 10^{(C/20)}} \ *$$

* Gain (K) will only affect Magnitude but not phase.

8) $T(s) = Ks^n$

$\Rightarrow T(j\omega) = K(j\omega)^n \quad \Rightarrow \quad |T(j\omega)| = K\omega^n$

$$\therefore \boxed{|T(j\omega)|_{dB} = (20n)\log(\omega) + 20\log K} \ *$$

9) $T(s) = \dfrac{1}{(s/\omega_c + 1)}$

$\Rightarrow T(j\omega) = \dfrac{1}{\left(\frac{j\omega}{\omega_c} + 1\right)} \quad \Rightarrow \quad |T(j\omega)| = \dfrac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$

$|T(j\omega)|_{dB} = -20\log\left(\sqrt{1 + \frac{\omega^2}{\omega_c^2}}\right) \quad ; \quad \angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$

* So, For low frequency region, $\omega \ll \omega_c \Rightarrow \frac{\omega^2}{\omega_c^2} \to 0$.

$$\therefore \boxed{|T(j\omega)|_{dB} = 0} \ * \qquad \boxed{\angle T(j\omega) = 0^\circ} \ *$$

* For $\omega = \omega_c$, $\omega/\omega_c = 1$

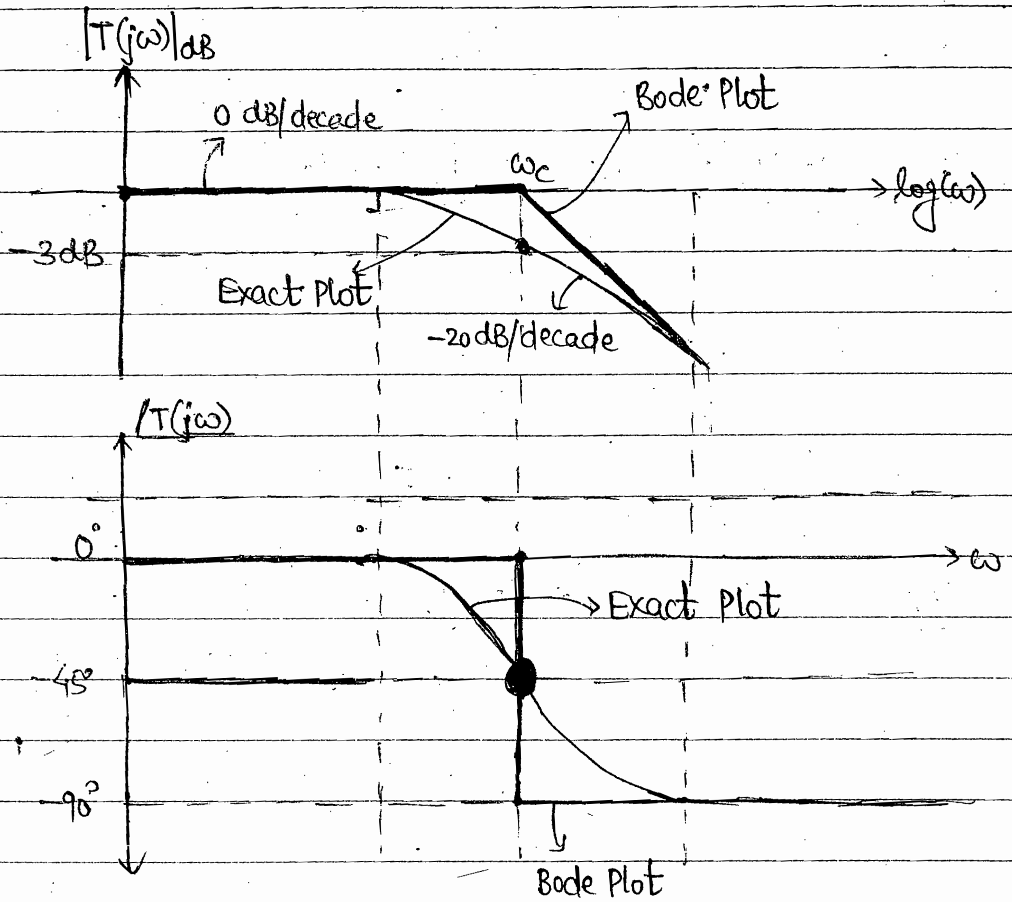
$$\therefore \boxed{|T(j\omega)|_{dB} = -3\,dB} \ * \qquad \boxed{\angle T(j\omega) = -45^\circ} \ *$$

* For Higher frequency region, $\omega \gg \omega_c$ or $\frac{\omega^2}{\omega_c^2} \gg 1 \simeq \infty$

$$\boxed{|T(j\omega)|_{dB} = -20\log(\omega) + 20\log(\omega_c)} \ * \qquad C = 20\log(\omega_c).$$

Also, $\boxed{\angle T(j\omega) \cong -90^\circ}$ *.

** Exact Plot :-



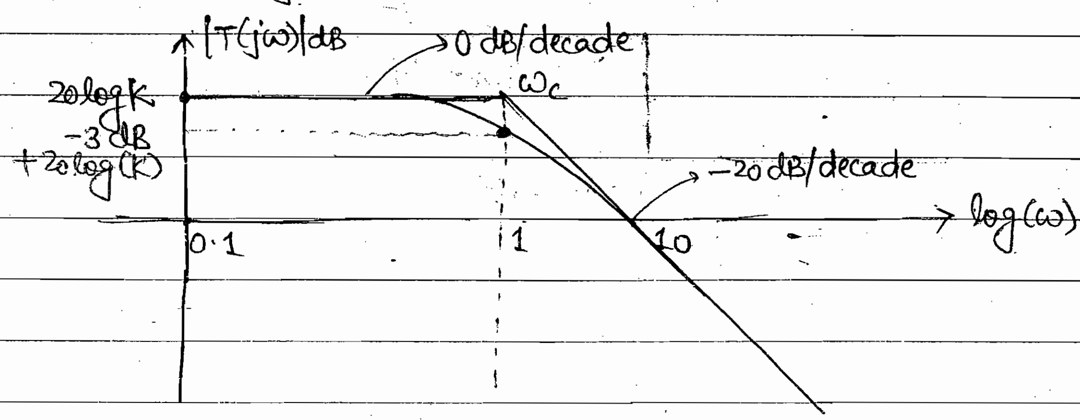
10) $$T(s) = \frac{K}{\left(\frac{s}{\omega_c}+1\right)}.$$

$$\Rightarrow T(j\omega) = \frac{K}{\left(\frac{j\omega}{\omega_c}+1\right)} \Rightarrow |T(j\omega)| = \frac{K}{\sqrt{1+(\omega/\omega_c)^2}}.$$

$$|T(j\omega)|_{dB} = 20\log(K) - 20\log\left(\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}\right)$$

- $\omega << \omega_c$, $\boxed{|T(j\omega)| = 20\log(K)}$ *
- $\omega = \omega_c$, $|T(j\omega)| = -3 + 20\log(K)$
- $\omega >> \omega_c$, $\boxed{|T(j\omega)| = 20\log(K\cdot\omega_c) - 20\log(\omega)}$ * $C = 20\log(\omega_c K)$

* Phase Angle plot will remain same.



Q.11) $T(s) = \left(\frac{s}{\omega_c} + 1\right)$

$\Rightarrow T(j\omega) = \left[\frac{j\omega}{\omega_c} + 1\right] \Rightarrow |T(j\omega)| = \sqrt{1 + (\omega^2/\omega_c^2)}$

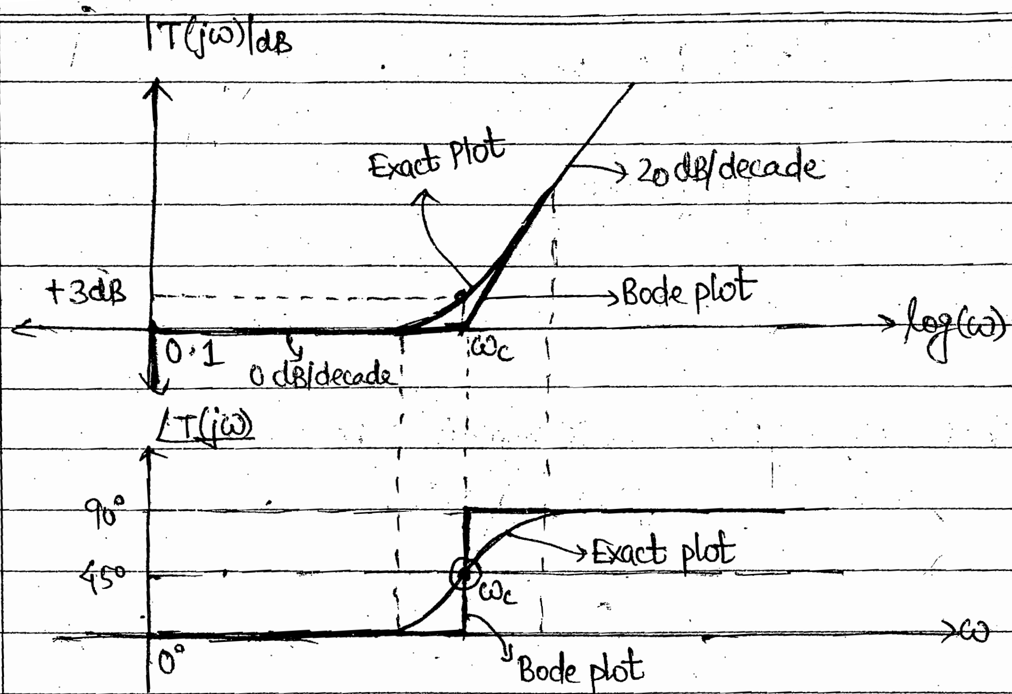
$\therefore |T(j\omega)|_{dB} = 20\log\left(\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}\right)$; $\angle T(j\omega) = \tan^{-1}(\omega/\omega_c)$

(i) $\omega << \omega_c$, $|T(j\omega)|_{dB} = 0$ dB. ; $\angle T(j\omega) = 0^\circ$

(ii) $\omega = \omega_c$, $|T(j\omega)|_{dB} = +3$ dB ; $\angle T(j\omega) = 45^\circ$.

(iii) $\omega >> \omega_c$; $\boxed{|T(j\omega)|_{dB} = 20\log(\omega) - 20\log(\omega_c)}$ * $\boxed{\angle T(j\omega) = 90^\circ}$

$\boxed{C = -20\log(\omega_c)}$ *.



12, $T(s) = K\left(\frac{s}{\omega_c}+1\right)$.

$$\Rightarrow T(j\omega) = K\left(\frac{j\omega}{\omega_c}+1\right) \Rightarrow |T(j\omega)| = K\sqrt{1+(\omega/\omega_c)^2}$$

$$|T(j\omega)|_{dB} = 20\log(K) + 20\log\left(\sqrt{1+(\omega/\omega_c)^2}\right)$$

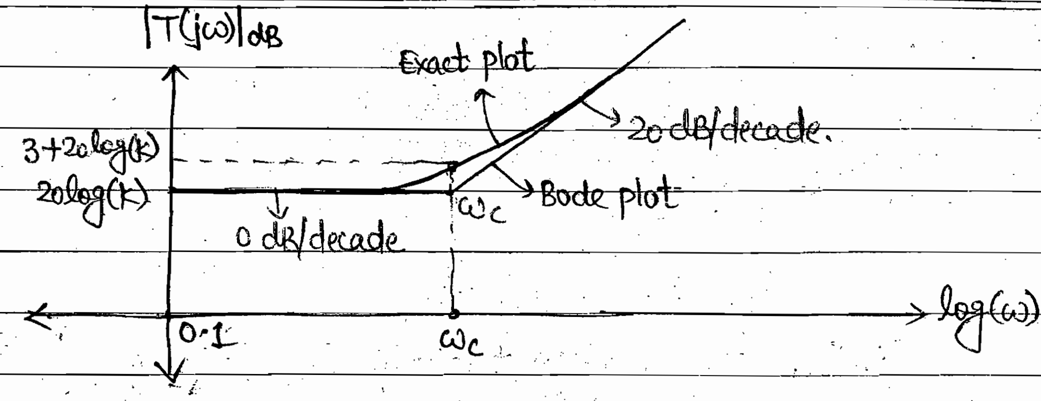
(i) $\omega << \omega_c$; $|T(j\omega)|_{dB} = 20\log(K)$.

(ii) $\omega = \omega_c$; $|T(j\omega)|_{dB} = 3\text{ dB} + 20\log(K)$.

(iii) $\omega >> \omega_c$; $\boxed{|T(j\omega)|_{dB} = 20\log(\omega) + 20\log(K/\omega_c)}$ *

$$C = 20\log(K/\omega_c)$$

** Phase Angle plot will remain same.



Date -08-2014

Determination of DC Gain from Bode Plot :-

$$T(s) = \frac{1}{s^2}$$

$$T(j\omega) = \frac{1}{-\omega^2} \Rightarrow |T(j\omega)| = 1/\omega^2$$

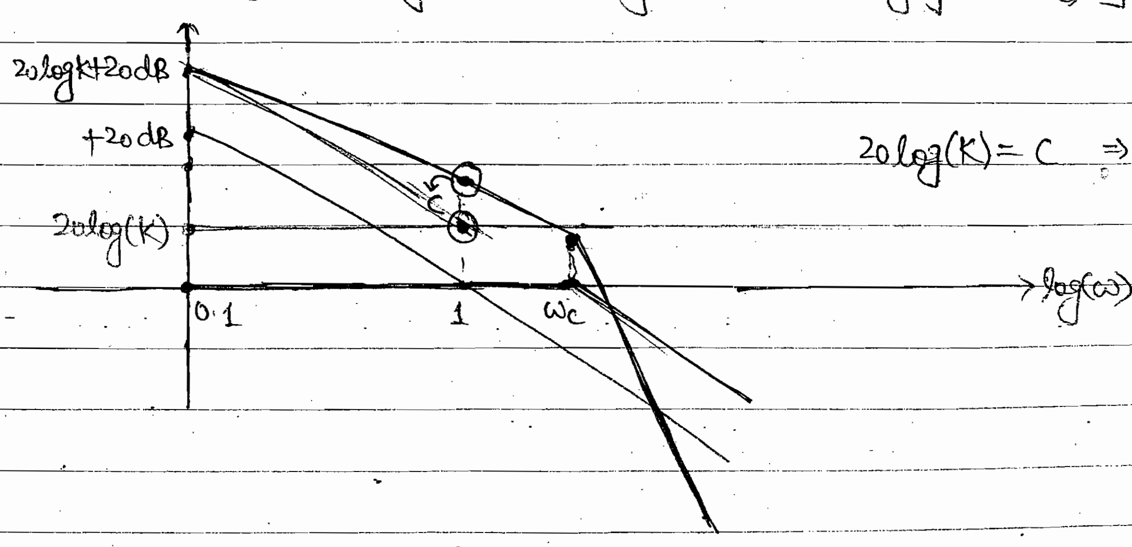
$$\boxed{|T(j\omega)|_{dB} = -40\log(\omega)} \ *$$

Case:-I) When first corner frequency $(\omega_{c_1}) > 1$ rad/sec. :-

$$T(s) = \frac{K}{s\left[s/\omega_c + 1\right]}$$

$$T(j\omega) = \frac{K}{j\omega\left[j\omega/\omega_c + 1\right]} \Rightarrow |T(j\omega)| = \frac{K}{\omega\sqrt{1+(\omega/\omega_c)^2}}$$

$$|T(j\omega)|_{dB} = 20\log(K) + (-20\log(\omega)) + \left[-20\log\sqrt{1+(\omega/\omega_c)^2}\right].$$

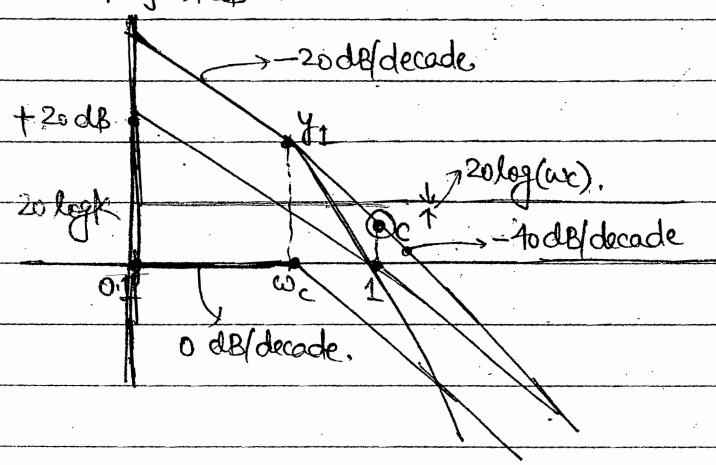


$$20\log(K) = C \Rightarrow K = (10)^{C/20}.$$

Case-III: If corner frequency $(\omega_c) < 1$ rad/sec

$$T(s) = \frac{K}{s\left[\frac{s}{\omega_c}+1\right]}$$

$$|T(j\omega)|_{dB} = 20\log(K) + (-20\log\omega) + \left[-20\log\left[\sqrt{1+(\omega/\omega_c)^2}\right]\right].$$



$$20\log(K) = c - [20\log\omega_c].$$

$$K = (10)^{(c-20\log\omega_c)/20}$$

Using equation of line,

$$\frac{20 + 20\log(K) - y_1}{\log(0.1) - \log(\omega_c)} = -20.$$

$$\Rightarrow \quad 20 + 20\log(K) - y_1 = -20\log(0.1) + 20\log(\omega_c).$$

$$\Rightarrow \quad \cancel{20} + 20\log(K) - y_1 = \cancel{20} + 20\log(\omega_c)$$

$$\Rightarrow \quad \boxed{y_1 = 20\log(K/\omega_c).} \; *$$

Again, $\quad c - [20\log(K) - 20\log(\omega_c)] = -40[\log 1 - \log\omega_c]$

$$\Rightarrow \quad c = 40\log(\omega_c) - 20\log(\omega_c) + 20\log(K)$$

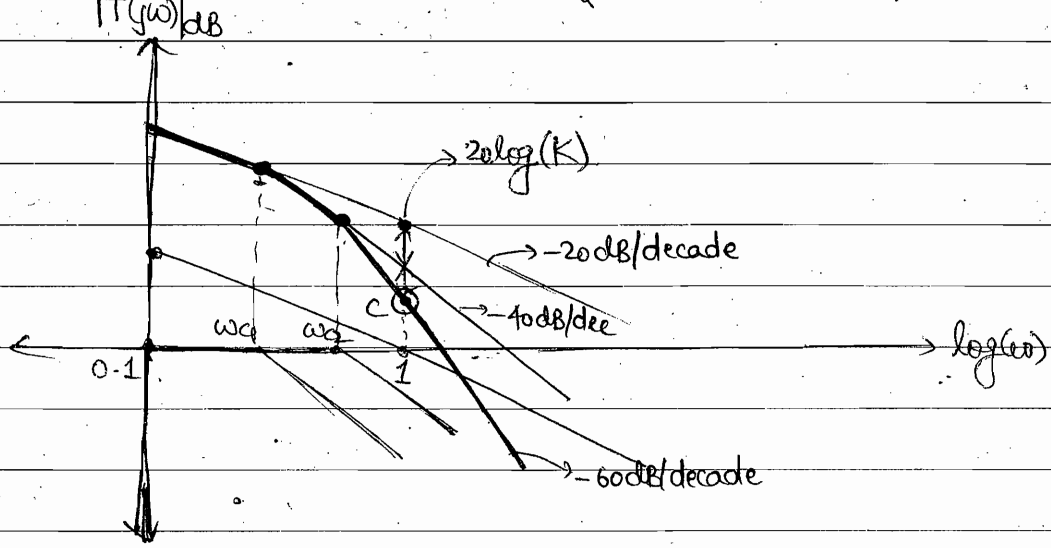
$$\therefore \quad c = 20\log(\omega_c) + 20\log(K).$$

$\therefore \Delta C = 20\log(K) - c$

$$\boxed{\Delta C = -20\log(\omega_c)} * . \quad (-ve \text{ is because } \omega_c < 1).$$

Case :- III : When more than one corner frequencies are there.

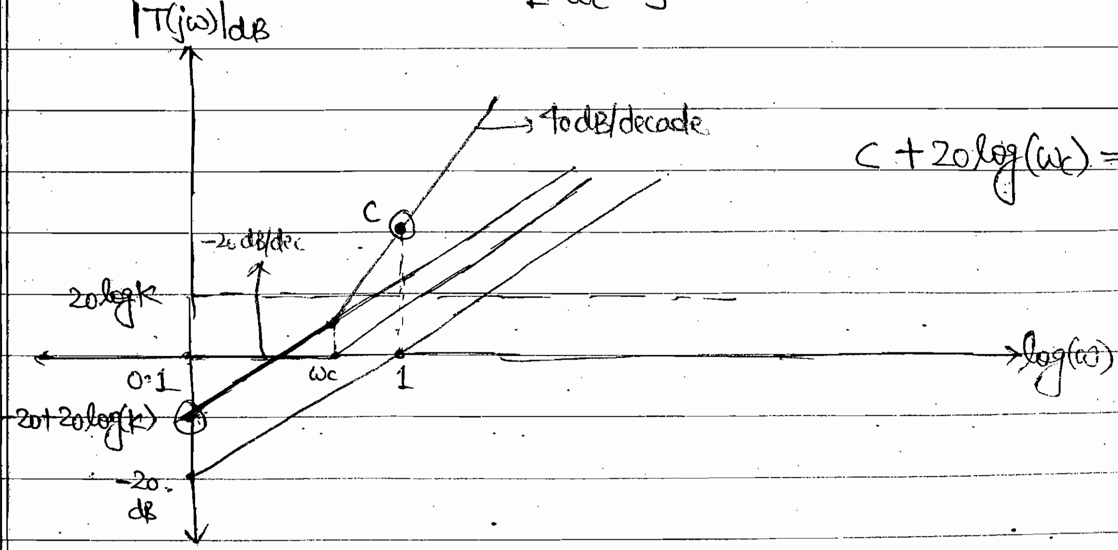
$$T(S) = \frac{K}{S\left[\frac{S}{\omega_{c_1}}+1\right]\left[\frac{S}{\omega_{c_2}}+1\right]}$$



$$\therefore 20\log(K) = c - [20\log(\omega_{c_1})] - [20\log(\omega_{c_2})].$$

Case : IV : when zero is also present in system :-

$$T(S) = K(S) \cdot \left[\frac{S}{\omega_c}+1\right].$$



$$c + 20\log(\omega_c) = 20\log(K)$$

GAIN MARGIN AND PHASE MARGIN :-

Gain Margin :-

- Gain Margin defines the stability of closed loop transfer function from its open loop plot. In Ratio, Gain Margin is inverse of Magnitude of open loop Transfer function at Phase crossover Frequency.

- Phase crossover frequency is that frequency where phase angle of OLTF becomes $-180°$.

* In dB, Gain Margin is negative of Magnitude at phase crossover frequency.
* If Open Loop system is Minimum phase system, for closed loop system to be stable, Gain Margin should be positive.

$$G(s)H(s) = \frac{K}{[s/\omega_{c_1}+1][s/\omega_{c_2}+1][s/\omega_{c_3}+1]}$$

$$|G(j\omega)H(j\omega)| = 20\log|G(j\omega)\cdot H(j\omega)|.$$

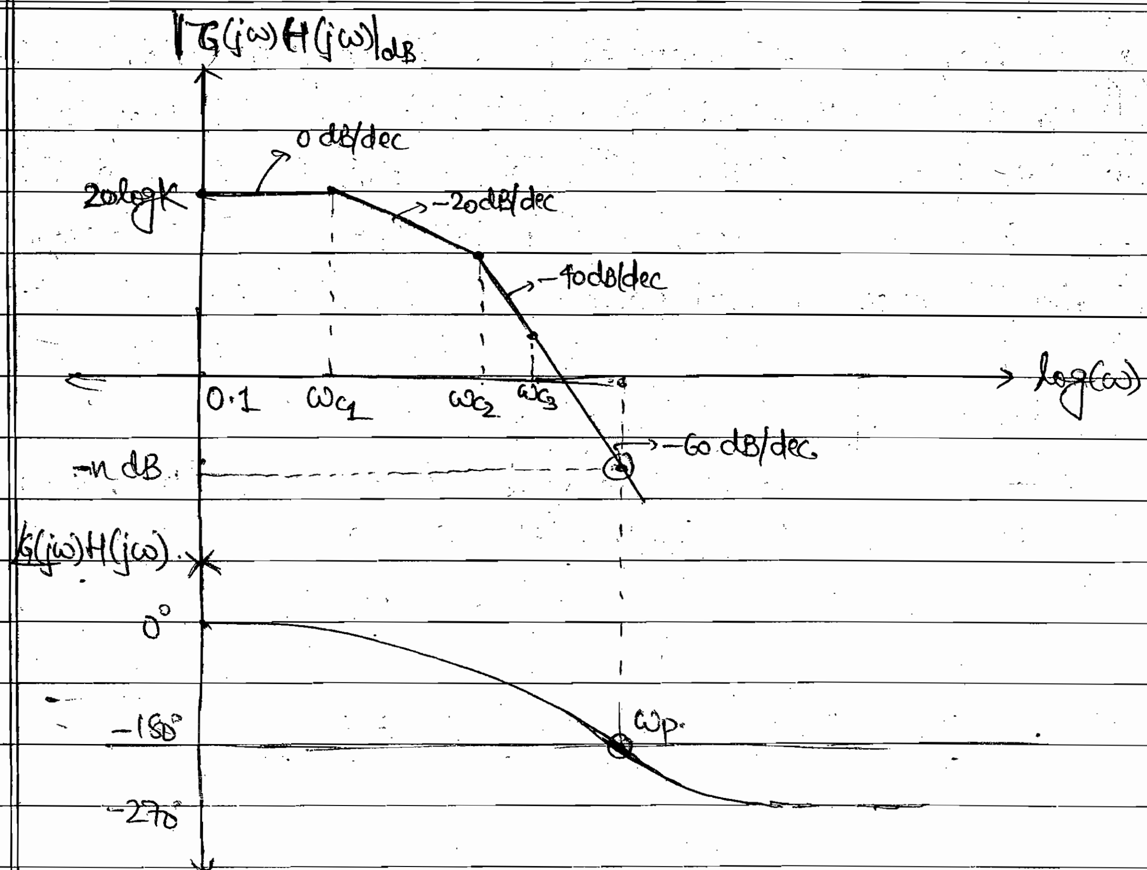
$$= 20\log(K) - 20\log\left(\sqrt{1-(\omega/\omega_{c_1})^2}\right) - 20\log\left(\sqrt{1-(\omega/\omega_{c_2})^2}\right).$$

$$-20\log\left(\sqrt{1-(\omega/\omega_{c_3})^2}\right).$$

$$\angle G(j\omega)\cdot H(j\omega) = -\tan^{-1}(\omega/\omega_{c_1}) - \tan^{-1}(\omega/\omega_{c_2}) - \tan^{-1}(\omega/\omega_{c_3}).$$

** From Bode Plot, Phase Margin cannot be calculated. So, we draw always Exact plot of Phase Angle plot, and this will give range of ω_p where $\omega_p = -180°$

i.e $\omega_{c_2} < \omega_p < \omega_{c_3}$ (In above case).



$$\text{Gain Margin} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_p}}$$

$$\boxed{\text{Gain Margin} = -20\log|G(j\omega_p)H(j\omega_p)| = +n \text{ dB}} *$$

* Now, to shift the Bode plot downwards,

$\therefore$ $20\log(K_1)$ = Final point − Initial point.

$$20\log(K_1) = [-n_1] - [-n] = n - n_1.$$

$$K_1 = (10)^{(n-n_1)/20} \qquad [K_1 < 1 \text{ as } (n-n_1) = -ve].$$

or $\boxed{K \cdot K_1 < 1}$ *.

So, $20\log(K_1)$ will get <u>added</u> to every Magnitude [Note: $K<1$].

So, here $GM = +n_1 \cdot$ dB.

- Gain Margin is a unique characteristic of particular DC Gain. No two different DC gain will have same Gain Margin.

* Gain Margin obtained from Bode plot is not exact Gain Margin. It is approximated Gain Margin as Bode Magnitude plot is approximated plot.

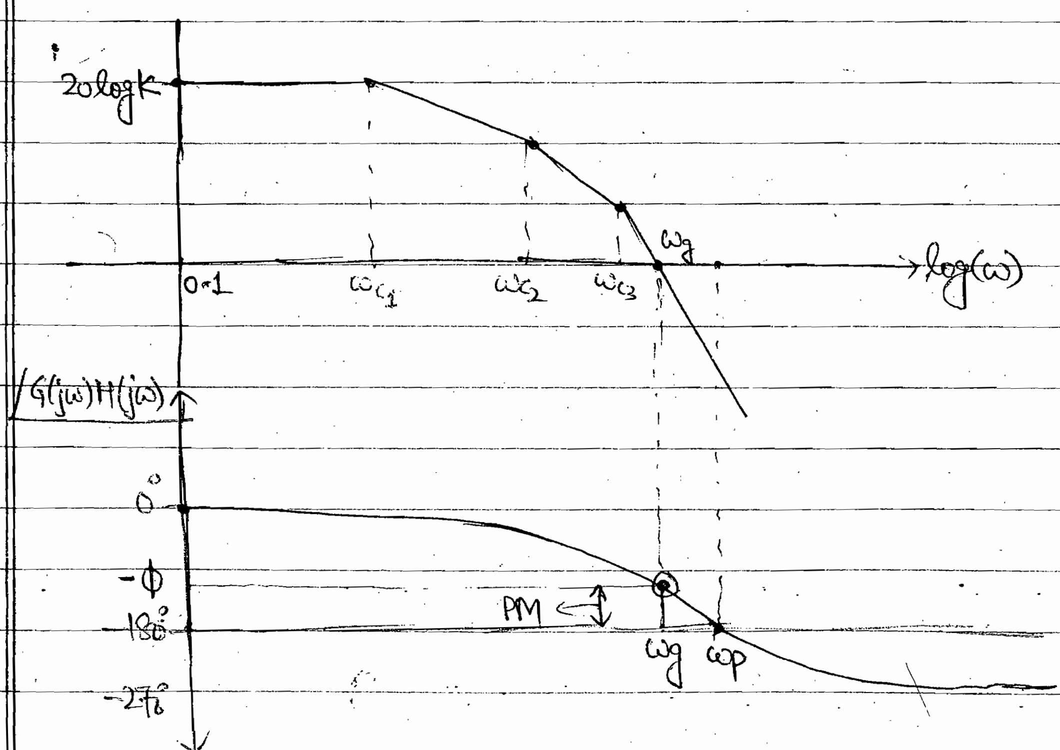
Phase Margin :-

Phase Margin defines the stability of Closed Loop transfer function from its open Loop plot and for closed loop system to be stable, Phase Margin should be positive.

$$\text{Phase Margin} = 180^\circ + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_g}$$

$\omega_g \rightarrow$ Gain crossover frequency.

- Gain Crossover Frequency is a frequency at which Gain of Open loop transfer function, in ratio, becomes equal to 1 and in dB, becomes 0 dB.

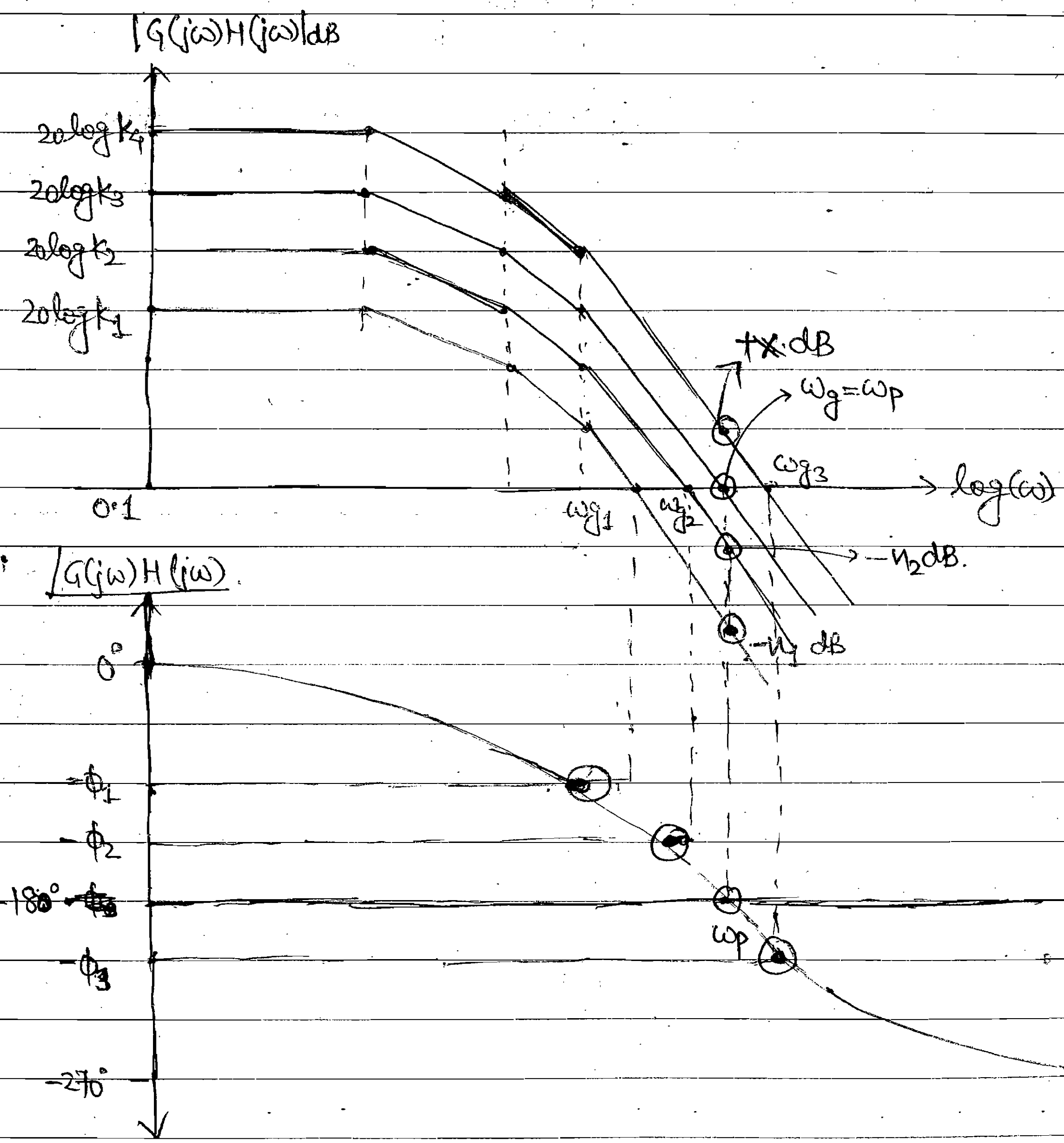


$$\angle G(j\omega)H(j\omega) = -\tan^{-1}(\omega/\omega_1) - \tan^{-1}(\omega/\omega_2) - \tan^{-1}(\omega/\omega_3) = -\phi.$$

$$\text{Phase Margin} = 180^\circ + \angle G(j\omega)H(j\omega)\big|_{\omega=\omega_g}.$$

$$\therefore \text{Phase Margin} = 180^\circ - \phi.$$

* ω_g can be changed by increasing or decreasing DC gain.
* As K decreases, PM and GM increases and vice-versa.
* Gain Margin follows Phase Margin and vice-versa.



$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{\left(\frac{s}{\omega_{c_1}}+1\right)\left(\frac{s}{\omega_{c_2}}+1\right)\left(\frac{s}{\omega_{c_3}}+1\right)+K}$$

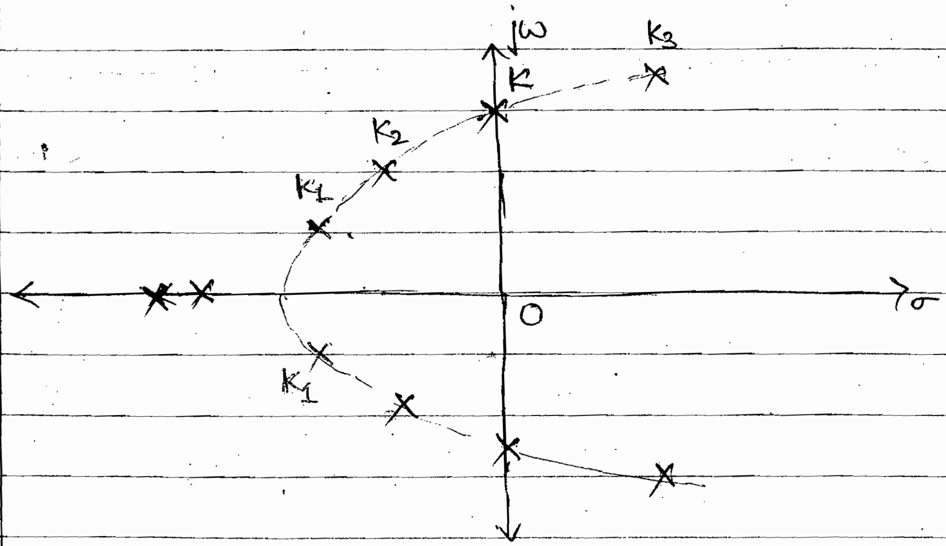
$$\Rightarrow \quad 1+G(s)H(s) = \left(\frac{s}{\omega_{c_1}}+1\right)\left(\frac{s}{\omega_{c_2}}+1\right)\left(\frac{s}{\omega_{c_3}}+1\right)+K = 0$$

$$\therefore \quad GM = -\left|G(j\omega_p)H(j\omega_p)\right|$$

$$PM = 180^\circ + \angle G(j\omega_g)H(j\omega_g).$$

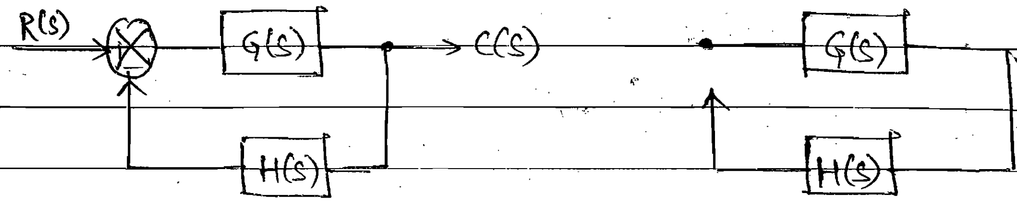
DC Gain	Crossover Frequency	GM	PM	Stability.
K_1	$\omega_p > \omega_{g_1}$	$+n_1$ dB	+ve	Stable. (More).
K_2	$\omega_p > \omega_{g_2}$	$+n_2$ dB	+ve	Stable
K	$\omega_p = \omega_g$	0 dB	0°	Marginally Stable.
K_3	$\omega_p < \omega_g$	$-x_1$ dB	−ve	Unstable.

$$K_3 > K > K_2 > K_1.$$



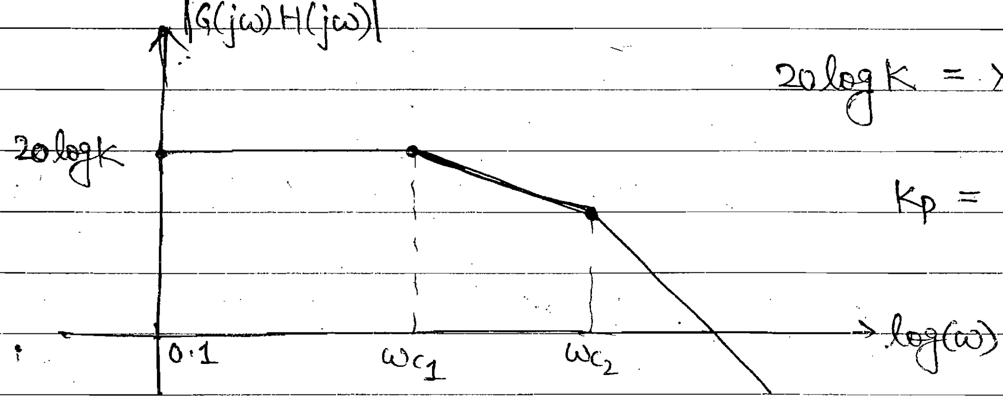
* Determination of Error-co-efficients from Bode Plot :

1. Positional Error-Coefficient :



$$CLTF = \frac{G(s)}{1+G(s)H(s)} \qquad OLTF = G(s)H(s)$$

$$\text{Let} \quad G(s)H(s) = \frac{K}{(s/\omega_{c_1}+1)(s/\omega_{c_2}+1)}$$



$$20\log K = X$$

$$K_p = (10)^{(X/20)} = K$$

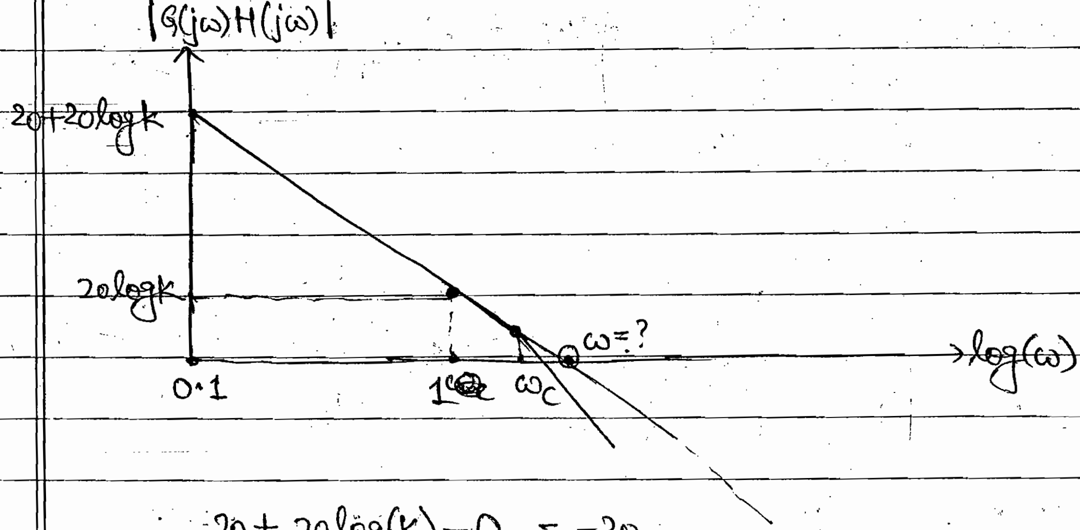
$$\text{Now,} \quad K_p = \lim_{s\to 0} \frac{K}{(s/\omega_{c_1}+1)(s/\omega_{c_2}+1)} = K.$$

$$\therefore \quad e_{ss} = \frac{1}{1+K_p}$$

$$\boxed{e_{ss} = \frac{1}{1+(10)^{X/20}}} \; *$$

2. Velocity Error-Coefficient :

$$G(s)H(s) = \frac{1}{s(s/\omega_c+1)}$$



$$\frac{20 + 20\log(K) - 0}{\log(0\cdot 1) - \log(\omega)} = -20$$

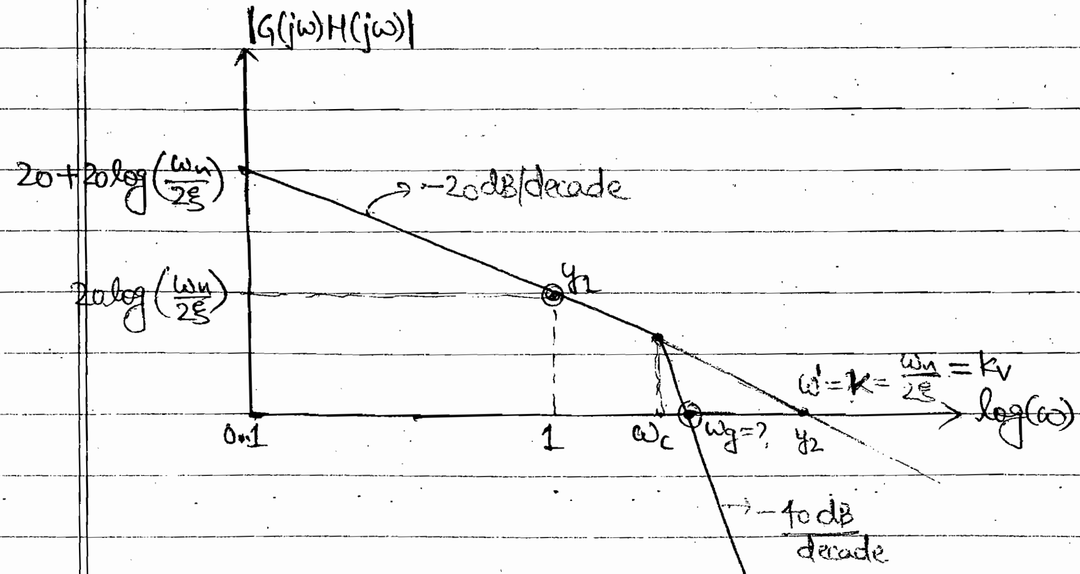
$$\Rightarrow \quad \cancel{20} + 20\log K = \cancel{20} + 20\log(\omega)$$

$$\Rightarrow \quad \boxed{\omega = K} \; *$$

** • Now, Let $G(s)\cdot H(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

* writing it in Standard Form, $G(s)H(s) = \dfrac{(\omega_n/2\xi)}{s\left[\frac{s}{2\xi\omega_n} + 1\right]}$

Comparing $\quad K = \dfrac{\omega_n}{2\xi}$



To find ω_g; $\dfrac{0-y_1}{\log\left(\frac{\omega_n}{2\xi}\right)-\log(2\xi\omega_n)} = -20.$ $[\omega_c = 2\xi\omega_n \text{ here}]$

$$\Rightarrow \quad y_1 = 20\log\left(\frac{1}{2\xi}\right)^2 \Rightarrow \boxed{y_1 = -40\log(2\xi)} *$$

Again, $\dfrac{0-(-40\log(2\xi))}{\log(\omega_g)-\log(2\xi\omega_n)} = -40.$

$$\Rightarrow \quad 40\log(2\xi) = -40\log(\omega_g) + 40\log(2\xi\omega_n)$$

$$\Rightarrow \quad 40\log\left(\frac{2\xi}{2\xi\omega_n}\right) = -40\log(\omega_g)$$

$$\Rightarrow \quad \text{or } \log(\omega_n/\xi) = \log(\omega_g)$$

$$\Rightarrow \quad \boxed{\omega_g = \omega_n} *$$

Now, we have frequencies as: $\omega_c = 2\xi\omega_n$, $\boxed{\omega_g = \omega_n}$, $\omega' = \left(\frac{2\xi}{\omega_n}\right)^{-1}$

$$\therefore \boxed{\xi = \frac{1}{2}\frac{\omega_c}{\omega_g}} *$$

* By only finding ξ and ω_n, we can find all parameters like t_s, t_p, $\%M_p$, t_r as they depend only on ξ and ω_n.

* <u>For e_{ss}</u> :- $K_v = \lim\limits_{s\to 0} s \times \dfrac{(\omega_n)/2\xi}{s\left(\frac{s}{2\xi\omega_n}+1\right)}$ $\Rightarrow K_v = \dfrac{\omega_n}{2\xi} = \omega'$

$$\therefore \boxed{e_{ss} = \frac{2\xi.A}{\omega_n}} *$$

$$\boxed{\omega_c\omega' = \omega_n^2 = \omega_g^2} * \Rightarrow \boxed{\omega_g = \sqrt{\omega_c.\omega'}} *$$

* **Note** * ω_g is Geometric Mean of ω_c and ω'.

* Now, by taking log both sides,

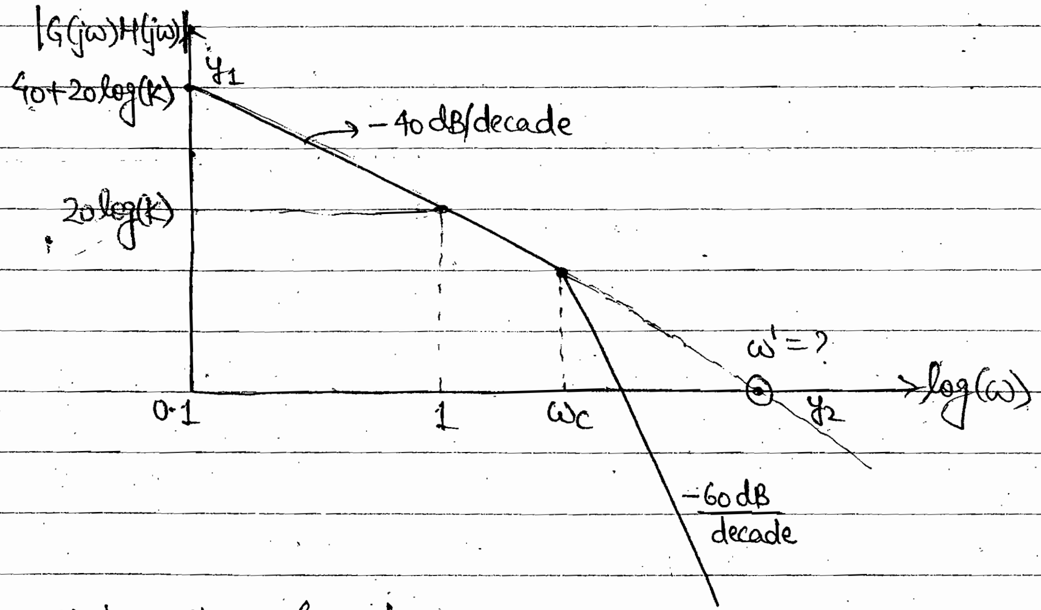
$$2\log(\omega_g) = \log(\omega) + \log(\omega')$$

$$\Rightarrow \boxed{\log(\omega_g) = \frac{\log(\omega) + \log(\omega')}{2}} \; *$$

* **Note:** $\therefore$ $\log(\omega_g)$ is Arithmetic Mean of $\log(\omega)$ and $\log(\omega')$.

3) **Acceleration-Error Co-efficient :-**

$$G(s)H(s) = \frac{K}{s^2\left[\frac{s}{\omega_c} + 1\right]} \; ; \; |G(j\omega)H(j\omega)| = 20\log(K) - 40\log(\omega) - 20\log\left(\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}\right)$$



Using slope formula,

$$\frac{0 - [40 + 20\log(K)]}{\log(\omega') - \log(0.1)} = -40$$

$$\Rightarrow -40 - 20\log K = -40\log(\omega') - 40$$

$$\boxed{\omega_n' = \sqrt{K}} \; * \quad \text{——— (1)}$$

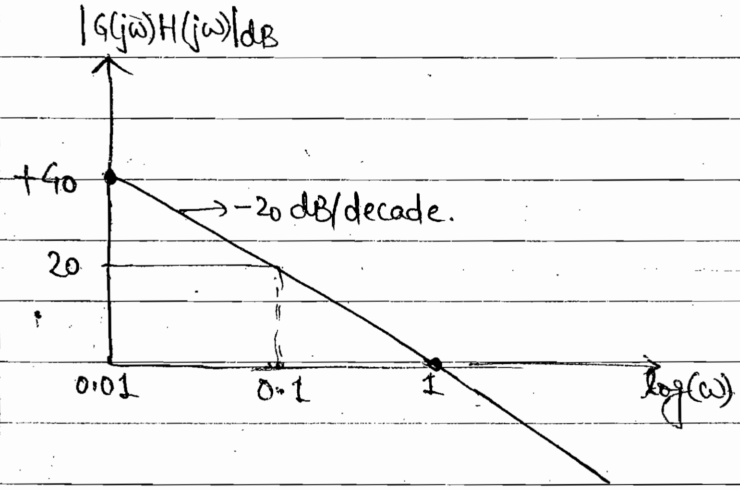
* For e_{ss} :- $K_a = \lim_{s \to 0} s^2 \cdot \frac{K}{s^2[s/\omega_c + 1]} = K = \omega'^2$ [Using eq-(1)]

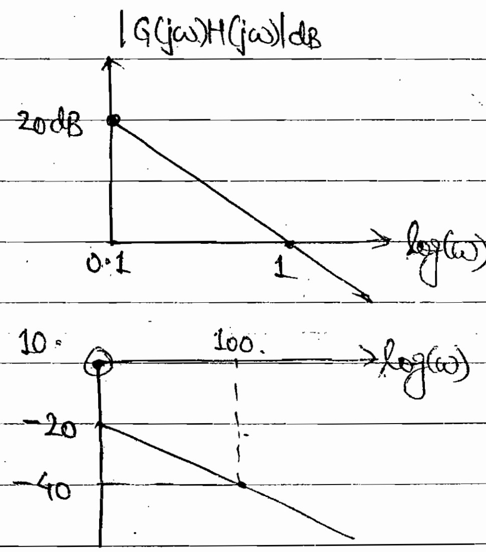
$\therefore \quad e_{ss} = \frac{A}{K_a}$

$$\therefore \quad \boxed{e_{ss} = \frac{A}{\omega'^2}} \quad *$$

- For $G(s)H(s) = Ks\left(s/\omega_c + 1\right)$, $\omega' = 1/K$.
- For $G(s)H(s) = Ks^2\left(s/\omega_c + 1\right)$, $\omega' = 1/\sqrt{K}$

→ * Bode plot of $1/s$ 'I





** On clipping, the frequencies that will get clipped must not contain corner frequency.

Case – I :→ <u>Initial slope and First corner frequency > 1 rad/sec</u> :↓

- If Initially slope is there of +20 dB/dec then at origin, Pole is present.
- If Line is having initial slope, that means Origin contains zero, for +20 dB/decade slope.

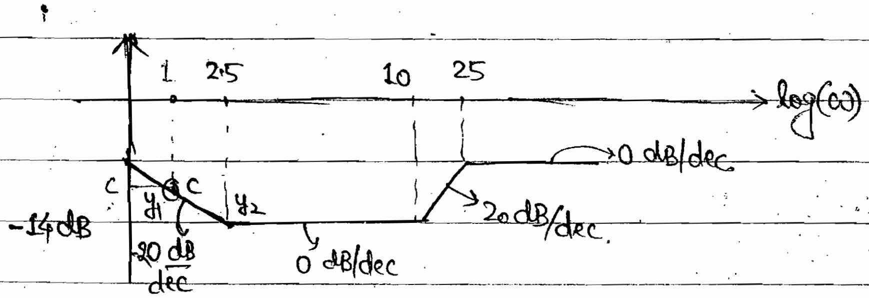
* Now, we will search for ω_c at which slope is changing. If change in slope i.e Final slope − Initial slope is −20 dB/decade, then that corner frequency is due to Pole and it will come into the denominator of TF ; $TF = \dfrac{K}{(s/\omega_c + 1)}$

* If Final slope − Initial slope = 20 dB/decade, than that ω_c is due to zero, then it will come in Numerator of Transfer function

$$TF = (s/\omega_c + 1)$$

* To calculate DC gain, we will calculate Magnitude at $\omega = 1$ and then, we'll equate that Magnitude with $20\log(K)$.

Ques:- (1)



Find Transfer Function.

Ans:- (1) Using slope Formula, $\dfrac{C - (-14)}{\log(1) - \log(2.5)} = -20.$

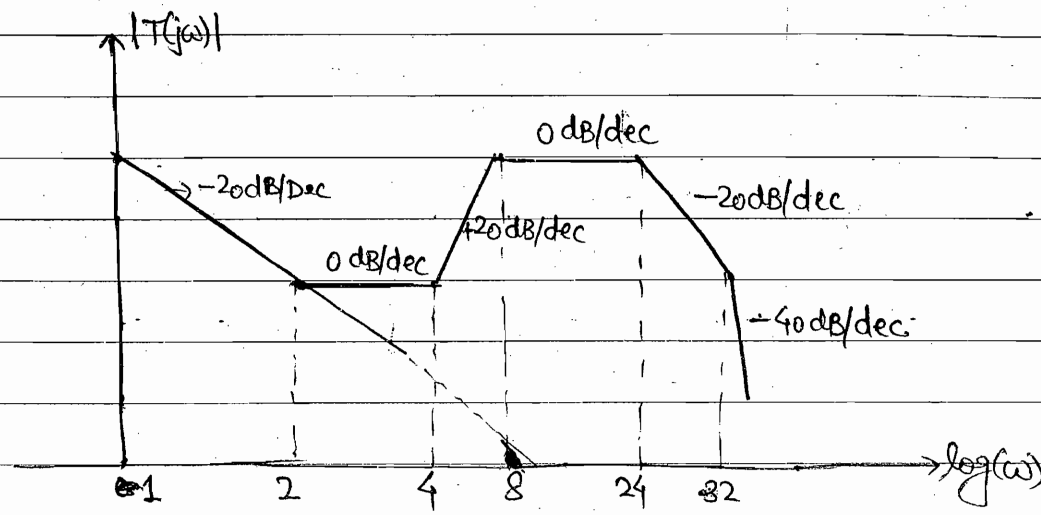
$\Rightarrow \quad C + 14 = 20\log(2.5). \quad \Rightarrow \quad \boxed{C = -6.04}$ *

Now, $20\log K = C \quad \Rightarrow \quad \boxed{K = 0.5}$ *.

$$\therefore \; T.F = \frac{0.5\left(\frac{s}{2.5}+1\right)\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{25}+1\right)}$$ Here, $0.5 =$ DC Gain.

$\Rightarrow \quad T.F =$

Ques:- (2) For the given bode plot, calculate its transfer function.



Ans:- (2) Here, DC Gain $= 8 = K$.

Now, $$T.F = \frac{8\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{s\left(\frac{s}{8}+1\right)\left(\frac{s}{24}+1\right)\left(\frac{s}{32}+1\right)}$$

$$T.F = \frac{\cancel{8}\left(\frac{s+2}{\cancel{2}}\right)\left(\frac{s+4}{\cancel{4}}\right)}{s\left(\frac{s+8}{8}\right)\left(\frac{s+24}{24}\right)\left(\frac{s+32}{32}\right)} = \frac{6144\,(s+2)(s+4)}{s(s+8)(s+24)(s+32)}$$

Case III: Origin contains pole and First corner frequency < 1 rad/sec :-

- For DC gain, we will magnitude at $\omega = 1$ and then we will use:

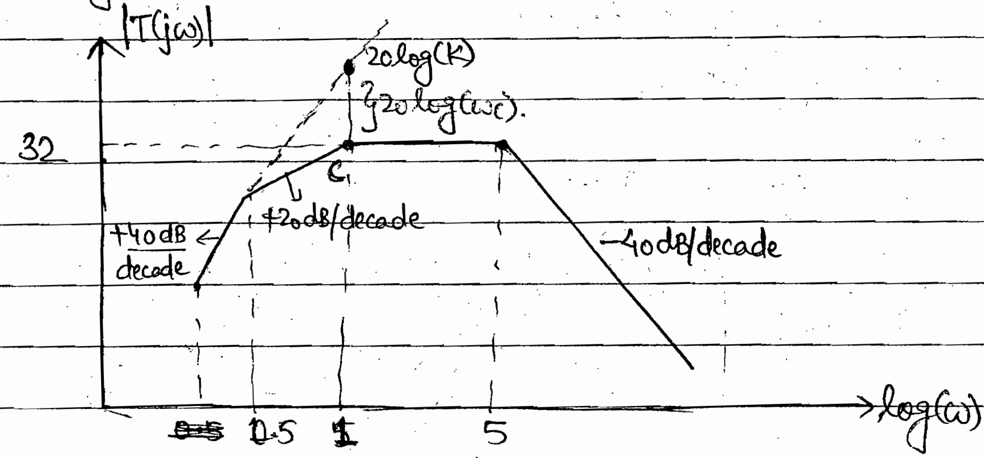
$$20\log(K) = C \pm 20\log(\omega_c).$$

where $C =$ Magnitude at $\omega = 1$.

$\omega_c =$ Corner frequency less than 1 rad/sec.

$K =$ DC Gain of the system.

Ques:- (3) For the given Bode plot, obtain its transfer function.



Ans:- (3) $20\log K = C - [20\log(\omega_c)]$.

$$20\log(K) = C - [20\log(0.5)] = C - [-6.02]$$

$$\Rightarrow 20\log(K) = 32 + 6.02 = 38.02.$$

$$K = (10)^{\frac{38.02}{20}} \Rightarrow \boxed{K \simeq 80} *$$

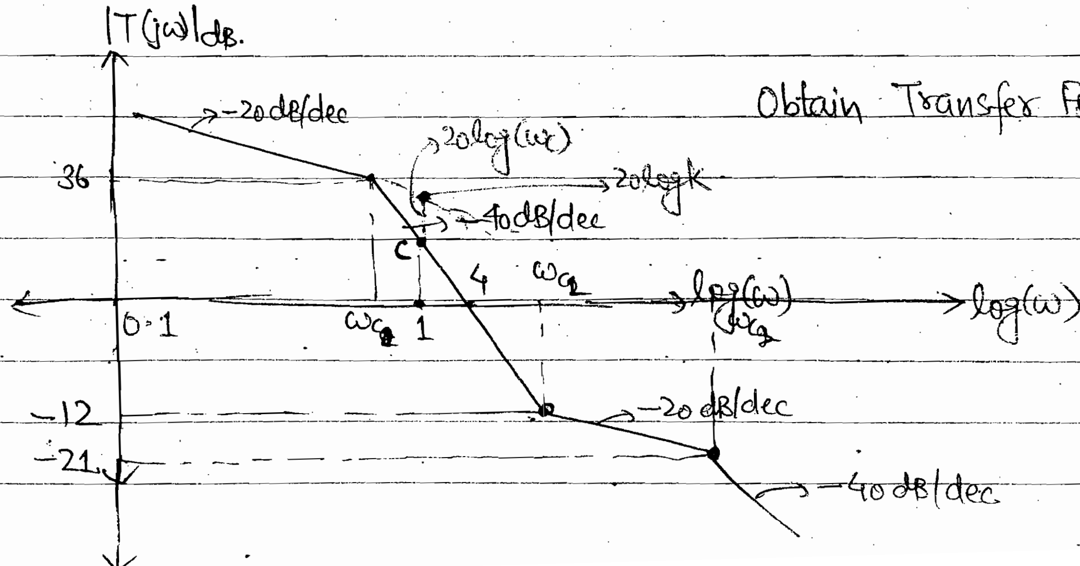
$$TF = \frac{80\, s^2}{\left(\frac{s}{0.5}+1\right)\left(\frac{s}{1}+1\right)\left(\frac{s}{5}+1\right)^2}$$

$$\therefore TF = \frac{80\, s^2 \times 25}{(2s+1)(s+1)(s^2+10s+25)}$$

$$T.F = \frac{2000\, s^2}{(2s+1)(s+1)(s^2+10s+25)}, \quad \text{Ans}$$

Ques:- (4)

Obtain Transfer Function.



Ans:- (4) Using slope Formula, $\dfrac{36-0}{\log(\omega_c)-\log(4)} = -40.$

$\Rightarrow \quad \omega_c = 0.5$ rad/sec. [Case: II applicable].

$$\frac{c-0}{\log(1)-\log(4)} = -40 \Rightarrow c = 24.08.$$

$$20\log(K) = c - 20\log(\omega_c) \Rightarrow 20\log(K) = 24.08 - 20\log(0.5)$$

$$\Rightarrow \quad 20\log(K) = 24.08-(-6.02)$$

$$\Rightarrow \quad 20\log(K) = 30.1 \Rightarrow K = (10)^{30.1/20}$$

$$\boxed{K = 32} \; *$$

$$T.F = \frac{32\left(\frac{s}{\omega_{c_1}}+1\right)}{s\left(\frac{s}{0.5}+1\right)\left(\frac{s}{\omega_{c_2}}+1\right)}$$

$\therefore$ For ω_{c_1} ; $\dfrac{0-(-12)}{\log(4)-\log(\omega_{c_1})} = -40.$

$$\Rightarrow \quad 12 = -40[\log(4)-\log(\omega_{c_1})]$$

$$\Rightarrow \quad \boxed{\omega_{c_1} \cong 8 \text{ r/s}} \; *$$

$\therefore$ For ω_{c_2} ; $\dfrac{-12-(-21)}{\log(8)-\log(\omega_{c_2})} = -20$

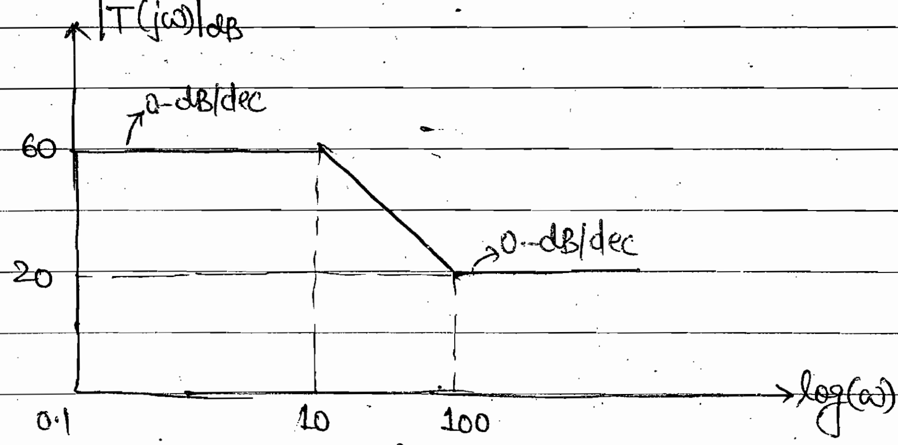
$$\boxed{\omega_{c_2} \simeq 22.5 \text{ rad/sec}} \; *$$

$$\therefore \quad T.F. = \frac{32\left(\frac{s}{8}+1\right)}{s\left(\frac{s}{0.5}+1\right)\left(\frac{s}{22.5}+1\right)} \quad \text{Ans}$$

Case:- III Initial line parallel to 0-dB axis:-

- In this case, Origin will contain any pole or Zero. In this case, we will calculate initial height of the line and then we will equate that height with $20\log(K)$. Rest of other calculations will remain same as it is in previous case.

Ques:- (5) For the given bode plot, obtain its transfer function.



Ans:- (5)

$$\therefore \quad \cancel{20}\log K = \cancel{60}$$

$$\Rightarrow \boxed{K = 1000.} \quad *$$

$$\frac{60-20}{\log(10)-\log(100)} = \text{Slope.}$$

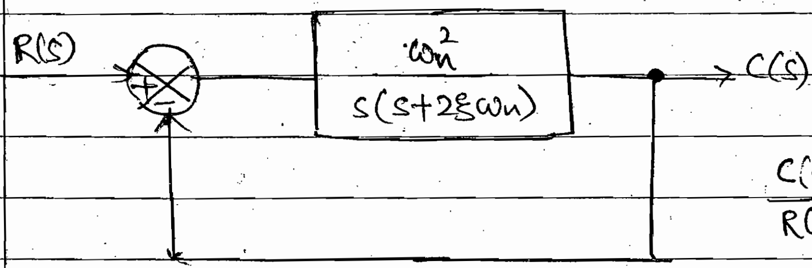
$$\therefore \quad \text{Slope} = \frac{40}{1-2}$$

$$\Rightarrow \boxed{\text{Slope} = -40 \text{ dB/decade}} \quad *.$$

$$\therefore \quad T.F. = \frac{1000.\left(\frac{s}{100}+1\right)^2}{\left(\frac{s}{10}+1\right)^2} = \frac{1000 \times \frac{1}{10000}\left(s^2+200s+10^4\right)}{\frac{1}{100}(s+10)^2}$$

$$\therefore \quad \boxed{T.F. = \frac{10^{+1}(s+100)^2}{(s+10)^2}} \quad * \quad \text{Ans}$$

* Bode Plot for Closed Loop Second Order Under-damped System :-



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Now, $$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

or $$T(s) = \frac{1}{(s/\omega_n)^2+2\xi(s/\omega_n)+1}$$ [In Standard Form]

or $$T(j\omega) = \frac{1}{\left[1-(\omega/\omega_n)^2\right]+\left[2\xi(\omega/\omega_n)\right]}$$

$$\therefore |T(j\omega)|_{dB} = -20\log\left[\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(4\xi^2\,\omega^2/\omega_n^2\right)}\right]$$

and $$\angle T(j\omega) = -\tan^{-1}\left[\frac{2\xi(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right]$$

(i) $\omega \ll \omega_n$:- $|T(j\omega)|_{dB} = -20\log\sqrt{1}$

$$\boxed{|T(j\omega)|_{dB} = 0 \text{ dB}} \ * \ ; \quad \boxed{\angle T(j\omega) = 0^\circ} \ *$$

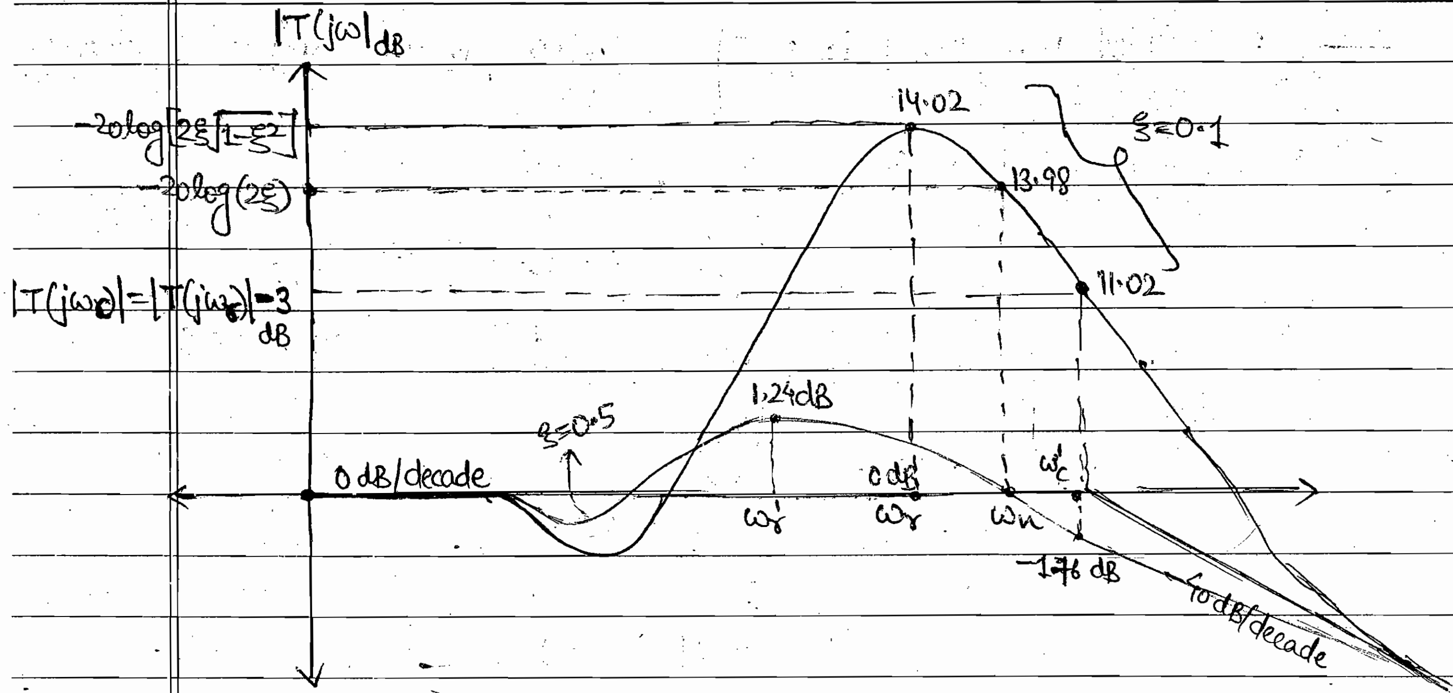
(ii) $\omega = \omega_n$:- $|T(j\omega)|_{dB} = -20\log(2\xi)$

$$\therefore \boxed{|T(j\omega)|_{dB} = -20\log(2\xi)} \ * \ ; \quad \boxed{\angle T(j\omega) = -90^\circ} \ *$$

(iii) $\omega \gg \omega_n$:- $|T(j\omega)|_{dB} = -20\log\left(\sqrt{\frac{\omega^4}{\omega_n^4}}\right)$

$$\therefore \boxed{|T(j\omega)|_{dB} = -40\log(\omega)+40\log(\omega_n)} \ * \quad \boxed{\angle T(j\omega) = -180^\circ} \ *$$

where, $c = 40\log(\omega_n)$.



(i) Resonant Frequency (ω_r):-

- It is defined as Frequency at which slope of Magnitude Response is 0 and Magnitude is maximum.

* To calculate Resonant Frequency, we will differentiate Magnitude Response with respect to ω, and putting $\frac{dT(j\omega)}{d\omega} = 0$

We will get, $\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$ *

Putting ω_r value in Transfer Function $T(j\omega)$,

$$\therefore \quad |T(j\omega_r)| = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

or $\boxed{|T(j\omega_r)|_{dB} = -20\log\left[2\xi\sqrt{1-\xi^2}\right]}$ *

$\boxed{\angle T(j\omega_r) = -90^\circ - \sin^{-1}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}$ *

(ii) Natural Frequency (ω_n):-

$$|T(j\omega_n)| = \frac{1}{2\xi}$$

$$\boxed{|T(j\omega_n)|_{dB} = -20\log(2\xi)} \ * \qquad \boxed{\angle T(j\omega_n) = -90^\circ} \ *$$

$$\omega_c = \text{Bandwidth} = \omega_n\sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}}$$

$$|T(j\omega_c)| = \frac{1}{\sqrt{2}}|T(j\omega_r)|$$

$$\Rightarrow \boxed{|T(j\omega_c)| = \frac{1}{\sqrt{2}}\left[\frac{1}{2\xi\sqrt{1-\xi^2}}\right]} \ *$$

$$\boxed{|T(j\omega_c)|_{dB} = |T(j\omega_r)|_{dB} - 3\text{ dB} = \left[-20\log\left(2\xi\sqrt{1-\xi^2}\right)\right] - 3\text{ dB}} \ *$$

* for $\xi = 0.1$,

$$\omega_r = \omega_n\sqrt{1-2\xi^2} = \sqrt{0.98}\,\omega_n$$

$$|T(j\omega_r)| = -20\log\left[2\times 0.1\times\sqrt{1-(0.1)^2}\right] = 14.02\text{ dB}$$

$$|T(j\omega_n)| = -20\log(2\times 0.1) = 13.98$$

$$\omega_c = \omega_n\sqrt{(1-2(0.1)^2)+\sqrt{4(0.1)^4-4(0.1)^2+2}} = 1.57\,\omega_n$$

$$|T(j\omega_c)| = |T(j\omega_r)|_{dB} - 3\text{ dB} = 11.02\text{ dB}$$

* for $\xi = 0.5$,

$$\omega_r = \omega_n\sqrt{1-2(0.5)^2} = \sqrt{0.5}\,\omega_n$$

$$|T(j\omega_r)| = -20\log\left[2\times 0.5\sqrt{1-(0.5)^2}\right] = 1.24\text{ dB}$$

$$|T(j\omega_n)| = -20\log(2\times 1) = 0\text{ dB}$$

$$\omega_c' = \omega_n \sqrt{(1-2(0.5)^2) + \sqrt{4(0.5)^4 - 4(0.5)^2 + 2}} = 1.27\,\omega_n.$$

$$|T(j\omega_c')| = |T(j\omega_r)|_{dB} - 3\,dB = 1.24 - 3\,dB = -1.76\,dB.$$

Note:- • We can observe that, when ξ is increased, ω_r and ω_c decreases. Sharpness of response reduces when ξ is increased.

* For what ξ, $|T(j\omega_r)| = 1$, or 0 in dB,

$$\frac{1}{2\xi\sqrt{1-\xi^2}} = 1 \;\Rightarrow\; 4\xi^2(1-\xi^2) = 1$$

$$\Rightarrow\; 4\xi^2 - 4\xi^4 = 1.$$

or $$4\xi^4 - 4\xi^2 + 1 = 0. \;\Rightarrow\; (2\xi^2-1)^2 = 0.$$

$$\boxed{\xi = 1/\sqrt{2} = 0.707 = -3\,dB} \;*$$

* For $\xi = 1/\sqrt{2} = 0.707$:-

$$\omega_r = \omega_n \sqrt{1 - 2\times 1/2} = 0.$$

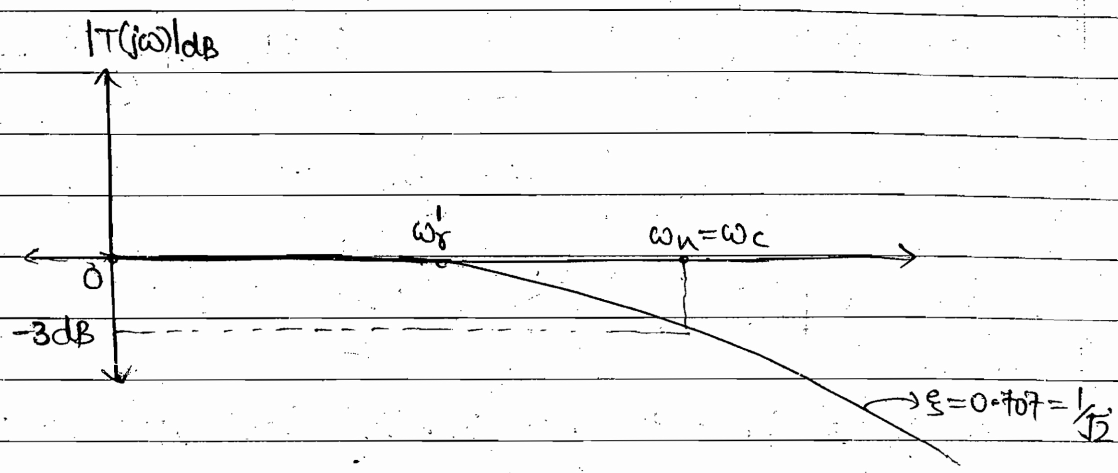
$$|T(j\omega_r)|_{dB} = -20\log\left[2\times 1/\sqrt{2}\sqrt{1-1/2}\right] = 1 = 0\,dB.$$

$$|T(j\omega_n)|_{dB} = -20\log(2\times 1/\sqrt{2}) = -3\,dB.$$

$$\omega_c = \omega_n \sqrt{\left(1 - 2\times\tfrac{1}{2}\right) + \sqrt{4\xi^4 - 4\xi^2 + 2}} = \omega_n.$$

$$|T(j\omega_c)|_{dB} = |T(j\omega_r)|_{dB} - 3\,dB = -3\,dB.$$

* For $\xi = 1/\sqrt{2}$, graph will be :-



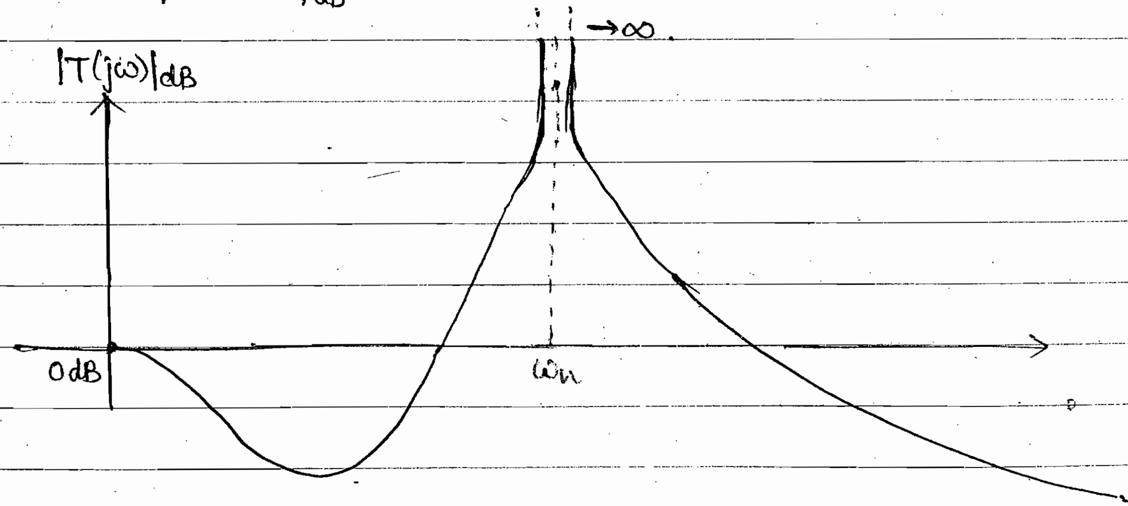
* For $\xi = 0$:-

$\boxed{\omega_r = \omega_n}$ * $\boxed{|T(j\omega_r)| = \infty}$ *

$|T(j\omega_n)|_{dB} = \infty.$

$\omega_c = \omega_n\left[\sqrt{1+\sqrt{2}}\right].$

$|T(j\omega_c)|_{dB} = |T(j\omega_r)|_{dB} - 3\,dB = \infty.$



- That means, in time-domain, if for $\xi = 0$, where Response oscillates, (that frequency), in Frequency-Response, on that frequency, Magnitude will become ∞.

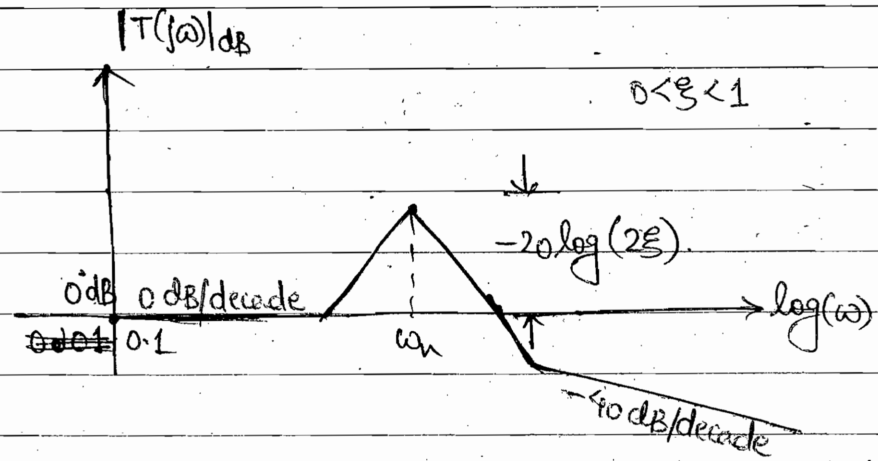
- For $\xi > 1/\sqrt{2}$, ω_r becomes Imaginary which is not possible.

So, $\boxed{0 < \xi < 1/\sqrt{2}}$ * [In Frequency-Response].

$\boxed{\omega_n > \omega_r > 0}$ *

$\boxed{\omega_n\sqrt{1+\sqrt{2}} > \omega_c > \omega_n}$ *.

Note: • When system tends towards stability, the sharpness of Bode Magnitude plot will decrease and its bandwidth will also decrease.



Note: • In Bode Plot, as we do not consider frequency ω_r, so there is no restriction on ξ.

$\therefore$ $\boxed{0 < \xi < 1}$ * For Bode Plot only.

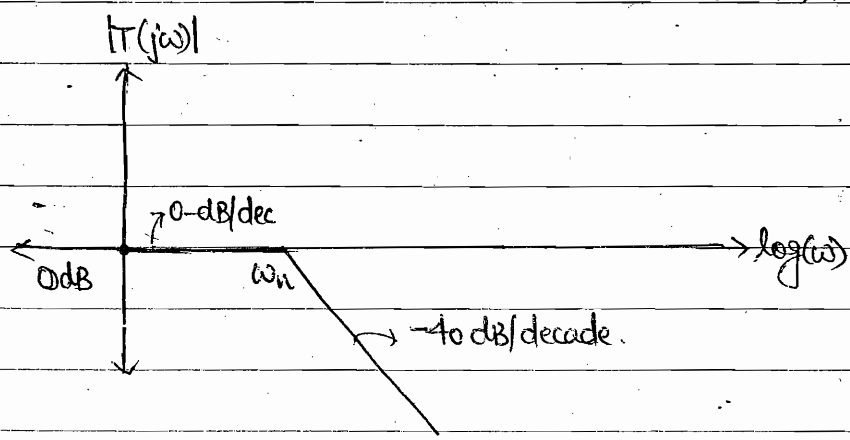
Note: * • For $\xi = 0$, Bode Plot is not possible as slope is not constant anywhere.

Bode Plot for Critically Damped system :-

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

- For Critically-damped system, $\xi = 1$,

$$\therefore \quad T(s) = \frac{\omega_n^2}{(s+\omega_n)^2} \Rightarrow T(s) = \frac{1}{(s/\omega_n + 1)^2}$$



Bode Plot for over-damped system :-

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

- For over-damped system, $\xi > 1$.

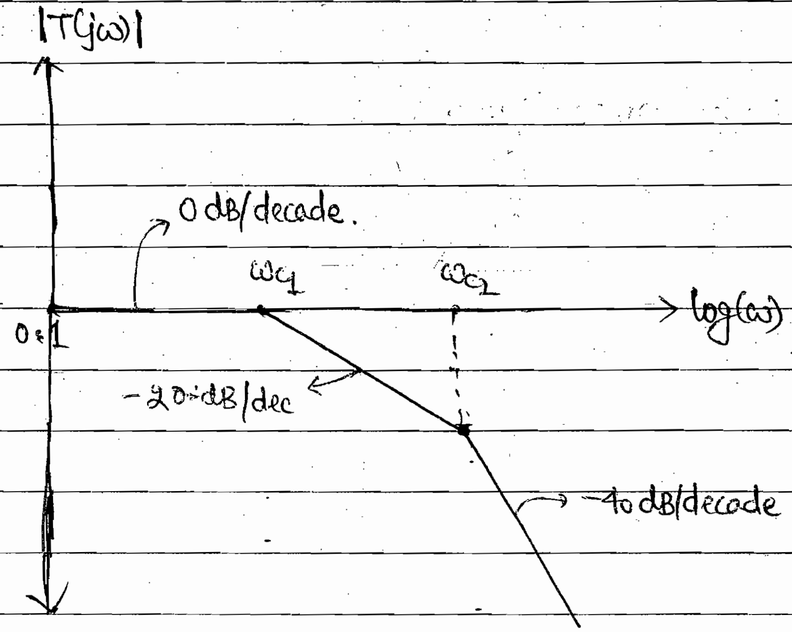
$$\therefore \quad T(s) = \frac{\omega_n^2}{\left[s + (\xi\omega_n + \omega_n\sqrt{\xi^2-1})\right]\left[s + (\xi\omega_n - \omega_n\sqrt{\xi^2-1})\right]}$$

$$\text{or} \quad T(s) = \frac{\omega_n^2}{\underbrace{(\xi\omega_n + \omega_n\sqrt{\xi^2-1})(\xi\omega_n - \omega_n\sqrt{\xi^2-1})}_{\omega_n^2}\left[\frac{s}{\xi\omega_n + \omega_n\sqrt{\xi^2-1}} + 1\right]\left[\frac{s}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}} + 1\right]}$$

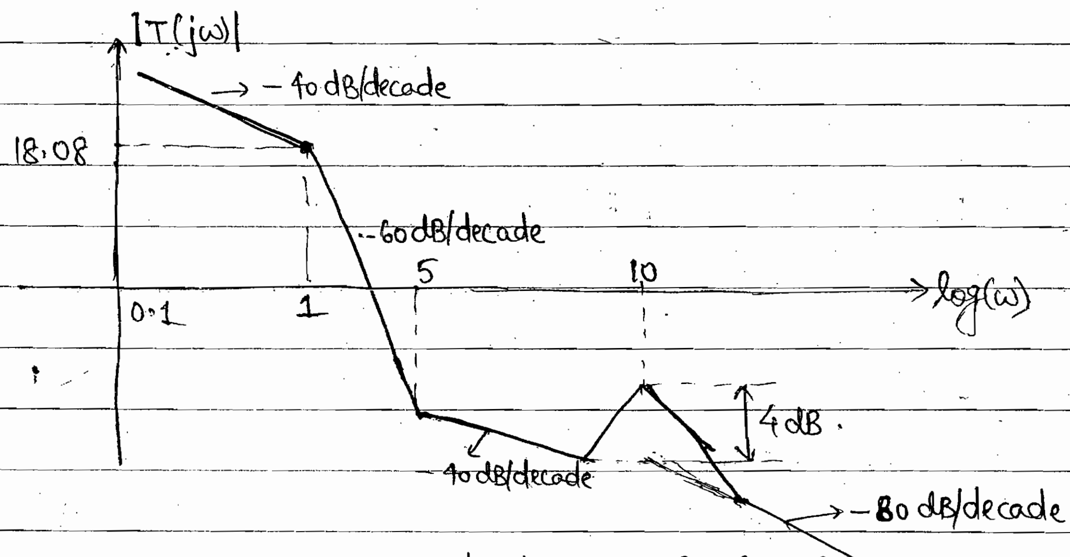
$$\text{or} \quad T(s) = \frac{\omega_n^2}{\omega_n^2\left[\frac{s}{\xi\omega_n + \omega_n\sqrt{\xi^2-1}} + 1\right]\left[\frac{s}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}} + 1\right]}$$

or $T(s) = \dfrac{1}{\left[\frac{s}{\omega_{c_1}}+1\right]\left[\frac{s}{\omega_{c_2}}+1\right]}$, $\omega_{c_1} = \xi\omega_n - \omega_n\sqrt{\xi^2-1}$

$\omega_{c_2} = \xi\omega_n + \omega_n\sqrt{\xi^2-1}$.



Ques:- 6



For given Bode Plot, find its Transfer function.

Ans:- 6 Case: I, $20\log(K) = 18.08 \Rightarrow K = 8$.

$$T.F. = \frac{\left(\frac{s}{5}+1\right)}{s^2\left(\frac{s}{1}+1\right)}$$

For Under-damped system, Standardized Transfer Function is:

$$T.F. = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\xi\left(\frac{s}{\omega_n}\right) + 1}$$

$$\text{or } T(s) = \frac{1}{(s/10)^2 + 2\xi(s/10) + 1}$$

as $|T(j\omega_n)| = -20\log(2\xi) \Rightarrow \boxed{\xi = 0.315}$ *

$[\because |T(j\omega_n)| = 4\text{ dB}]$

$$\therefore T.F = \frac{(s/5 + 1)}{s^2(\frac{s}{1}+1)([s/10]^2 + 0.063s + 1)}$$ Ans

Chapter: 7

ROOT LOCUS TECHNIQUE

* In case of Root Locus, the variable is Gain of Transfer Function and we calculate the locus of pole for Closed Loop System when Gain of Open Loop Transfer Function changes.

* So, for Root Locus, stability is defined in terms of Location of Pole. Hence, Root Locus defines Absolute Stability.

* **Example:** $G(s) \cdot H(s) = \dfrac{K}{s(s+2)}$, K is a variable here.

$$\therefore \quad T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s^2+2s+K} \qquad [0<K<\infty].$$

$$\therefore \quad q(s) = s^2+2s+K=0.$$

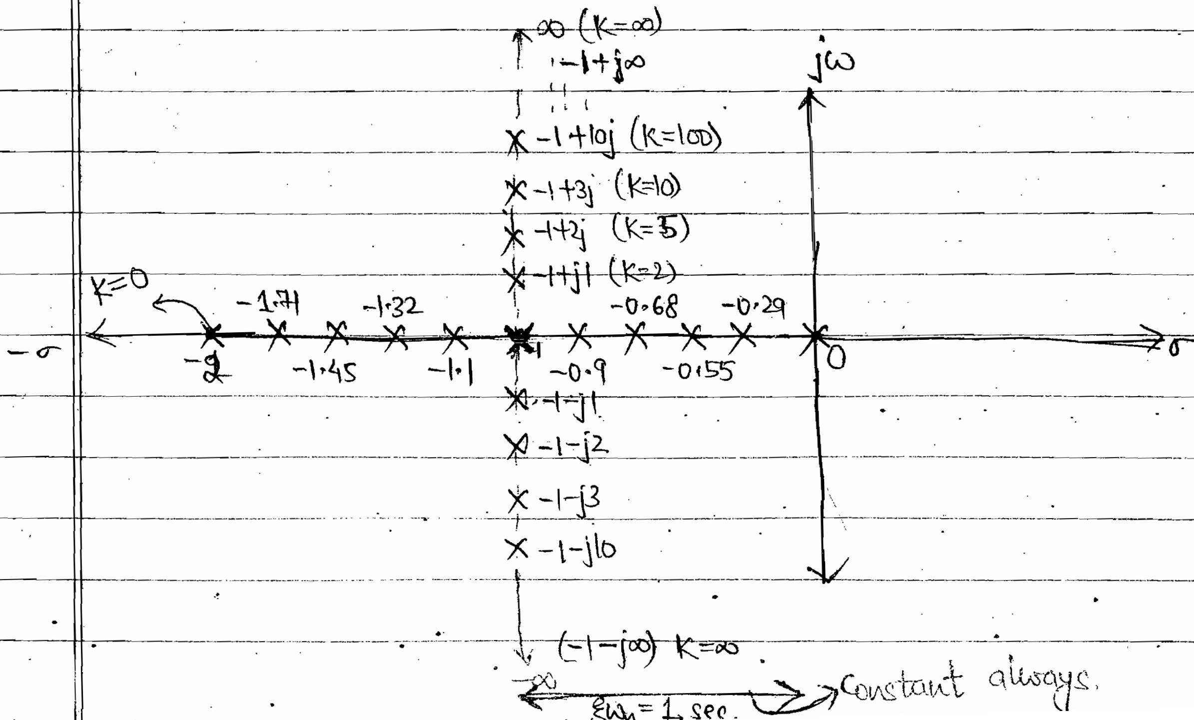
$K=0$; $s^2+2s=0$, $\therefore s=0,\ s=-2.$

$K=0.5$; $s^2+2s+0.5=0$; $s=-0.29,\ s=-1.71.$

$K=0.8$; $s^2+2s+0.8=0$; $s=-0.55,\ s=-1.45$

$K=0.9$; $s^2+2s+0.9=0$; $s=-0.68,\ s=-1.32.$

$K=0.99$; $s^2+2s+0.99=0$; $s=-0.9,\ -1.1.$



$K = 1$; $s^2 + 2s + 1 = 0$; $s = -1, -1$.

$K = 2$; $s^2 + 2s + 2 = 0$; $s = -1 \pm j1$

$K = 5$; $s^2 + 2s + 5 = 0$; $s = -1 \pm j2$

$K = 10$; $s^2 + 2s + 10 = 0$; $s = -1 \pm j3$.

$K = 100$; $s^2 + 2s + 100 = 0$; $s = -1 \pm j10$.

$K = \infty$; $s^2 + 2s + \infty = 0$; $s = -1 \pm j\infty$.

- for $0 < K < 1$; Over-damped ; $\xi > 1$

  $K = 1$; Critically-damped. ; $\xi = 1$

  $1 < K < \infty$; Underdamped ; $\xi < 1$.

- This system is highly stable.
- For given system,

$$s^2 + 2s + K = 0.$$

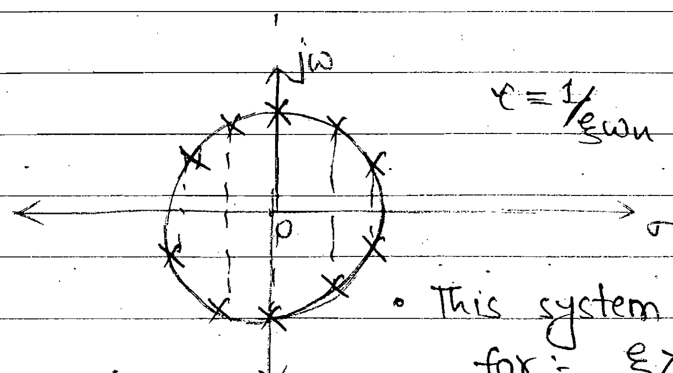
$$\therefore \omega_n = \sqrt{K}, \quad \xi\omega_n = 1 \quad \text{and} \quad \therefore \xi = 1/\sqrt{K} = \cos\phi.$$

$$\boxed{\phi = \cos^{-1}\left(1/\sqrt{K}\right)} \ *.$$

Now, $t_s = \dfrac{4}{\xi\omega_n}$; $t_s = 4$ sec, $\tau = 1$ second.

- For $T(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

Here, ξ is variable, ω_n is constant.



$\tau = 1/\xi\omega_n$, as ξ varies, τ varies.

- This system is conditionally stable for :- $\xi > 0$.

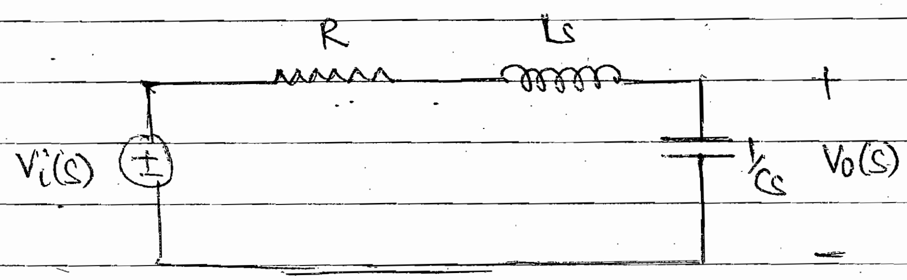
Now, for system :- $T(s) = \dfrac{K \cdot \omega_n^2}{s^2 + 2\xi\omega_n s + K\omega_n^2}$ — ①

Here, $\xi\omega_n$ is constant, K is variable.

Undamped Natural Frequency $= \sqrt{K} \cdot \omega_n$ (Variable).

- But this circuit will be High stable for any value of K lying between 0 and ∞.

** Consider a RLC circuit :-



Now, $\dfrac{V_o(s)}{V_i(s)} = \dfrac{1/Cs}{R + Ls + 1/Cs} = \dfrac{1}{RCs + LCs^2 + 1}$

or $\dfrac{V_o(s)}{V_i(s)} = \dfrac{(1/LC)}{s^2 + R/L\, s + 1/LC}$

Here, $\boxed{\omega_n = \dfrac{1}{\sqrt{LC}}}$ * $\boxed{\xi\omega_n = \dfrac{R}{2L}}$ *

$\boxed{\xi = \dfrac{R}{2}\sqrt{\dfrac{C}{L}}}$ *

- To keep ξ as variable and ω_n as constant, we can only change R but not L as if we change L, ω_n also gets varied.

- As $-\infty < R < \infty$, so $-\infty < R < 0$ and $0 < R < \infty$.
  $-\infty < \xi < 0$ and $0 < \xi < \infty$
  $\hookrightarrow$ Unstable $\hookrightarrow$ Stable

• As R is power dissipation passive element, so $0<R<\infty$ is stable.

* Now, for ω_n to be variable but $\xi\omega_n$ to be constant, we can only vary capacitor.
  - This system will be highly stable, hence, Gain Margin should be $+\infty$.

Note:
- If Time constant is constant and ω_n is variable, system is highly stable.
- If R is varied, system won't be highly stable.
- If C is varied, ($\xi\omega_n$ keeping constant), system will be highly stable.
- If L is varied, then also system will be highly stable.

** Poles start, here, from Open Loop pole and tends towards Infinity.

Note: ** In any system, Order of pole = Order of zero always. In transfer functions, only those poles and zeros are visible which is located at finite distances and those poles and zeros which are not visible, they are located at ∞, so, their individual TFs are 1.
Only, Root Locus, explains that in a system, Order of Pole equals Order of zero, i.e

$$G(s)H(s) = \frac{(K/2)}{s[s/2+1]}, \text{ Actually, it is :- } G(s)\cdot H(s) = \frac{(K/2)[s/\omega_{c_1}+1][s/\omega_{c_2}+1]}{s[s/2+1]}$$

Here, $\omega_{c_1} = \omega_{c_2} = \infty$, $\therefore \left[\frac{s}{\infty}+1\right] = 0+1 \Rightarrow 1$.

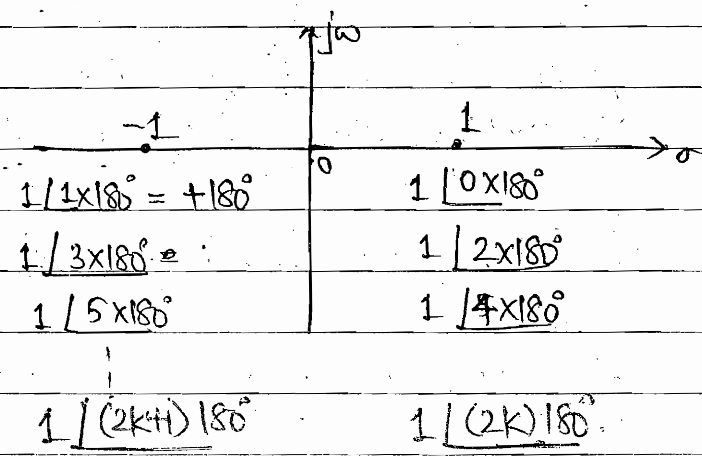
$$\therefore G(s)H(s) = \frac{(K/2)\times 1\times 1}{s[s/2+1]}$$

$$\therefore G(s)H(s) = \frac{(K/2)}{s[s/2+1]}$$

- $q(s) = 1 + G(s)H(s) = 0$

$$\therefore \boxed{G(s)H(s) = -1} \; *$$

** This implies that, closed loop poles will lie on s-plane on only those locations, where when we substitute value of s and k in Open Loop Transfer function, the values of $G(s)H(s)$ or open loop TF must be -1.



** Note: $\therefore$ For Closed Loop poles to lie on (-1), Phase must be: $(2K+1)180°$.

* Analysis: $G(s=-0.29)\,H(s=-0.29) = \dfrac{0.5}{(0.29)(2-0.29)} = -1.$

$$G(s=-1.45)\,H(s=-1.45) = \frac{0.8}{(-1.45)(2-1.45)} = -1.$$

$\therefore$ At $K = 0.5, 0.8$ and $s = -1.45, -0.29$, closed loop poles are there as $G(s)H(s) = -1$ for these values.

Now, at $s=1$, $G(s=1)H(s=1) = \dfrac{K}{(1)(2+1)} = \dfrac{K}{3}$.

$\therefore$ For any positive K, $G(s=1)H(s=1) \neq -1$; so, closed loop poles does not lie at $s=1$.

- At $s=-3$, $G(s=-3)H(s=-3) = \dfrac{K}{(-3)(-3+2)} = \dfrac{K}{3}$

$\therefore$ Here, also $s=-3$, no pole is there.

- At $s=-0.5$, $G(s=-0.5)H(s=-0.5) = \dfrac{K}{(-0.5)(2-0.5)} = -\dfrac{K}{7.5}$

* In Root Locus, Phase Angle condition is dominant. If any location satisfies Phase Angle condition, then for finite value of K in $0<K<\infty$, it will also satisfy Magnitude condition.

→ **Origination Point :-**

* Root Locus originates either from Open Loop pole or Infinity but priority will be given to Open Loop pole and value of K at origination point is zero.

→ **Termination Point :-**

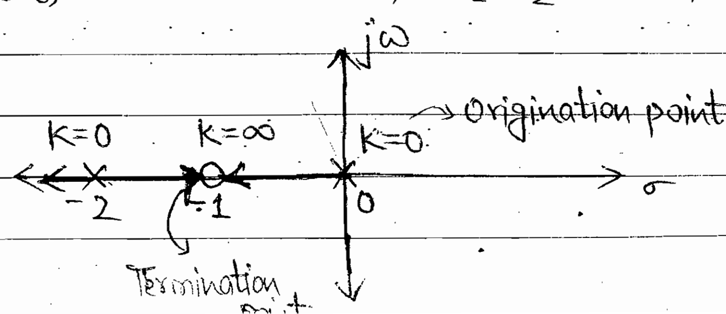
* Root Locus terminates either at Open Loop Zero or Infinity but priority will be given to Open Loop Zero and value of K at termination point will be ∞.

Example:- $G(s)H(s) = \dfrac{K(s+1)}{s(s+2)}$

$$1+G(s)H(s) = 0 \Rightarrow \frac{K(s+1)}{s(s+2)} = -1.$$

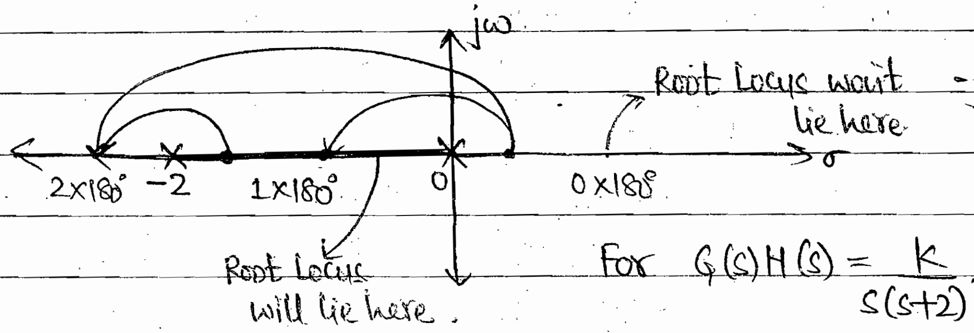
$$\frac{K(s+1)}{s(s+2)} = -1 \Rightarrow K = -\frac{s(s+2)}{(s+1)}$$

$$\therefore K_{(s=0)} = -0 = 0, \quad K_{s=-2} = 0, \quad K_{s=-1} = \infty.$$



* Existence of Root Locus on Real-axis :-

- Root Locus will exist only at that location on Real-axis whose overall angle subtended by that location with Open Loop pole and zero is : Odd Integral Multiple of 180° or in other words, Root Locus will exist only in that section of Real-axis which contains odd number of Open Loop poles and zeros towards its Right side.



For $G(s)H(s) = \dfrac{K}{s(s+2)}$.

Here, 0×180° represents, for any value of s here, in this location, value of G(s)H(s) will be Positive.

$$\therefore \quad G(s=1)H(s=1) = \frac{K}{1(1+2)} = K/3 = +ve.$$

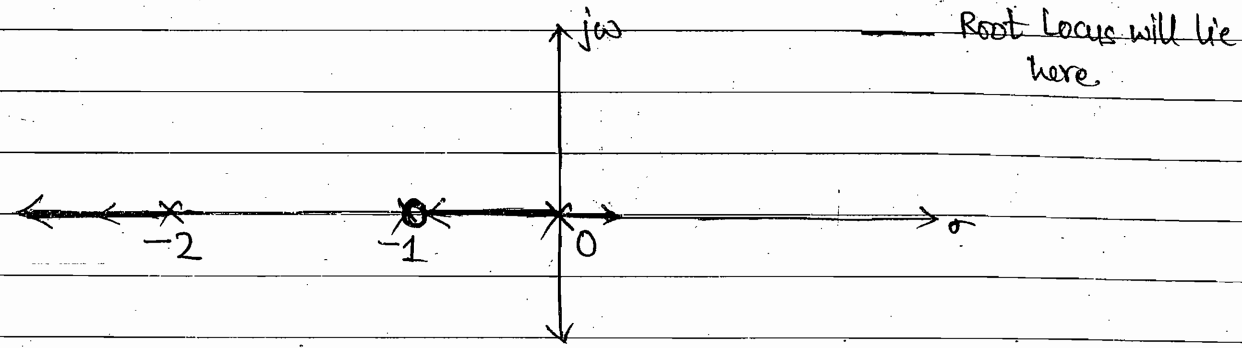
- 1×180° represents, for any value of s, one term will be negative. Hence, for a specific K, G(s)H(s) = −ve

$$\therefore \quad G(s)H(s)\Big|_{s=-1} = \frac{K}{(-1)(2-1)} = \frac{K}{(-1)} = -ve.$$

- 2×180° represents, for any value of s, two term will be negative, finally giving G(s)H(s) = +ve.

$$G(s)H(s)\Big|_{s=-3} = \frac{K}{(-3)(2-3)} = \frac{K}{3} = +ve.$$

• For $G(s)H(s) = \dfrac{K(s+1)}{s(s+2)}$

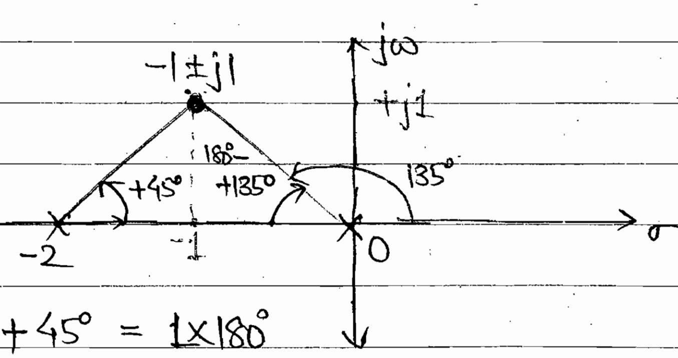


Existence of Root Locus on Complex-plane :-

• Root Locus will exist only at that location on complex-plane whose overall subtended angle from all open loop poles and zeros is : odd Integral Multiple of 180° or in other words, we substitute the given complex location in characteristics equation and then we will calculate the value of K.

* If K is real and positive, for given complex-location, then that location is valid location and closed loop pole will exist at that location.

* If K is either, Imaginary, complex or Negative, then that location will be Invalid location and closed loop pole won't exist at that location.



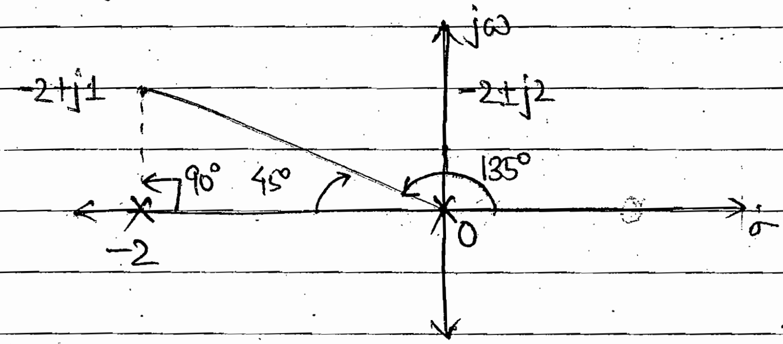
$\therefore 135° + 45° = 1 \times 180°$

* Other Method :- $K\Big|_{s=-1+j1} = -s[s+2]\Big|_{s=-1+j1} = [(-1+j1)[-1+j1+2]]$

$K = -(-2) \Rightarrow \boxed{K\Big|_{s=-1+j1} = 2}$ *

- As $K = 2$ (+ve and real), $s = -1 \pm j1$ lies on Root Locus.

Similarly,



$\therefore$ $135° + 90° = 225°$.

- Other Method :- $K|_{-2+j2} = -\left[(-2+j2)[\cancel{-2}+j2+\cancel{2}]\right]$

$$\boxed{K|_{-2+j2} = 4(1+j)} *$$

- As K = Complex, $s = -2+j2$ does not lie on Root Locus.

*→

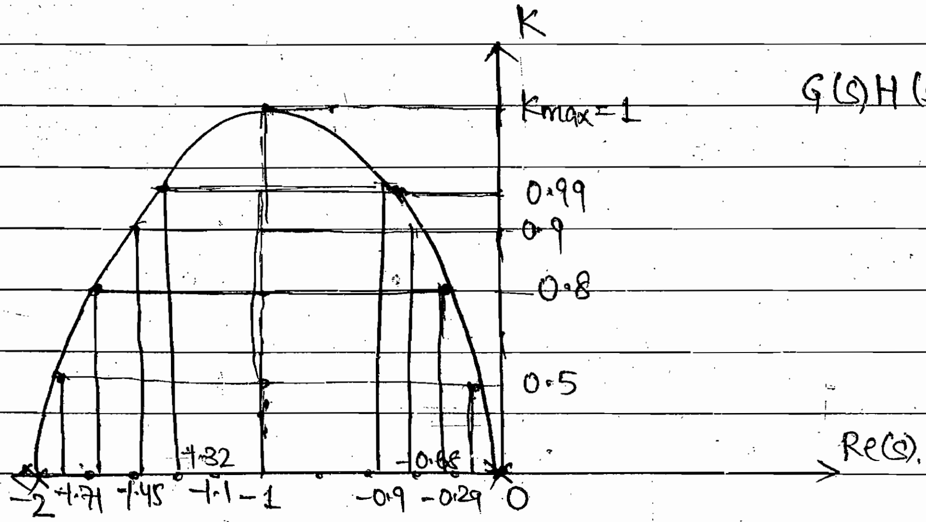
- Break point is location in s-plane where two pole coincides simultaneously. It is of two type :-

(i) Break-away point (ii) Break-in point.

(i) Break-away point :-

- It is a point in s-plane where the value of K is Maximum. Thus, whenever Root Locus shifts from Real-axis to complex plane, always Break-away point will exist. To calculate, location of Break-away point, we will differentiate K in characteristics equation with respect to s and then set,

$$\frac{dK}{ds} = 0.$$



$$G(s)H(s) = \frac{K}{s(s+2)}$$

- So, breakaway point is Maximum value of K for Root Locus to be on the Real-axis. If K exceeds this Maximum value, then Root Locus will enter into Complex-plane. Thus, whenever Root Locus shifts from Real-axis to Complex-plane, always Break-away point exists. To calculate, location of break-away, do;

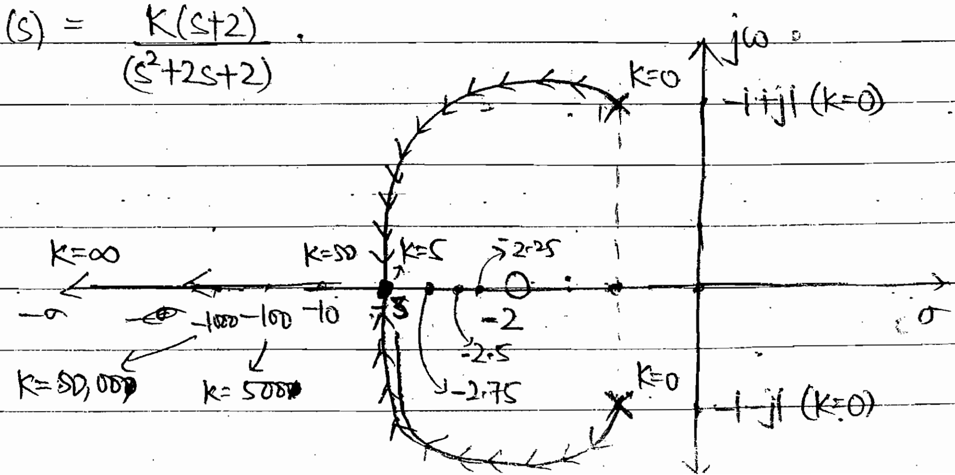
$$\frac{dK}{ds} = 0.$$

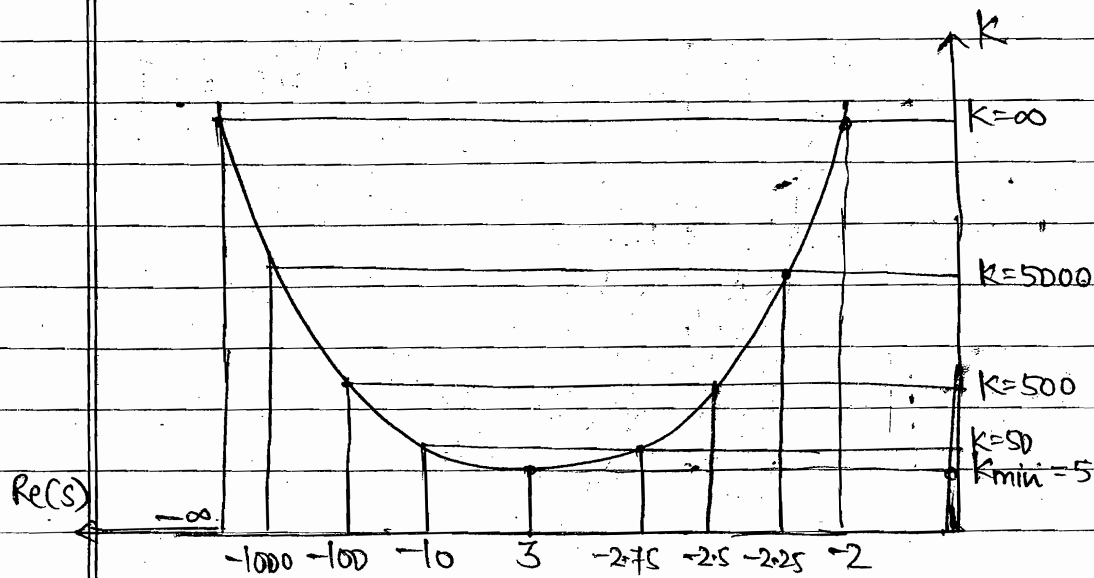
(ii) Break-in Point :-

- It is the Minimum value of K for Root Locus to be on the Real-axis. When K is less than this minimum value, then Root Locus will remain in complex-plane. Thus, whenever Root Locus shifts from Complex-plane to real-axis, always Break-in point exists. To calculate location of break-in point, do;

$$\frac{dK}{ds} = 0.$$

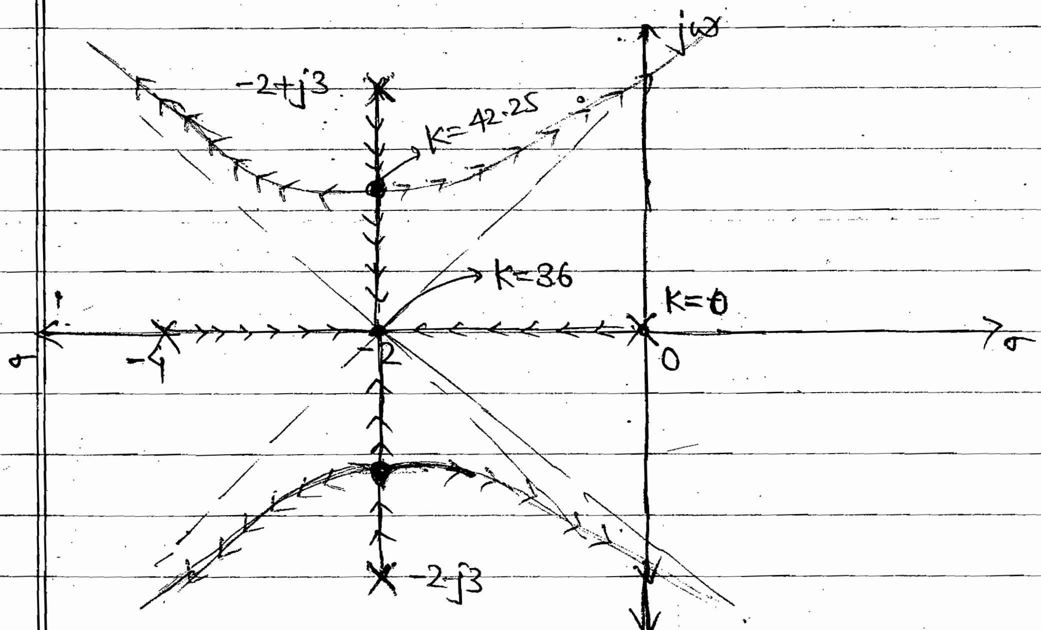
* Example:- $G(s) \cdot H(s) = \frac{K(s+2)}{(s^2+2s+2)}$.

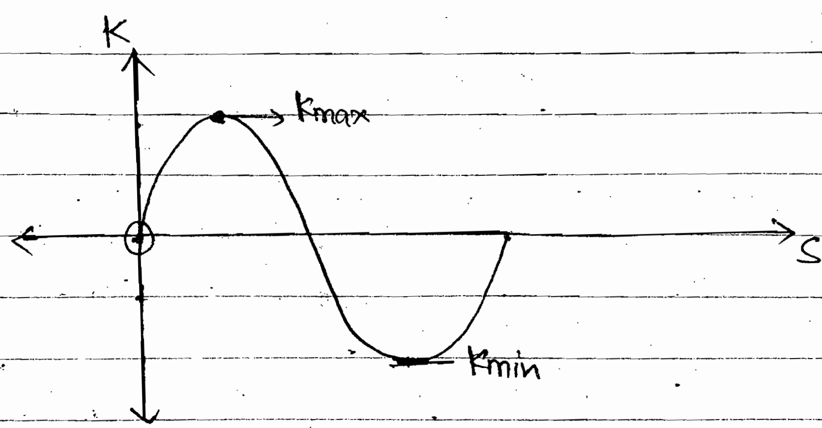




• It is not symmetrical graph.

* Example :- $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+13)}$





Centroid :-

- It is the origin of Asymptotic line. The centroid (X) will be :

$$\boxed{X = \frac{\Sigma P - \Sigma Z}{P - Z}}$$ * ΣP = Sum of the location of Open Loop pole.

ΣZ = Sum of the location of Open Loop Zero.

P = Total number of open Loop poles.

Z = Total number of Open Loop zero.

Angle of Asymptotes :-

- We will calculate Asymptotic angle only for that branch of Root Locus which is either originating or terminating to or from Infinity.

Case:-I → P > Z :-

From p number of open loop poles, p branch of Root Locus ~~and~~ branch will originate and out of p branches, z number of branches will terminate on z number of open loop zeros. and remaining (p-z) number of branch will terminate at ∞. We will calculate asymptotic angle only for (p-z) number of branches which will terminate at ∞.

$$\boxed{\theta_K = \frac{(2K+1)180^\circ}{P-Z}}$$ * $K = 0, 1, 2 \dots K_{max}$

$\boxed{K_{max} = P - Z - 1}$ *.

- For $G(s)H(s) = \frac{K}{s(s+2)}$

$\Rightarrow \Sigma p = -2 \quad , \quad \Sigma Z = 0.$

$\therefore X = \frac{-2-0}{2-0} \Rightarrow X = -1$

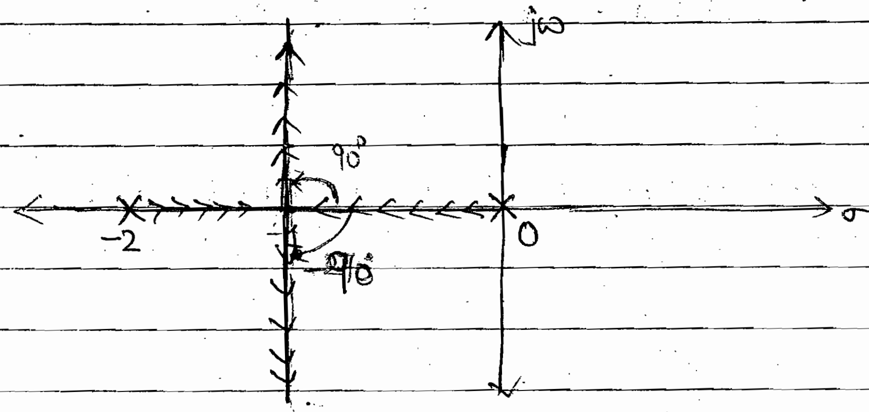
Now, $K_{max} = P-Z-1 = 2-0-1 = 1$

$\therefore K_{max} = 0+1 = 1.$

$\therefore K = 0, 1.$

Now, $\theta_K = \dfrac{(2K+1)180^\circ}{P-Z} \Rightarrow \theta_0 = \dfrac{180^\circ}{2} \Rightarrow \theta_0 = 90^\circ.$

$\theta_1 = \dfrac{3 \times 180^\circ}{2} = 270^\circ.$



* **Example:-** $G(s)H(s) = \dfrac{K(s+2)}{s(s+2)+2}$

$\therefore P = 2, \quad Z = 1, \quad K_{max} = 2-1-1 = 0.$

$\therefore K = 0.$

$\Sigma P = -2, \quad \Sigma Z = -2$

$P = 2 \quad Z = 1. \quad \therefore \theta_K = \dfrac{(2K+1)180^\circ}{P-Z}$

$X = \dfrac{-2+2}{2-1}$ $\quad \boxed{\theta_0 = 180^\circ}$ *

$\therefore \boxed{X = 0.}$ *.

- Root Locus of this OLTF is drawn earlier.

• For, $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+13)}$.

Now, $K_{max} = 4-0-1 = 3$, $K = 0,1,2,3$.

$\therefore \theta_0 = 45^\circ$, $\theta_1 = 135^\circ$, $\theta_2 = 225^\circ$, $\theta_3 = 315^\circ$.

$\Sigma P = -8$ $\Sigma Z = 0$.

$\therefore X = \dfrac{-8-0}{4} = -2$.

* Case:- II :- $Z > P$.

• Here, from p number of open loop poles, p branch of Root Locus will originate and all p branch will terminate at p number of zeros. For termination of Remaining $(Z-P)$ number of branch, Root Locus will originate from Infinity and we will calculate Angle of Asymptotes only for those branches of Root Locus which are originating from ∞.

$$\boxed{\theta_K = \frac{(2K+1)180^\circ}{Z-P}}, \quad K = 0,1,2,\ldots(Z-P-1).$$

* Consider :- $G(s)H(s) = \dfrac{Ks(s+1)(s+3)}{(s+2)}$

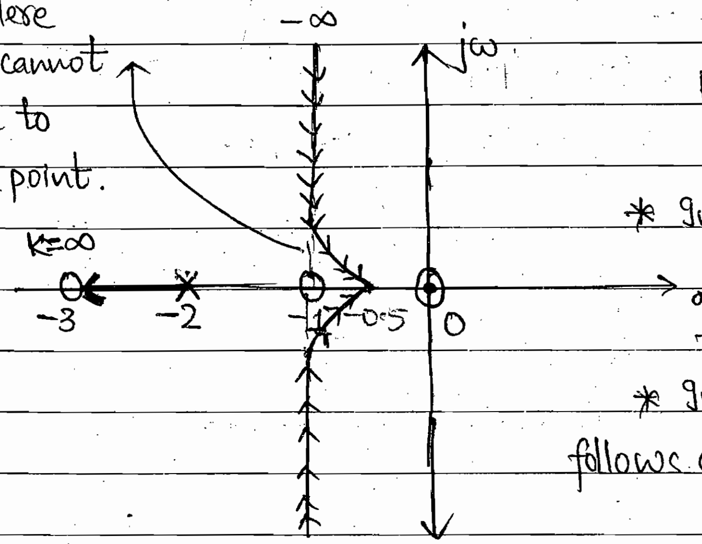
$\Rightarrow Z = 3$, $P = 1$, $\Sigma Z = -4$, $\Sigma P = -2$.

$\therefore X = \dfrac{-4+2}{2} = -1$

$\therefore K_{max} = (3-1)-1 = 1$, $K = 0,1$.

$\therefore \theta_0 = 90^\circ$, $\theta_1 = 270^\circ$.

- Here Centroid cannot be equal to Break-in point.



Here, Break-point is : 0.5

* In first case, (P-Z), Root Locus follows Asymptotic line for higher value of K.

* In second case (Z>P), Root Locus follows asymptotic line for lower value of K.

*→ GAIN MARGIN :-

$$\boxed{\text{Gain Margin} = \frac{K_{\text{Marginally stable}}}{K_{\text{Desired}}}} \;*$$

- To calculate $K_{\text{Marg.stable}}$, we will use R-H criterion. In R-H criterion, we will calculate that value of K for which odd row becomes 0 and then, we will substitute that value of K in even row just above that odd row, then we will calculate location of poles. If poles are located at Imaginary-axis for this value of K, then that K will be

* For Closed Loop system to be Stable should be >1 and in dB, it should be positive provided OLTF is Minimum phase system.

* If OLTF is Non-minimum phase system, then for closed loop system to be stable, Gain Margin in ratio should be <1 and in dB, it should be Negative.

Angle of Departure :-

- We will calculate Angle of Departure only for that open Loop pole which lies at complex-conjugate location.

$$\boxed{\theta_d = 180^\circ - [\phi_p - \phi_z]}$$

* ϕ_p = Total angle subtended by remaining pole towards that pole whose departure angle to be calculated.

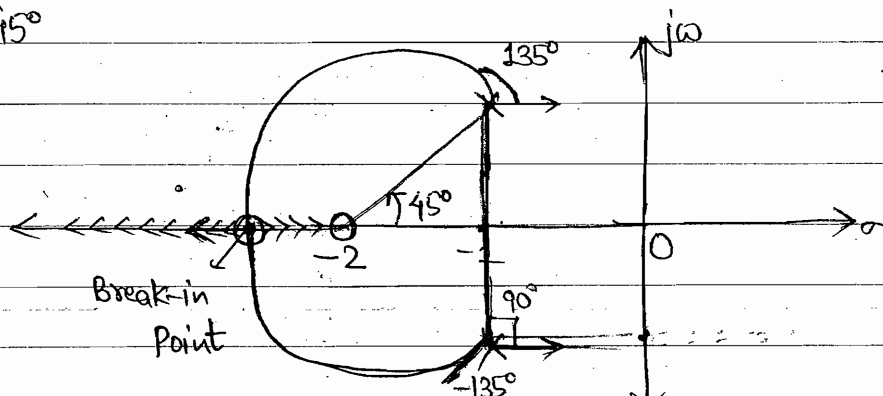
ϕ_z = Total angle subtended by odd open Loop zeros towards that complex-pole whose departure angle, we are calculating.

* Example :- $G(s)H(s) = \dfrac{K(s+2)}{s^2+2s+2}$

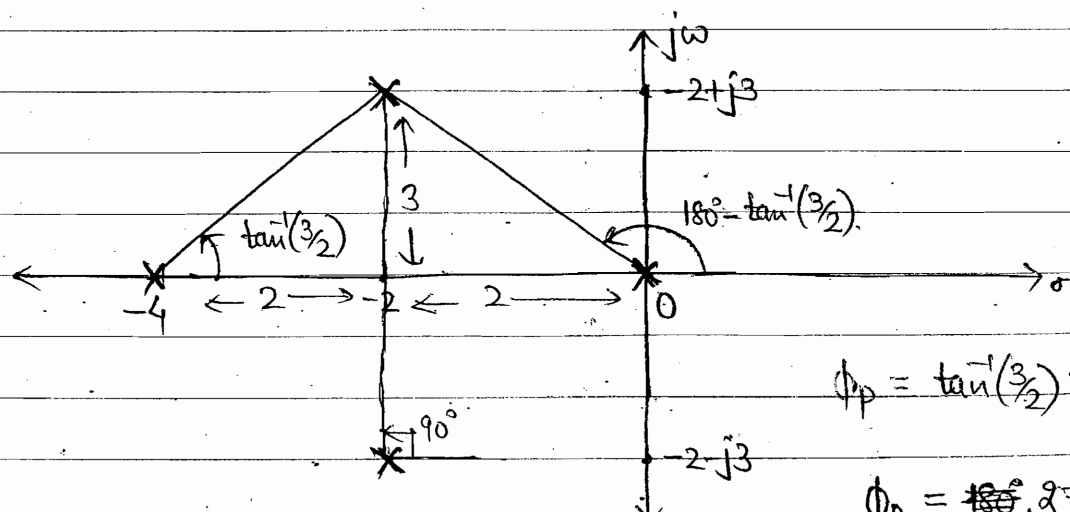
$\phi_p = 90^\circ$

$\phi_z = 45^\circ$

$\phi_d = 135^\circ$



* Example :- $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+13)}$



$\phi_p = \tan^{-1}(3/2) + 180^\circ - \tan^{-1}(3/2) + 90^\circ$

$\phi_p = \cancel{180}\ 270^\circ$

$\phi_z = 0°$, $\boxed{\phi_d = 180°}$ * or $\boxed{\phi_d = 0}$ *

* → **Angle of Arrival :-**

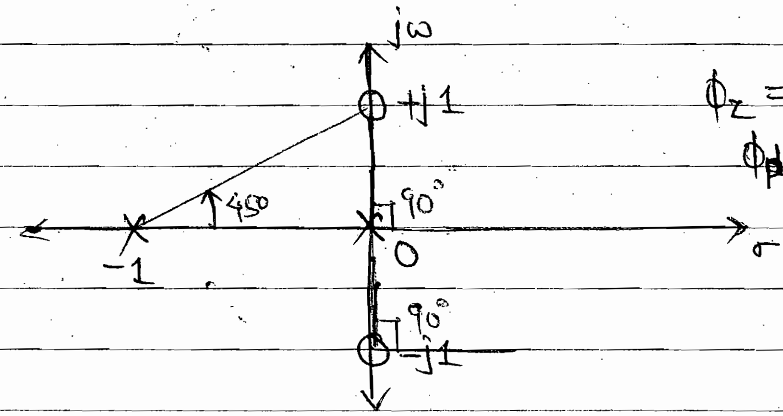
We calculate Angle of Arrival only for complex-zero.

$$\boxed{\theta_a = 180° - [\phi_z - \phi_p]}$$

* ϕ_z = Total angle subtended by remaining zero towards that complex-zero whose arrival angle we are to calculate.

ϕ_p = Total angle subtended by all open loop poles towards that zero whose arrival angle we are to calculate.

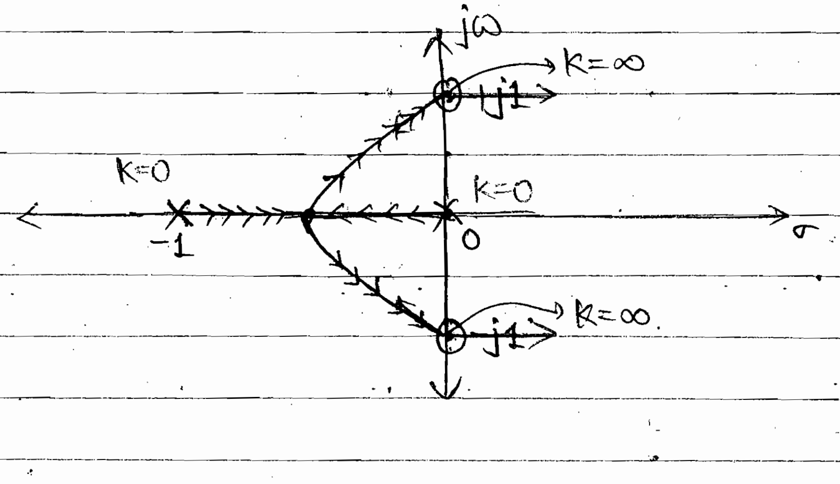
* Consider :- $G(s)H(s) = \dfrac{K(s^2+1)}{s(s+1)}$



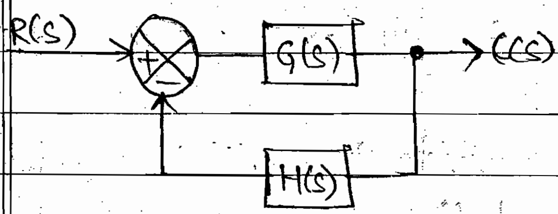
$\phi_z = 90°$

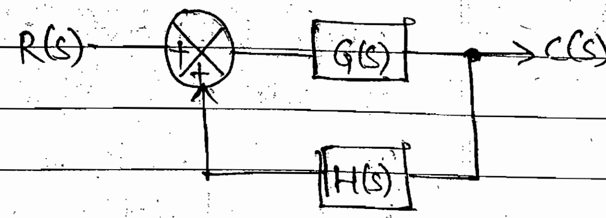
$\phi_p = 135°$

$\phi_a = 180° - [90° - 135°] = 225°$



⁎→ Root Locus Plot For Positive Feedback system :-





$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+\{G(s)H(s)\}} \qquad\qquad \frac{C(s)}{R(s)} = \frac{G(s)}{1+\{-G(s)H(s)\}}$$

$\therefore\ G(s)H(s) = -1$ $\qquad\qquad$ $G(s)\cdot H(s) = 1$

or $G(s)H(s) = 1 \angle (2k+1)\,180^\circ$. $\qquad\qquad$ or $G(s)H(s) = 1 \angle (2k)\,180^\circ$.

- If we provide phase lead or phase lag of 180° in Negative feedback system, then it converts to Positive Feedback System.

- $G(s)H(s) = \dfrac{+K}{s(s+2)}$, $\qquad\qquad$ - $G(s)H(s) = \dfrac{-K}{s(s+2)}$,

$0 < K < \infty$ $\qquad\qquad$ $-\infty < K < 0$.

implies Negative Feedback system. $\qquad\qquad$ implies Positive Feedback system (with −ve sign already used, range of K is: $0 < K < \infty$).

⁎→ Existence of Root Locus on Real-axis :-

- Root Locus will exist only at that section of Real axis where total angle subtended by all open loop poles and zeros towards observation point is :- Even Integral Multiple of 180°.

Or in other words, Root Locus exists only at that section of Real-axis which contains OL poles and zeros towards its Right side.

Existence of Root Locus on Complex-plane :-

- Root Locus will exist only at that location in complex plane where total angle subtended by all open loop poles and zeros towards observation point is : Even integral multiple of 180° or in other words, we will substitute the given complex-location in characteristics equation and then we will calculate the value of K. If K is real and positive, for the given complex location, then that location is valid location and closed loop poles will exist at that location.

* If K is either Imaginary, Negative or complex for the given location, then that location will be invalid location and closed loop poles will not exist at that location.

Angle of ~~Departure~~ :- Asymptotes :-

Case :- I :- $P > Z$:-

$$\theta_K = \frac{(2K)180^\circ}{P-Z}$$

$$K = 0, 1, 2 \ldots (P-Z-1)$$

Case : II :- $Z > P$:-

$$\theta_K = \frac{(2K)180^\circ}{Z-P}$$

$$K = 0, 1, 2 \ldots (Z-P-1)$$

Angle of Departure :-

$$\boxed{\theta_d = 0^\circ - (\phi_p - \phi_z)} \ *$$

Angle of Arrival :-

$$\boxed{\theta_a = 0^\circ - (\phi_z - \phi_p)} \ *$$

Numericals

Ques:- (1) The open loop TF of a system is:- $G(s)H(s) = \frac{K}{s(s+4)}$, $0 < K < \infty$

Draw its Root Locus and determine all possible damping conditions for different value of K. Also calculate value of K for $s = -2+j2$.

Ans:- (1) $s_{P_1} = 0$, $s_{P_2} = -4$

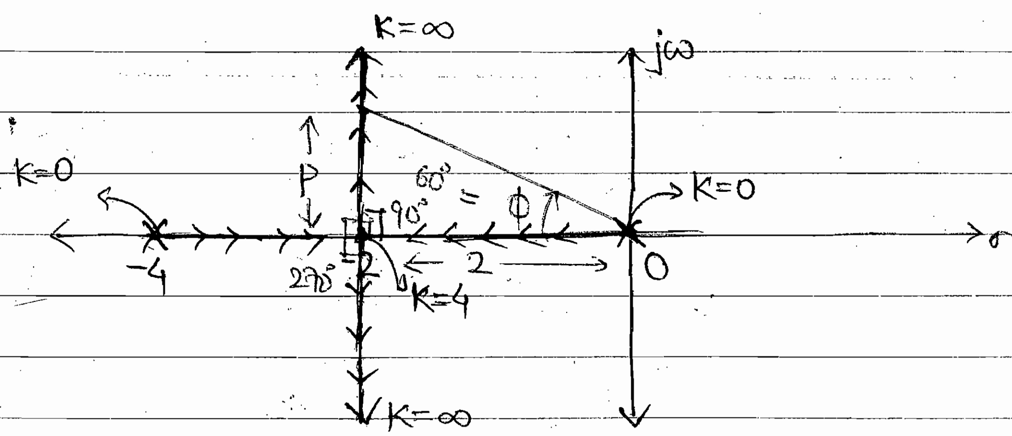
$$q(s) = 1 + G(s)H(s) = 0.$$

$$1 + \frac{K}{s(s+4)} = 0.$$

$\therefore$ $s^2 + 4s + (4+K) = 0.$ $\Rightarrow -K = s^2 + 4s.$

$$\text{or } -\frac{dK}{ds} = 2s + 4 = 0.$$

$\therefore$ $\boxed{s = -2}$ *



Now, $K\big|_{s=-2} = -[4-8] = 4.$ $\therefore K\big|_{s=-2} = 4.$

Now, $K_{max} = 2-0-1 = 1.$ $\therefore K = 0, 1.$

$\therefore$ $\theta_0 = 90°$, $\theta_1 = 270°$.

$$X = \frac{\Sigma P - \Sigma Z}{P-Z} = \frac{-4-0}{2} = -2.$$

As $q(s) = s^2 + 4s + (k) = 0.$

s^2	1	k
s^1	4	
s^0	k	

• As odd row does not become 0, so Root Locus does not cut Imaginary axis.

* Here, as for any value of K, as closed loop poles are on LHS of s-plane, so system will be: Highly stable [GM : +∞].

* $0 < K < 4$; Overdamped system ($\xi > 1$).
$K = 4$; Critically damped system ($\xi = 1$).
$K > 4$; Under-damped system ($\xi < 1$).
and system will never be undamped system.

As $K = -s^2 + 4s.$

$$\therefore K\Big|_{(-2+j2)} = -\left[(-2+j2)\left[-2+j2+4\right]\right] = -\left[(-2+j2)(2+j2)\right]$$

$$K\Big|_{(-2+j2)} = -\left[-4-4\right]$$

$$\boxed{K\Big|_{s=-2+j2} = 8} *.$$

* for $\xi = 0.5$, calculate K.

$\therefore \xi = \cos\phi \Rightarrow \cos\phi = 1/2$

$$\therefore \boxed{\phi = 60^\circ} *.$$

$$\tan\phi = \frac{P}{2} \Rightarrow \boxed{P = 2\sqrt{3}} *.$$

Ques:-(2) The OLTF of a system: $G(s)H(s) = \frac{-K}{s(s+4)}$.

- Draw its Root Locus and comment on stability.

Ans:-(2) $S_{p_{1,2}} = 0, -4.$

$$q(s) = 1 - G(s)H(s) = 0$$

$$\text{or } 1 - \frac{K}{s(s+4)} = 0.$$

$$\text{or } \frac{dK}{ds} = 2s+4 \quad \therefore \boxed{s=-2} *.$$

$K_{max} = (2-0)-1 = 1 \quad \therefore K = 0, 1.$

$\therefore \theta_0 = 0°, \quad \theta_1 = 180°.$

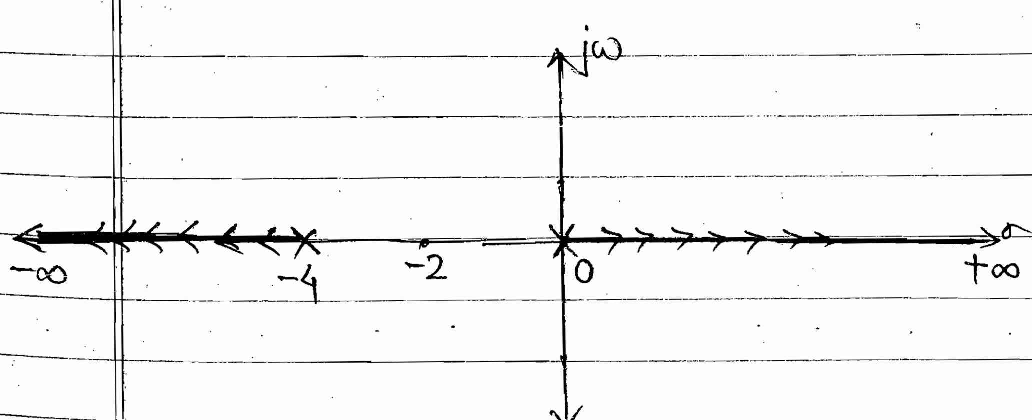
$\therefore X = \frac{\Sigma P - \Sigma Z}{P-Z} \Rightarrow X = \frac{-4-0}{2} = -2.$

$\therefore$ The breakpoint $s=-2$ is invalid break-point.

$q(s) = s^2+4s-K = 0.$

s^2	1	$-K$
s^1	4	
s^0	$-K$	

$\therefore$ Pole on Right-half of s-plane = 1.



$\therefore$ This system is highly unstable, so GM is $-ve\ \infty$.

Ques:(3) The open loop TF of a system is :- $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$, $0<K<\infty$.

Draw its Root Locus and determine Gain Margin at $K=6$.

Ans:-(3) $S_{p} = 0, -1, -3.$

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \frac{K}{s(s+1)(s+3)} = -1.$$

$$\Rightarrow K = -s[s+1][s+3]$$

$$\therefore s = -0.45, -2.21.$$

$$K\Big|_{s=-0.45} \equiv 0.63$$

As $\theta_K = \frac{(2K+1)}{180°(P-Z)} \Rightarrow \theta_K = \frac{(2K+1)180°}{3}$

$$\therefore \theta_0 = 60°, \theta_1 = 180°, \theta_2 = 300°.$$

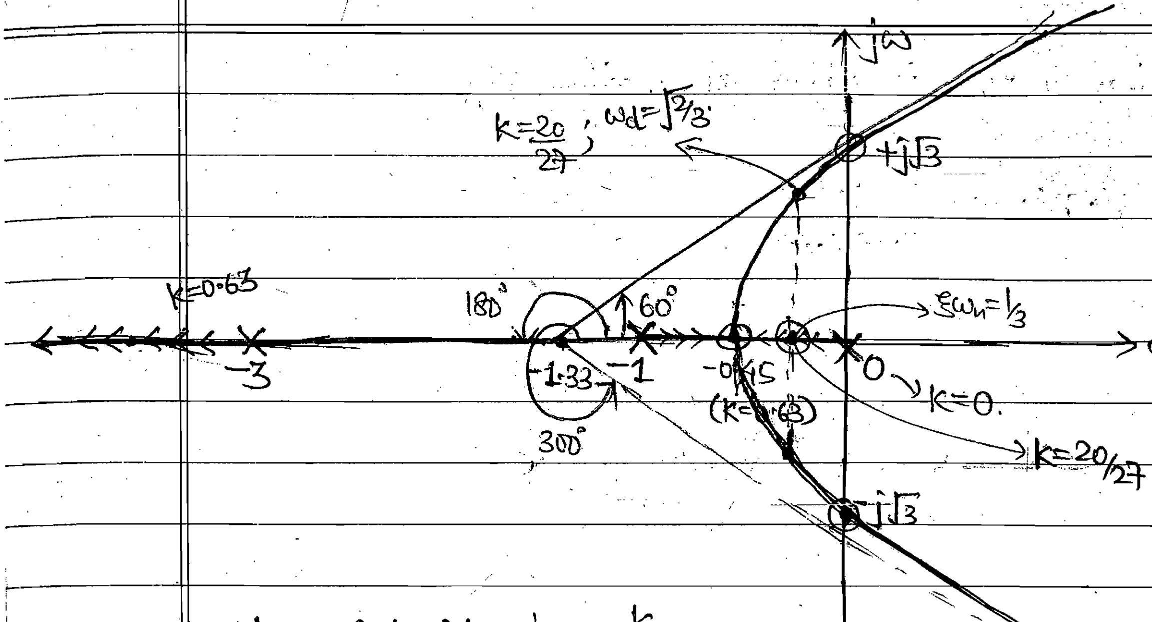
Now $X = \frac{\Sigma P - \Sigma Z}{P-Z} = \frac{-4-0}{+3} = -4/3 = -1.33$

Now, $q(s) = s^3+4s^2+3s+K = 0.$

s^3	1	3
s^2	4	K
s^1	$\frac{(12-K)}{4}$	
s^0	K	

$\Rightarrow$ $\boxed{K=12}$ * for system to be Marginally stable.

$\therefore 4s^2 + K = 0 \Rightarrow 4s^2 = -12 \Rightarrow s = \pm j\sqrt{3}.$



$\omega_n = \frac{\sqrt{7}}{3}$ and it is not constant here.

Now, Gain Margin $= \frac{K_{M.S}}{K_{desired}}$

$\therefore GM = \frac{12}{6}$

$\therefore \boxed{GM = 2}$ * $\qquad GM(dB) = 20\log(2)$

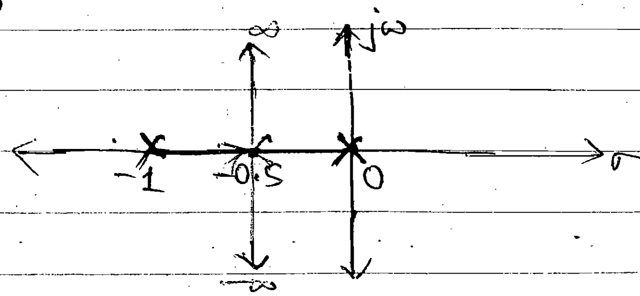
$\boxed{GM(dB) = 6.02 \text{ dB}}$ * (stable system).

- For $K_{desired} = 24$, $\qquad GM = \frac{12}{24} = \frac{1}{2}$.

$\therefore \boxed{GM(dB) = -6.02 \text{ dB}}$ * (Unstable system).

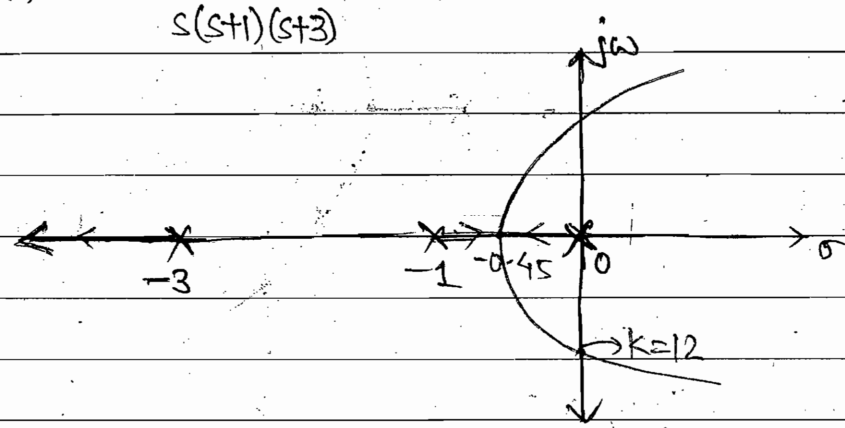
*→ Effect of Adding Pole in any system:-

* Example:- $G(s)H(s) = \frac{K}{s(s+1)}$.



- Now, when a pole is added at location $s=-3$.

So, $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$



* **Note**:- * With addition of pole in any system, its stability decreases because Root Locus which is locus of pole of closed Loop system will shift towards Right-side.

As stability depends on t_s, hence when stability will decrease, t_s will increase, $\% M_p$ will also increase.

Ques:- (4) The OLTF of a system is:- $G(s)H(s) = \dfrac{-K}{s(s+1)(s+3)}$; $0<K<\infty$.

- Draw its Root Locus and comment on the stability of the system.

Ans:- (4) $S_p = 0, -1, -3$, $G(s)H(s) = -1$, $K\big|_{s=-2.21} = 2.11$.

$\therefore \dfrac{dK}{ds} = 0$ gives : $s = -0.45, -2.21$.

$s = -2.21$ will be valid point.

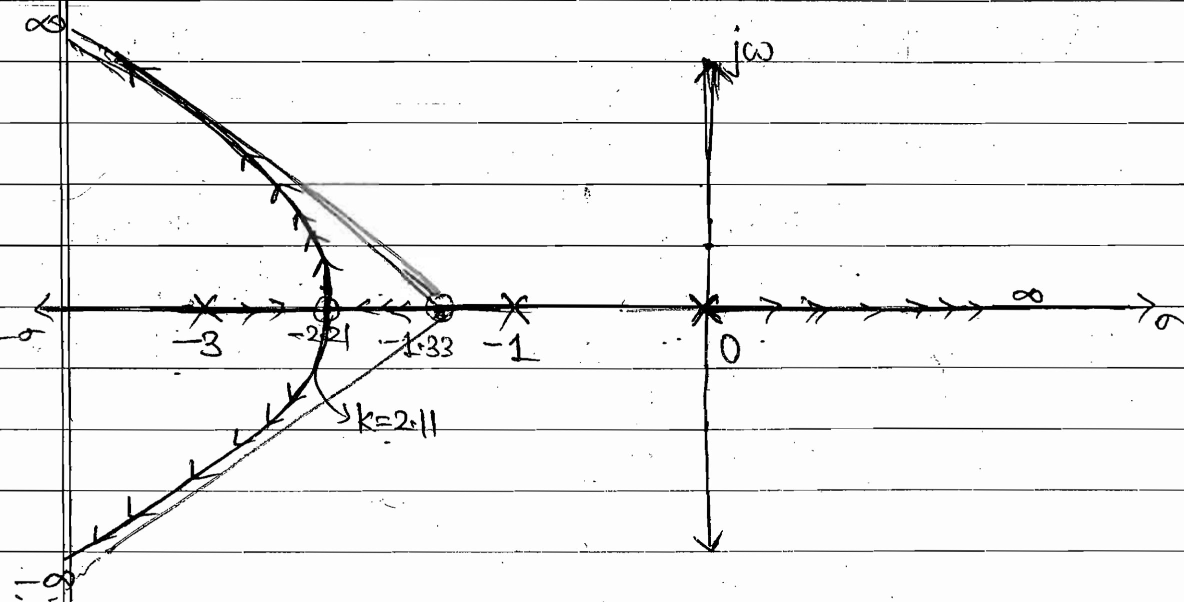
$\theta_0 = 0^\circ$, $\theta_1 = 120^\circ$, $\theta_2 = 240^\circ$ and $X = -1.33$.

$\therefore q(s) = s^3 + 4s^2 + 3s - K = 0.$

s^3	1	3
s^2	4	$-K$
s^1	$(12+K/4)$	
s^0	$-K$	

$\therefore$ $\boxed{K=-12}$ * (For Marginally Stable)

$\therefore$ It is invalid value of K as $0<K<\infty$.



Ques:- (5) The OLTF of a system is: $G(s)H(s) = \frac{K(s+1)^2}{(s+2)^2}$, $0<K<\infty$

Determine the possible damping conditions for different value of K.

Ans:- (5) $S_p = -2, -2$, $S_z = -1, -1$.

Now, $K = -\frac{(s+2)^2}{(s+1)^2}$

or $\frac{dK}{ds} = \frac{2(s+1)^2(s+2) - 2(s+2)^2(s+1)}{(s+1)^4} = 0$.

$\Rightarrow \cancel{2}(s+1)^{\cancel{2}}\cancel{(s+2)} = \cancel{2}(s+2)^{\cancel{2}}\cancel{(s+1)}$

$\Rightarrow (s+1) = (s+2)$. or $(s+1)(s+2) = 0$

$s = -1, -2$.

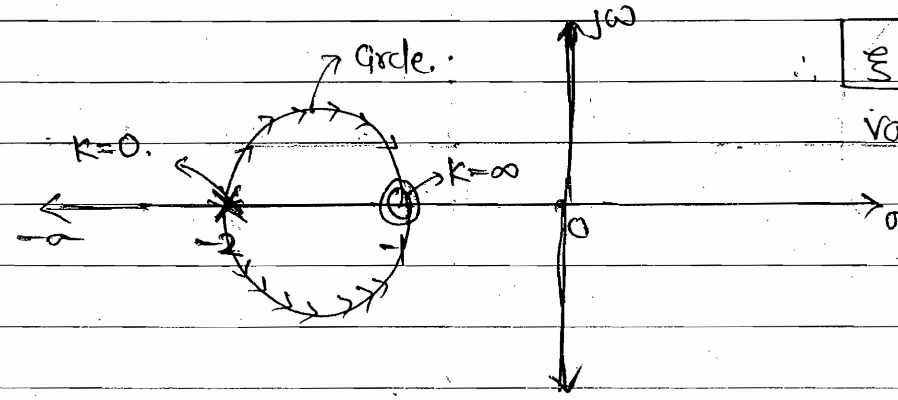
$K|_{s=-1} = \infty$ $K|_{s=-2} = 0$.

$$\Rightarrow \; q(s) = (s+2)^2 + K(s+1)^2 = 0.$$

$$\text{or} \quad (1+K)s^2 + (4+2K)s + (4+K) = 0.$$

s^2	$(1+K)$	$4+K$
s^1	$(4+2K)$	
s^0	$K+4$.	

$\boxed{K=-2}$ * For Marginally stable.
↳ It is invalid value.



∴ $\boxed{\xi < 1}$ * For all values of K, system will be underdamped.

Ques:- 6 The OLTF of a system is : $G(s)H(s) = \dfrac{-K(s+1)^2}{\cancel{(s+1)^2}(s+2)^2}$. Draw its Root Locus and determine the range of K for system to be stable.

Ans:- 6 $s = -1, -2$ here.

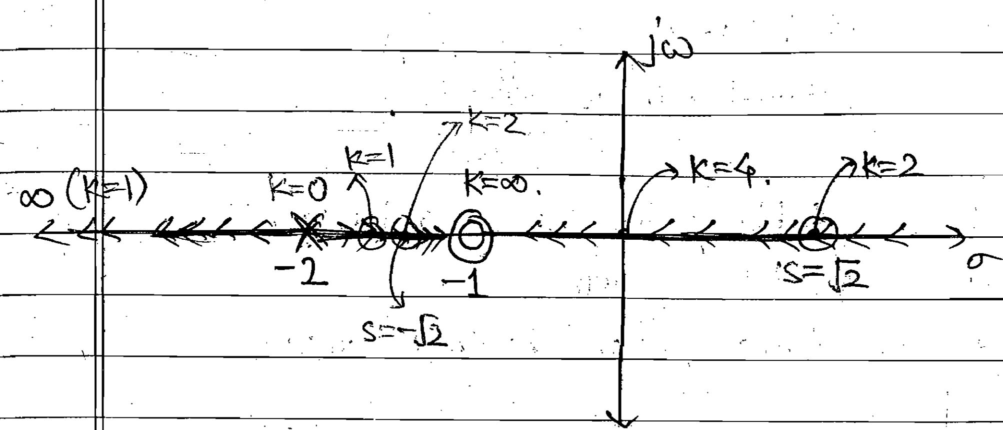
$$q(s) = (s+2)^2 - K(s+1)^2 = 0 = (1-K)s^2 + (4-2K)s + (4-K) = 0.$$

$$\therefore \quad 2K - 4 = 0 \qquad \boxed{K=2}$$ * For Marginally stable.
↳ Valid K.

$$s^2(1-K) + (4-K) = 0$$

$$\Rightarrow s^2(-1) + 2 = 0 \Rightarrow s^2 = +2$$

~~$s = \pm j1$.~~ $s = \pm\sqrt{2} + j0$



* Explanation :- $K = 0.1$

$$0.9s^2 + 3.8s + 3.9 = 0 \Rightarrow s = -1.75, -2.46.$$

$K = 0.5$, $\quad 0.5s^2 + 3s + 3.5 = 0$

$$s = -1.58, -4.41.$$

$K = 0.8$, $\quad 0.2s^2 + 2.4s + 3.2 = 0.$

$$s = -1.52, -10.47.$$

$K = 0$,

$$0.1s^2 + 2.2s + 3.1 = 0.$$

$$s = -1.51, -20.48.$$

$K = 1$,

$$(2s+3) = 0 \quad , \quad s = -1.5.$$

$K = 1.5$, $\quad -0.5s^2 + s + 2.5 = 0.$

$$s = -1.44, +3.44.$$

$K = 2$, $\quad -s^2 + 2 = 0$

$$s = \pm\sqrt{2}.$$

$K = 4$, $\quad -3s^2 - 4s = 0. \Rightarrow s(3s+4) = 0$

$$s = 0, -4/3.$$

$K = 5, \quad -4s^2 - 6s - 1 = 0$

$s = -1.3, -0.19.$

$\therefore 0 < K < 1$:- Stable system cum over-damped. $(\xi > 1)$.

$1 < K < 4$:- Unstable system

$K > 4$:- Stable system and Over-damped. $(\xi > 1)$.

Note:- • The above case of switching root locus from $-\infty$ to $+\infty$ will exist only under one condition when in open loop transfer function, Order of Pole = Order of zero and Feedback is Positive Feedback.

* A Positive feedback system cannot be highly stable but Negative feedback system may be
  • Positive Feedback system may remain Stable under finite range of K but it will always become unstable for range of K.

* A positive feedback system may be highly unstable.

Ques: 7 The OLTF of a system is: $G(s)H(s) = \dfrac{K(s+1)}{(s^2+0.4s+0.4)}, \; 0 < K < \infty.$

Draw its Root Locus and determine the value of K when system will be critically-damped and also determine the range of K when system is under-damped and over-damped.

Ans:- 7 $S_p = -0.2 \pm j0.6 \quad , S_z = -1.$

$q(s) = 1 + G(s)H(s) = 0 \Rightarrow K = \dfrac{-(s^2+0.4s+0.4)}{(s+1)}$

$$dK/ds = \frac{-[(s+1)(2s+0.4) - (s^2+0.4s+0.4)(1)]}{(s+1)^2}$$

$s = 0, -2. \quad \therefore K\big|_{s=0} = -0.4.$

$K\big|_{s=-2} = +ve \rightarrow \cancel{+3.6}$

$\theta_0 = 180^\circ$, $\alpha = \frac{(-0.2-0.2)-(-1)}{1} \Rightarrow \alpha = 0.6.$

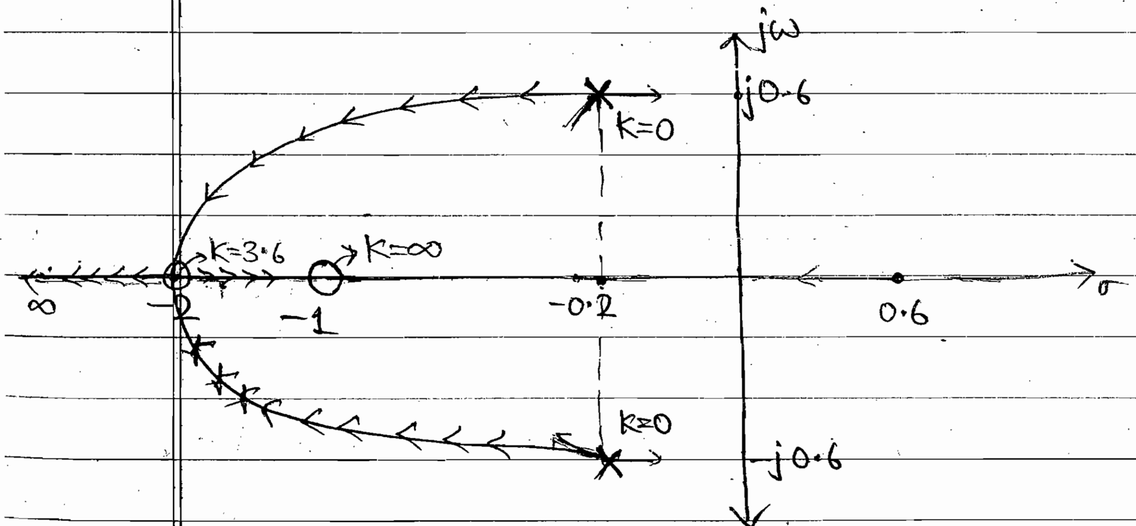
Now, $\phi_d = 180^\circ - [\phi_p - \phi_z]$

$$\phi_{d(2)} = 180^\circ - [90^\circ - 37^\circ] = 127^\circ$$

$$\phi_{d(1)} = -127^\circ.$$

$$q(s) = s^2 + 0.4s + 0.4 + K(s+1) = 0$$
$$q(s) = s^2 + s(0.4+K) + (0.4+K) = 0.$$

$\therefore$ $\boxed{K = -0.4}$ * for Marginally stable.
↳ Invalid point.



$0 < K < 3.6$:- $\zeta < 1$; Under-damped.
$K = 3.6$:- $\zeta = 1$; critically-damped.
$K > 3.6$:- $\zeta > 1$; Over-damped.

Ques:(8) The open Loop TF is $G(s)H(s) = \frac{-K(s+1)}{s^2+0.4s+0.4}$, $0<K<\infty$.

Plot its Root Locus and determine range of K for system to be stable.

Ans:-(8) $s_p = -0.2 \pm j0.6$, $s_z = -1$.

$dK/ds = 0 \Rightarrow$ ~~$s = -0.4, +0.36$~~. $s = 0, -2$.

$K|_{s=0} = -0.4$.

$\theta_0 = 0°$, $x = 0.6$.

$\theta_{d(2)} = 0 - [\phi_p - \phi_z] = -53°$.

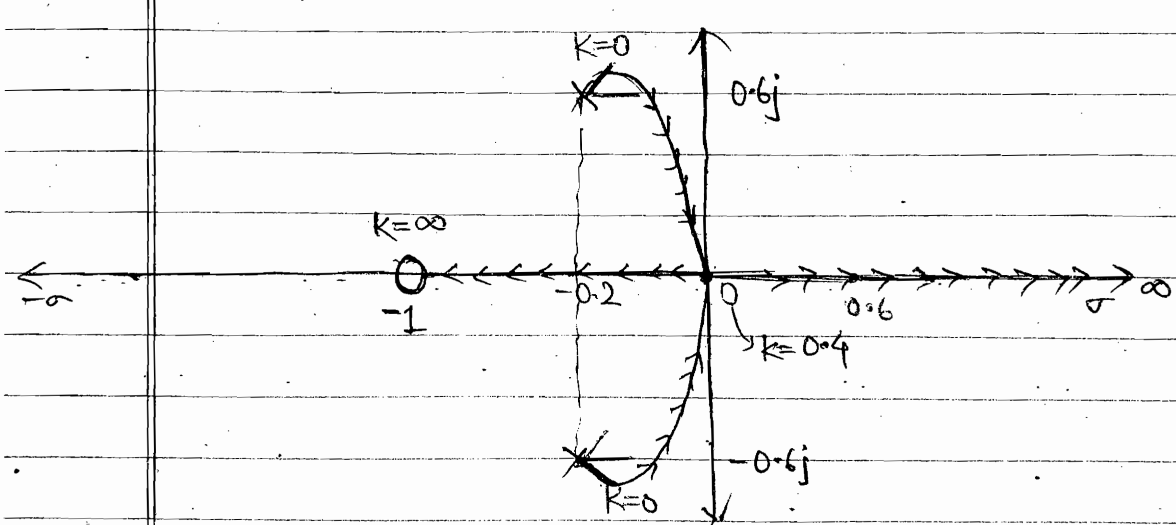
$\theta_{d(1)} = 53°$.

$q(s) = s^2 + (0.4-K)s + (0.4-K) = 0$.

$\therefore$ $\boxed{K = 0.4}$ * For Marginally stable.

↳ valid K.

$\therefore$ $s^2 + 0 = 0$ $s = 0$. [which is also breakpoint].



** • If any OLTF contains ~~0~~ zero in the right-half side, then closed loop system will always become unstable for higher value of k.

** • If any OLTF contains ~~zero~~ Pole in the right-half side, then its closed loop system will always remain unstable for lower values of K.

Ques:- (9) The OLTF of a system is: $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+13)}$. Draw its Root Locus and determine the value of K for system to be Marginally stable. Also, calculate undamped Natural frequency.

Ans:- (9) $S_p = 0, -4$, $-K = (s^2+4s)(s^2+4s+13)$.

$s = -2$; $s = -2 \pm j(1.57)$.

• For break-away point, to calculate it from complex-poles, use Angle condition.

$$\phi = -90^\circ + 90^\circ + \tan^{-1}\left(\frac{1.57}{2}\right) + 180^\circ - \tan^{-1}\left(\frac{1.57}{2}\right) = 180^\circ$$

As ϕ = Odd multiple of 180°, so, $s = -2 \pm j(1.57)$ may be the break-point.

$\theta_0 = 45^\circ$, $\theta_1 = 135^\circ$, $\theta_2 = 225^\circ$, $\theta_3 = 315^\circ$

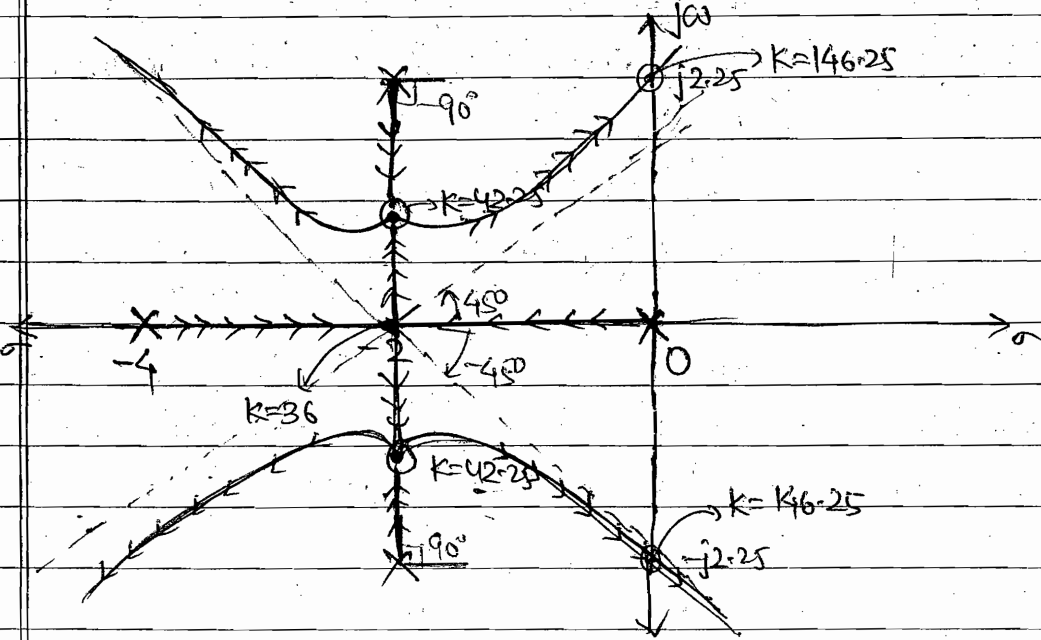
$$x = \frac{-8-0}{4-0} = -2.$$

$$\phi_d = 180^\circ - [\phi_p - \phi_z] = 180^\circ - [270^\circ - 0^\circ] = -90^\circ.$$

$$q(s) = (s^2+4s)(s^2+4s+13) + K = 0$$

$\therefore$ $\boxed{K = 146.25}$ * (For Marginally stable) valid

$\therefore \quad s = \pm j2.55$



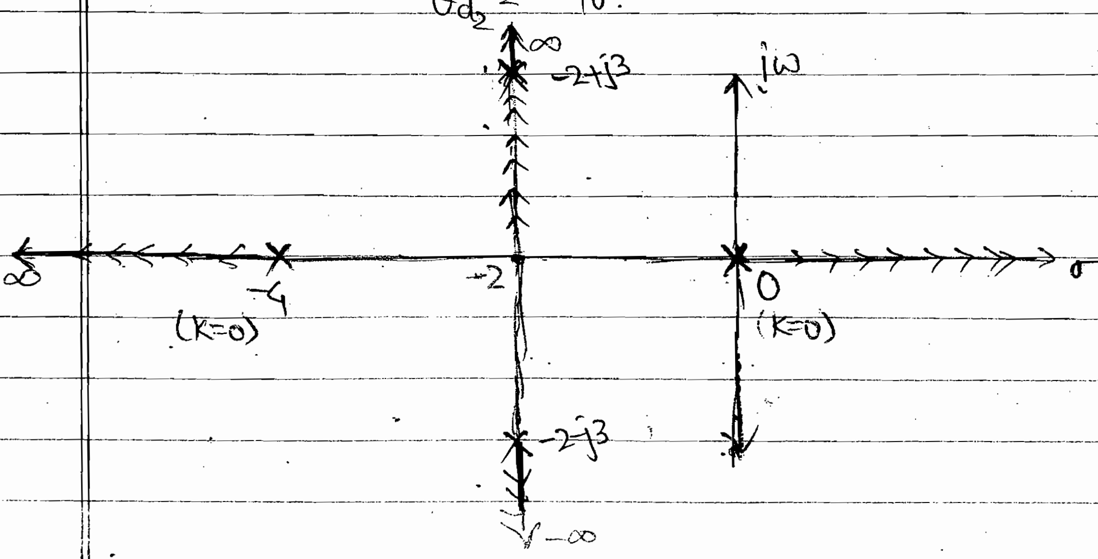
Ques:-(10) The OLTF is : $G(s)H(s) = \dfrac{-K}{s(s+4)(s^2+4s+13)}$, $0 < K < \infty$. Draw its Root Locus and comment on stability.

Ans:-(10) $s = -2 \; ; \; -2 \pm j(1.57)$.

$\theta_0 = 0°, \quad \theta_1 = 90°, \quad \theta_2 = 180°, \; \theta_3 = 270°.$

$X = -2 \quad , \quad \theta_{d_1} = 0 - [\phi_p - \phi_z] = -270° \cong +90°.$

$\theta_{d_2} = -90°.$



Ques:- 11 The closed loop transfer function of a system is:

$$\frac{C(s)}{R(s)} = \frac{1}{s^2+(4-K)s+3}$$

Draw its Root locus.

Ans:- 11

$$\frac{C(s)}{R(s)} = \frac{1}{(s^2+4s+3)+(-Ks)}$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{1/(s^2+4s+3)}{\left[1+\left\{\frac{-Ks}{(s+1)(s+3)}\right\}\right]}$$

$\therefore$ Comparing, $G(s) = \dfrac{1}{(s+1)(s+3)}$, $H(s) = -Ks$.

$$\therefore \quad G(s)H(s) = \frac{-Ks}{(s+1)(s+3)}.$$

Now, $S_p = -1, -3,$ $S_z = 0.$

$\boxed{\theta_0 = 0^\circ}$ * $\therefore$, $K = +\dfrac{(s+1)(s+3)}{s}$

$$\Rightarrow dk/ds = 0 \Rightarrow \frac{dk}{ds} = 0$$

$$\therefore \quad s = \pm\sqrt{3} \quad (\text{valid}).$$

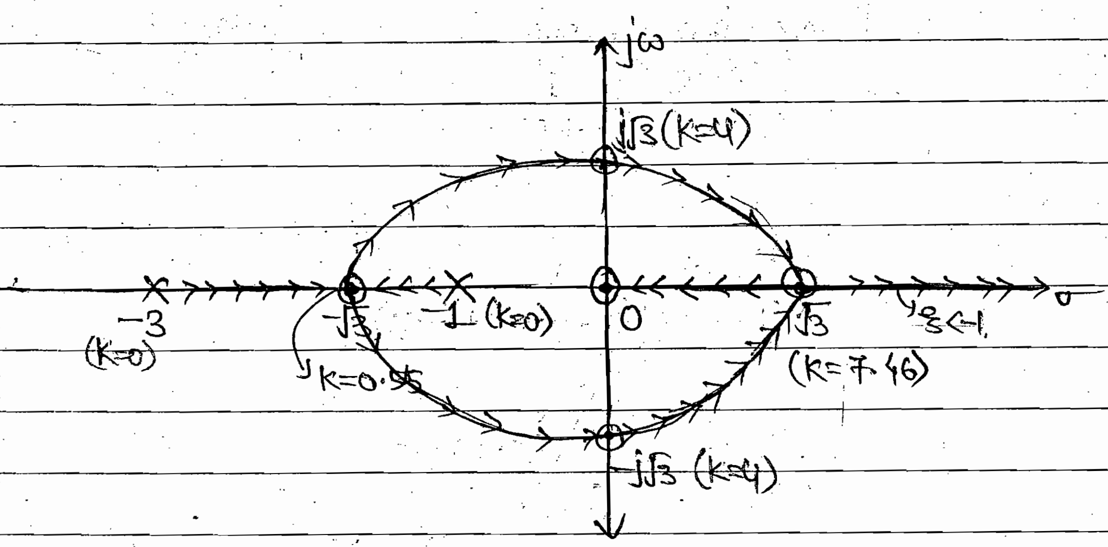
$$\therefore \quad K\Big|_{s=\sqrt{3}} = \frac{+(\sqrt{3}+1)(\sqrt{3}+3)}{\sqrt{3}} = +7.46. \quad (\text{Valid})$$

$$K\Big|_{s=-\sqrt{3}} = \frac{(-\sqrt{3}+1)(-\sqrt{3}+3)}{-\sqrt{3}} = 0.55. \quad (\text{Valid})$$

$$X = \frac{-4-0}{1} = -4.$$

$$q(s) = s^2+(4-K)s+3 = 0.$$

Here, $\omega_n = \sqrt{3}$, $\xi\omega_n = \dfrac{(4-K)}{2}$ or $\xi = \dfrac{(4-K)}{2\sqrt{3}}$



$q(s) = s^2 + (4-K)s + 3 = 0.$

$\therefore \quad 4 - K = 0 \Rightarrow \boxed{K=4}$ * For Marginally stable.
(valid).

$\therefore \quad s^2 + 3 = 0, \quad s = \pm j\sqrt{3}. \quad , \omega_n = \sqrt{3}$ rad/sec.

- Here, as ω_n is variable, $\xi\omega_n$ is variable and ξ is also variable, $\therefore$ system is unstable.

$0 < K < 0.55$; $\xi > 1$; over-damped.
$0.55 < K < 4$; $\xi < 1$; Under-damped.
$K > 7.46$; ~~$-1 < \xi < 1$~~ ; $\xi < -1$; (Unstable)
$K = 0.55$; $\xi = 1$; critically damped.
$K = 7.46$; $\xi = -1$; (Unstable)
$4 < K < 7.46$; $\xi < -1$; (Unstable).

* System will be unstable but can't be highly stable.

Ques:- (12) The OLTF is: $G(s)H(s) = \dfrac{K \cdot \omega_n^2}{s(s+2\xi\omega_n)}$, $0 < K < \infty$.

Ans:- (12) $S_p = 0, -2\xi\omega_n$.

or $K = \dfrac{-s(s+2\xi\omega_n)}{\omega_n^2}$

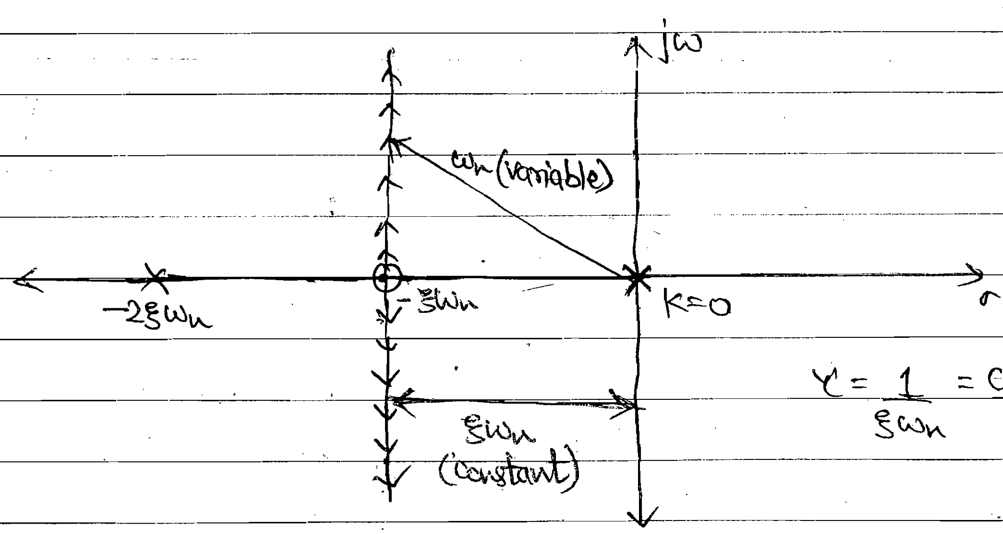
$\therefore dK/ds = 0$,

$s = -\xi\omega_n$. $\therefore K|_{s=-\xi\omega_n} = \xi^2$.

$\theta_K = \dfrac{(2K+1)180°}{P-Z} \Rightarrow K = 0, 1$

$\therefore \theta_0 = 90°$, $\theta_1 = 270°$.

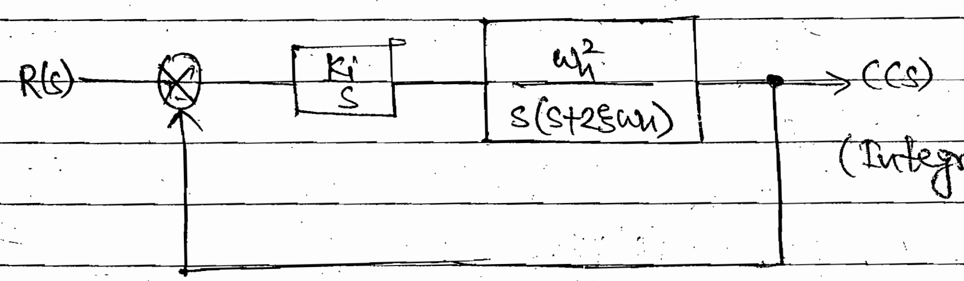
$x = \dfrac{\Sigma p - \Sigma z}{P-Z} = -\xi\omega_n$



$\tau = \dfrac{1}{\xi\omega_n}$ = Constant.

- So, system will be always highly stable.

* Understanding PI controller with Root Locus :-



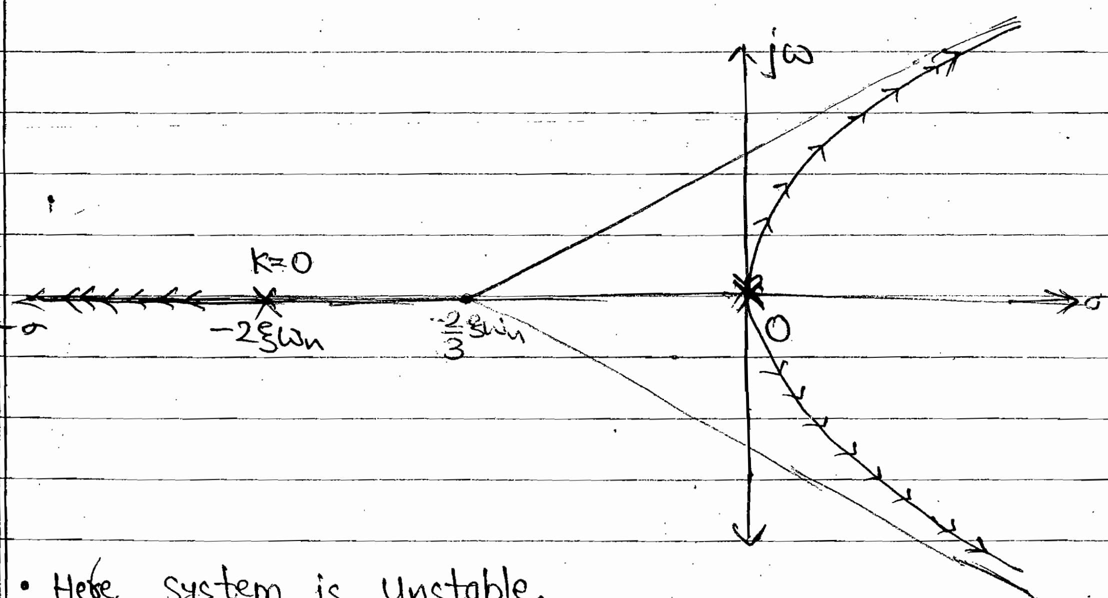
(Integral Controller)

$$G(s)H(s) = \frac{K_i \cdot \omega_n^2}{s^2(s+2\xi\omega_n)}$$

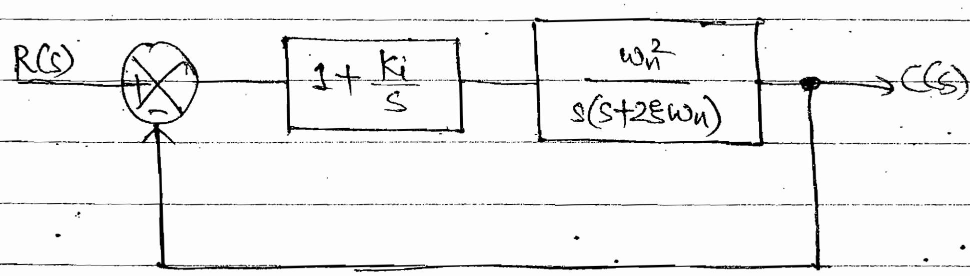
Here, $S_{Break} = 0$, $K_i\big|_{S_B=0} = 0$

$\theta_0 = 60^\circ$, $\theta_1 = 180^\circ$, $\theta_2 = 300^\circ$ [K=0,1,2 here].

$x = \frac{2}{3}\xi\omega_n$.



- Here, system is unstable.
  So, to overcome this, we use PI controller.



$$\therefore G(s)H(s) = \frac{(s+k_i)\cdot\omega_n^2}{s^2(s+2\xi\omega_n)}$$

Its, closed Loop Transfer function is:

$$T(s) = \frac{(s+k_i)\omega_n^2}{(s^3+2\xi\omega_n s^2+\omega_n^2 s)+k_i\omega_n^2}$$

Now, $$T(s) = \frac{\dfrac{(s+k_i)\omega_n^2}{s^3+2\xi\omega_n s^2+\omega_n^2 s}}{1+\dfrac{k_i\omega_n^2}{s[s^2+2\xi\omega_n s+\omega_n^2]}}$$

$$\therefore G(s)H(s) = \frac{k_i\cdot\omega_n^2}{s[s^2+2\xi\omega_n s+\omega_n^2]}$$

Here, $S_p = 0, -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$dk/ds = 0 \Rightarrow s =$ Invalid break-point.

$\theta_0 = 60^\circ, \quad \theta_1 = 180^\circ, \quad \theta_2 = 300^\circ$.

$x = -\frac{2}{3}\xi\omega_n$ (Centroid).

Using R-H Criterion: $q(s) = s^3+2\xi\omega_n s^2+\omega_n^2 s+k_i\omega_n^2 = 0$.

$$\therefore \begin{array}{c|cc} s^3 & 1 & \omega_n^2 \\ s^2 & 2\xi\omega_n & k_i\omega_n^2 \\ s^1 & \frac{2\xi\omega_n^3-k_i\omega_n^2}{2\xi\omega_n} & \\ s^0 & k_i\omega_n^2 & \end{array}$$

$\Rightarrow (2\xi\omega_n - k_i)\,\omega_n^2 = 0$

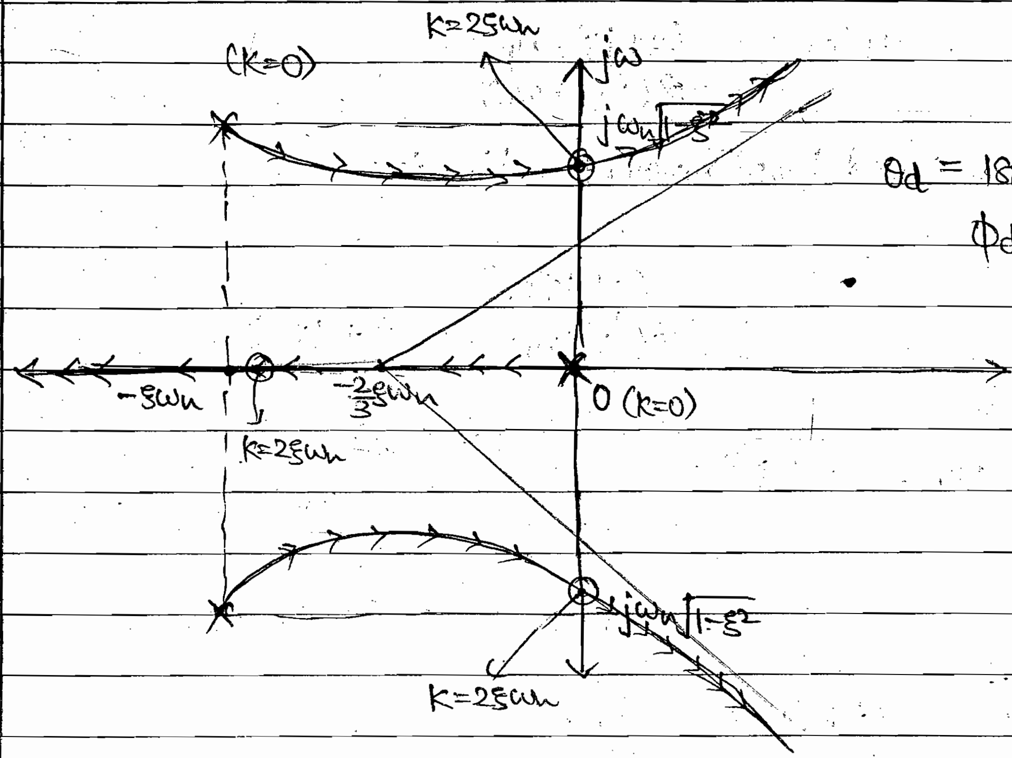
$\therefore$ ~~$\omega_n = 0, ; \omega_n\ k_i$~~

$\Rightarrow k_i = 2\xi\omega_n$.

$\therefore 2\xi\omega_n s^2 + k_i\omega_n^2 = 0$.

$\Rightarrow \cancel{2\xi\omega_n} s^2 = -k_i\omega_n^2 = -2\xi\omega_n^3$

$\boxed{s = \pm j\omega_n}$ *



$\theta_d = 180^\circ - [270 - 0 - \phi]$

$\phi_d = -90^\circ + \phi.$

$0 < K < 2\xi\omega_n$:- Stable system

$K = 2\xi\omega_n$:- Marginally stable system

$K > 2\xi\omega_n$:- Unstable system.

Chapter :- 8

NYQUIST PLOT

** In Nyquist Plot, the variable is entire s-plane while in Bode-plot, variable is positive frequency line.

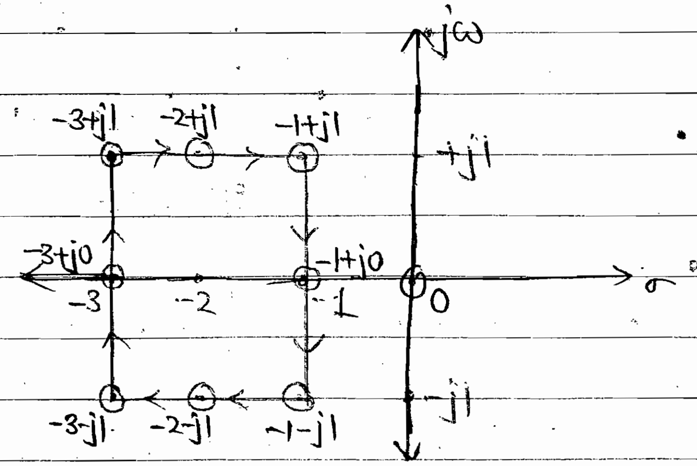
- If Open Loop Transfer function is Minimum Phase system, then for closed loop system to be stable (Minimum phase), both Gain Margin and Phase Margin should be Positive.
- If Open Loop TF is Non-minimum phase system (unstable system), then for closed loop system to be stable, both Gain Margin and Phase Margin should be Negative.

* Bode-plot defines only Relative Stability while Nyquist Plot defines both Absolute as well as Relative Stability.
* Gain Margin and Phase Margin calculated from Bode-plot is approximate GM and PM while Gain Margin and Phase Margin calculated from Nyquist Plot is Exact GM and PM.

*→ G(s)H(s) PLANE :-

- Consider an Arbitrary plane :-

Let $G(s)H(s) = \dfrac{K}{(s+2)}$



$$G(s)H(s)\Big|_{s=-3+j1} = \frac{1}{\{(-3+j1)+2\}} = \frac{1}{(-1+j1)} = \frac{1}{\sqrt{2}}\angle -\tan^{-1}\left(\frac{1}{-1}\right)$$

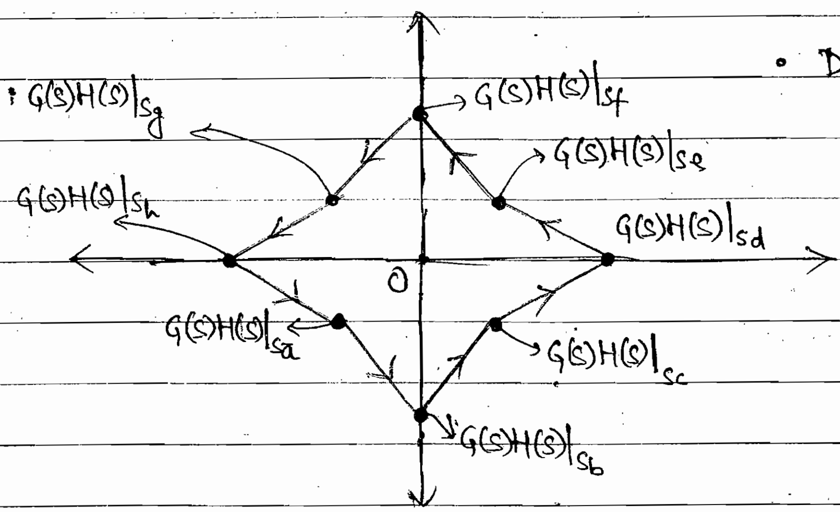
$$= \frac{1}{\sqrt{2}}\angle -135^\circ \text{ or } \angle +225^\circ.$$

$$G(s)H(s)\Big|_{s=-2+j1} = \frac{1}{\{(-2+j1)+2\}} = 1\angle -\tan^{-1}(1/0) = 1\angle -90^\circ$$

$$G(s)H(s)\Big|_{s=-1+j1} = \frac{1}{\{(-1+j1)+2\}} = \frac{1}{(1+j1)} = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$G(s)H(s)\Big|_{s=-1+j0} = \frac{1}{\{-1+2\}} = 1\angle 0^\circ.$$

$$G(s)H(s)\Big|_{s=-1-j1} = \frac{1}{\{(-1-j1)+2\}} = \frac{1}{(1-j1)} = \frac{1}{\sqrt{2}}\angle 45^\circ.$$



- Diverging graph is because of Pole as pole makes TF infinite.

- For one pole encirclement in clockwise direction on s-plane, G(s)H(s) plane will traverse one encirclement in Anti-clockwise direction.

(ii) $G(S)H(S) = (S+2)$

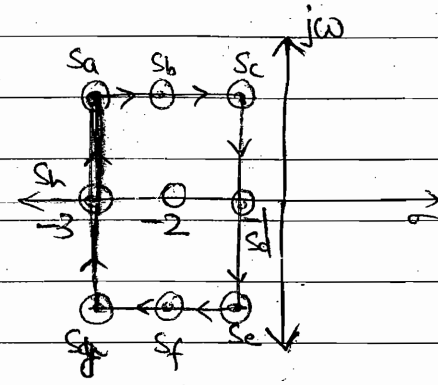
$$G(S)H(S)\Big|_{S_a=-3+j1} = (-3+j1+2) = \sqrt{2}\,\angle{+135^\circ} \text{ or } (-225^\circ)$$

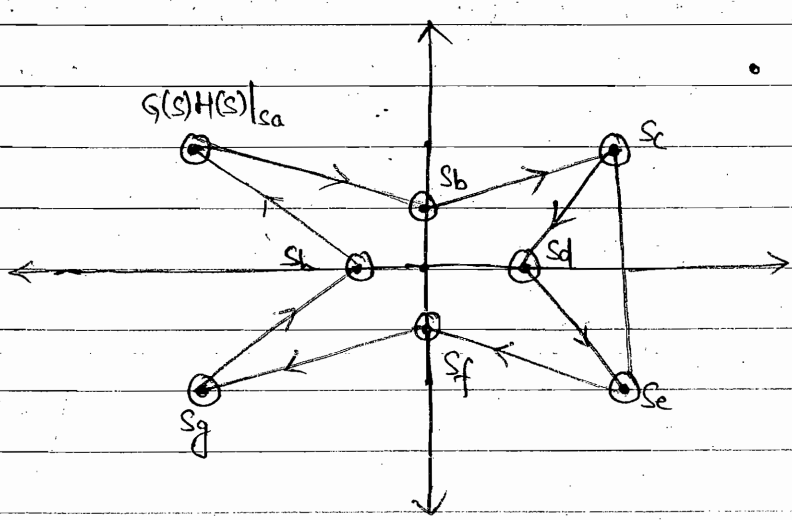
$$G(S)H(S)\Big|_{S_b=-2+j1} = (-2+j1)+2 = 1\,\angle{90^\circ}$$

$$G(S)H(S)\Big|_{S_c=-1+j1} = (-1+j1)+2 = \sqrt{2}\,\angle{45^\circ}$$

$$G(S)H(S)\Big|_{S_d=-1+j0} = (-1)+2 = 1\,\angle{0^\circ}$$

$$G(S)H(S)\Big|_{S_e=-1-j1} = (-1-j1+2) = \sqrt{2}\,\angle{-45^\circ}$$

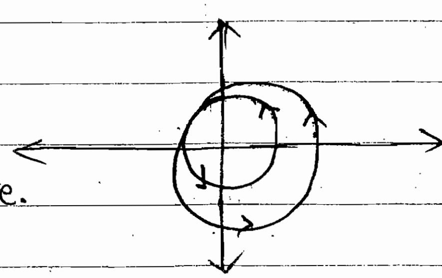




- Zero will have converging graph as it makes TF zero.

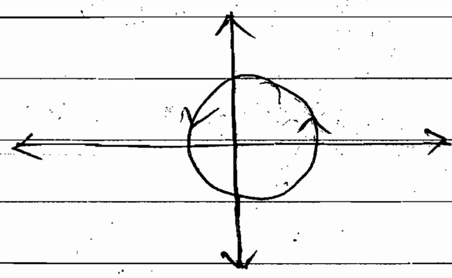
(iii) $G(S)H(S) = \dfrac{1}{(S+2)(S+3)}$

- As two poles are there, so, two encirclements in Anti-clockwise directions will be there.



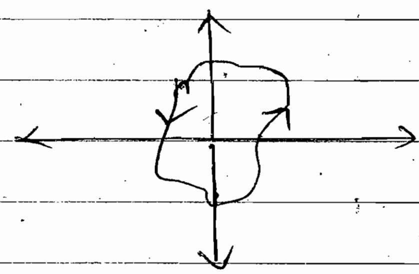
(iv) $G(s)H(s) = \dfrac{(s+1.5)}{(s+2)(s+2.5)}$

- As one zero and one-pole encirclements will get cancelled so, anti-clockwise direction of one pole will remain in graph.

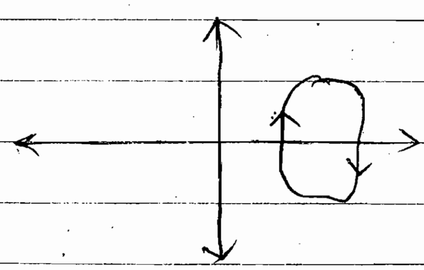


(v) $G(s)H(s) = \dfrac{1}{(s+2)(s+4)}$

- As chosen contour does not contain pole $s=-4$ in it. so, only one encirclement is there i.e due to $s=-2$ pole.



(vi) $G(s)H(s) = \dfrac{s+1.5}{s+2.5}$



* • Here, net encirclement around origin is 0, so encirclement will not there about origin.
* • Direction, here, is clockwise direction as to origin, pole is far away. So, zero will be dominant.

* • If chosen contour contains p-number of poles and in specified contour on traversing in clockwise direction in s-plane, then its $G(s)H(s)$ plane will encircle its origin p-number of times in Anti-clockwise direction and vice-versa.

* If chosen contour contains z-number of zeros and in specified contour, we traverse in Clockwise direction in s-plane, then its $G(s)H(s)$ plane encircles its origin z-number of times in Clockwise direction and vice-versa.

* If specified contour contains p-number of poles and z-number of poles and in specified contour, on traversal in clockwise direction in s-plane, then its $G(s)H(s)$ plane encircles its origin $(p-z)$ times in Anti-clockwise direction $(p>z)$ and $(z-p)$ times in Clockwise direction.

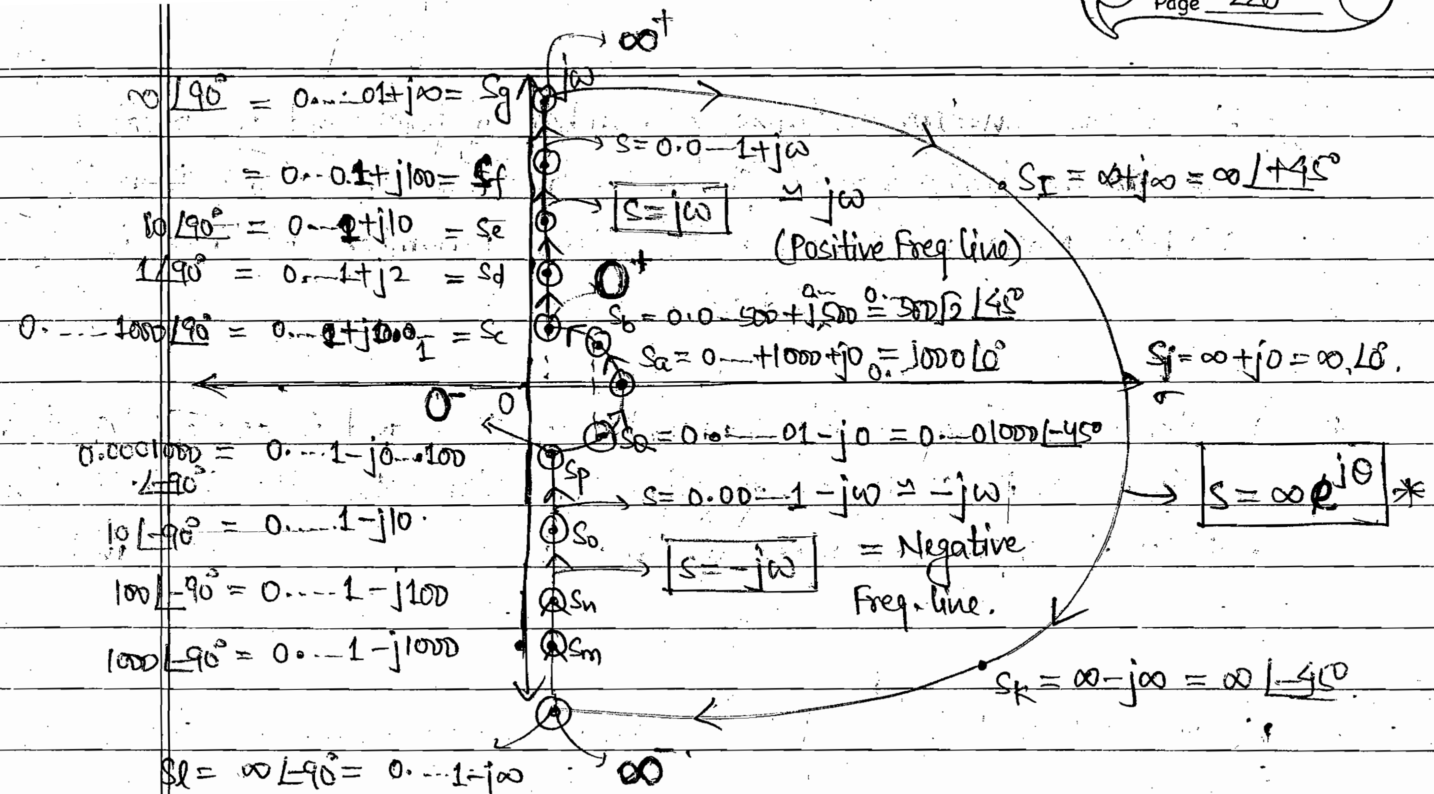
* Clockwise encirclement will be taken as negative while Anti-clockwise encirclement will be taken as positive.

*→ Generalized Specified Contour :-

$$s = \sigma + j\omega \text{ (In Rectangular)}$$

$$r = \sqrt{\sigma^2+\omega^2} \; ; \quad \theta = \tan^{-1}(\omega/\sigma)$$

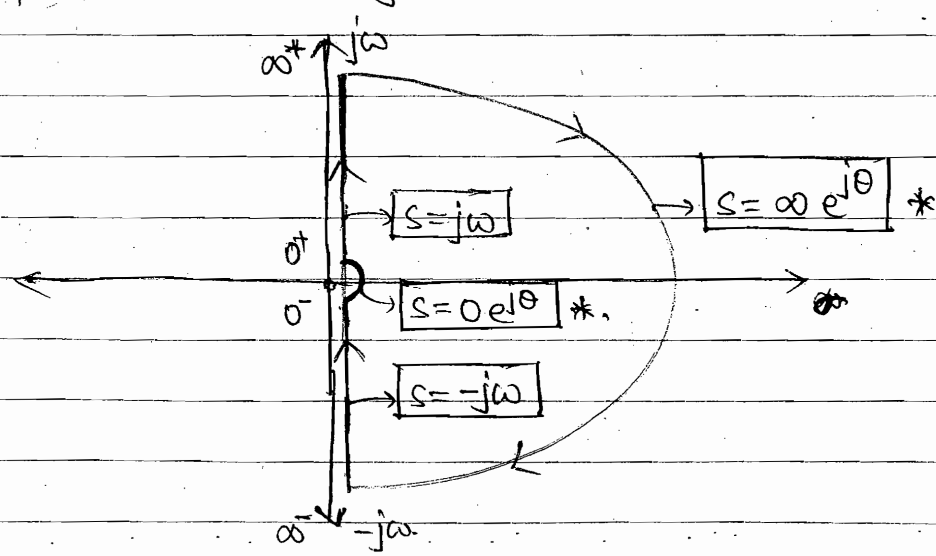
$$s = re^{j\theta} \quad \text{(In Polar Form)}$$



Here, $0^+ \Rightarrow$ 0 represents $0.0\ldots\ldots1000$

+ represents Positive Frequency line

* Origin is not included as pole at origin does not show instability of system. System is Marginally stable when pole is at origin.



$(s=j\omega) \Rightarrow (s=0^+)$ to $(s=\infty^+) \Rightarrow G(s=0^+)H(s=0^+) \rightarrow G(s)H(s)\big|_{s=\infty}$

$(s=\infty e^{j\theta}) \Rightarrow (s=\infty^+)$ to $(s=\infty^-) \Rightarrow G(s)H(s)\big|_{s=\infty^+} \rightarrow G(s)H(s)\big|_{s=\infty^-}$

$(s=-j\omega) \Rightarrow (s=\infty^-)$ to $(s=0^-) \Rightarrow G(s)H(s)\big|_{s=\infty^-}$ to $G(s)H(s)\big|_{s=0^-}$

$(s=0e^{j\theta}) \Rightarrow (s=0^-)$ to $(s=0^+) \Rightarrow G(s)H(s)\big|_{s=0^-}$ to $G(s)H(s)\big|_{s=0^+}$

$\Rightarrow$ $(s=j\omega)$ and $(s=-j\omega)$ will be Mirror Image about real axis.

***→ Relation between Open Loop Transfer Function, characteristics Equation and Closed Loop Transfer Function :-**

Say, $G(s)H(s) = \dfrac{s+1}{s+5}$

$\therefore S_{zo} = -1, \quad S_{po} = -5.$

$q(s) = \dfrac{2s+6}{s+5}$

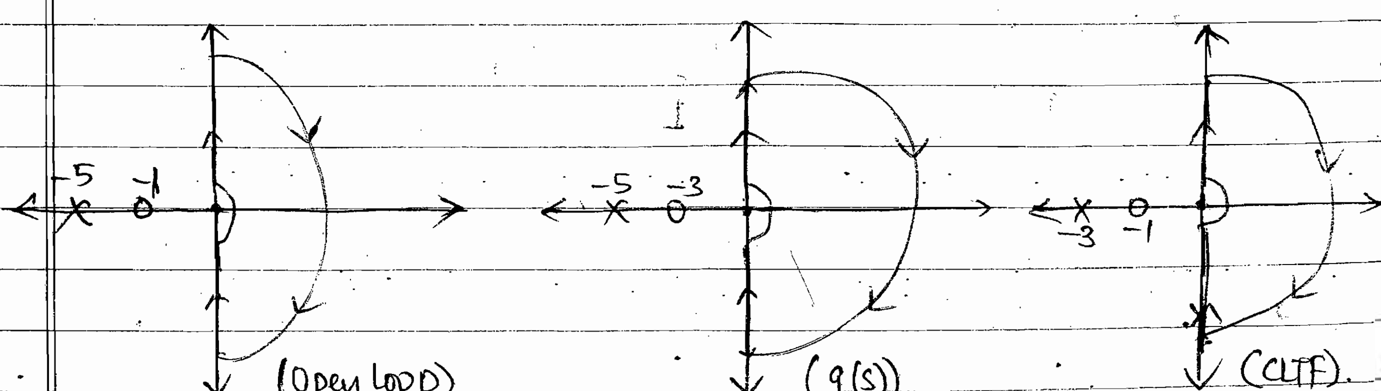
$S_{pc} = -5, \quad S_{zc} = -3.$

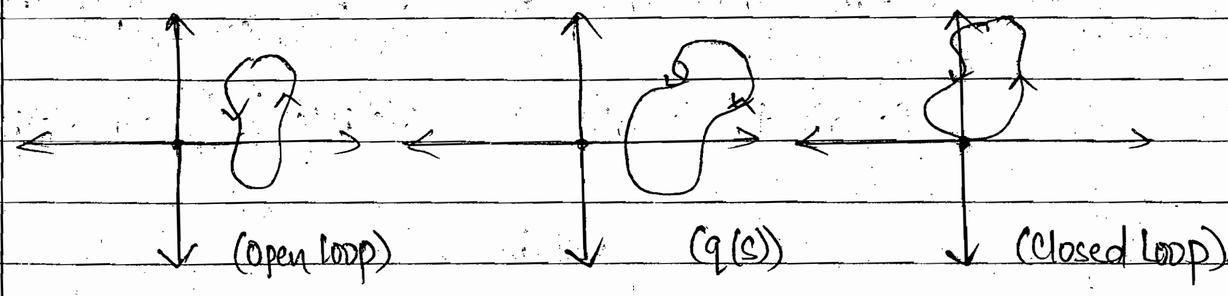
$T(s) = \dfrac{G(s)}{1+G(s)H(s)} = \dfrac{(s+1)}{2s+6}.$

$S_{pT} = -3, \quad S_{zT} = -1$

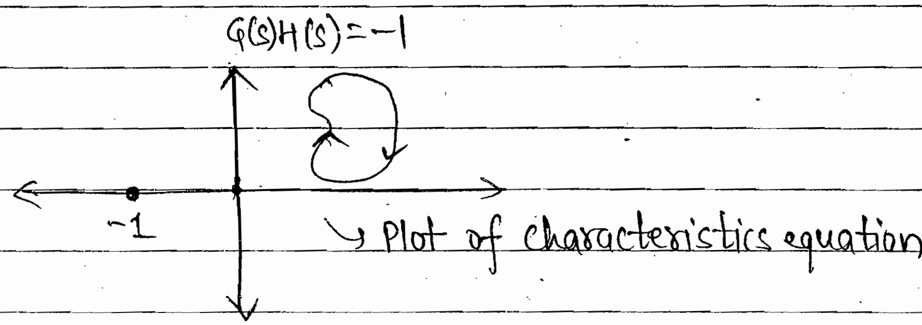
$\therefore$ $\boxed{S_{po} = S_{pc}}$ * $\therefore$ Open loop pole = Pole of characteristics Equation.

$\boxed{S_{zc} = S_{pT}}$ * $\therefore$ Zero of characteristics Equation = Pole of closed loop T.F.

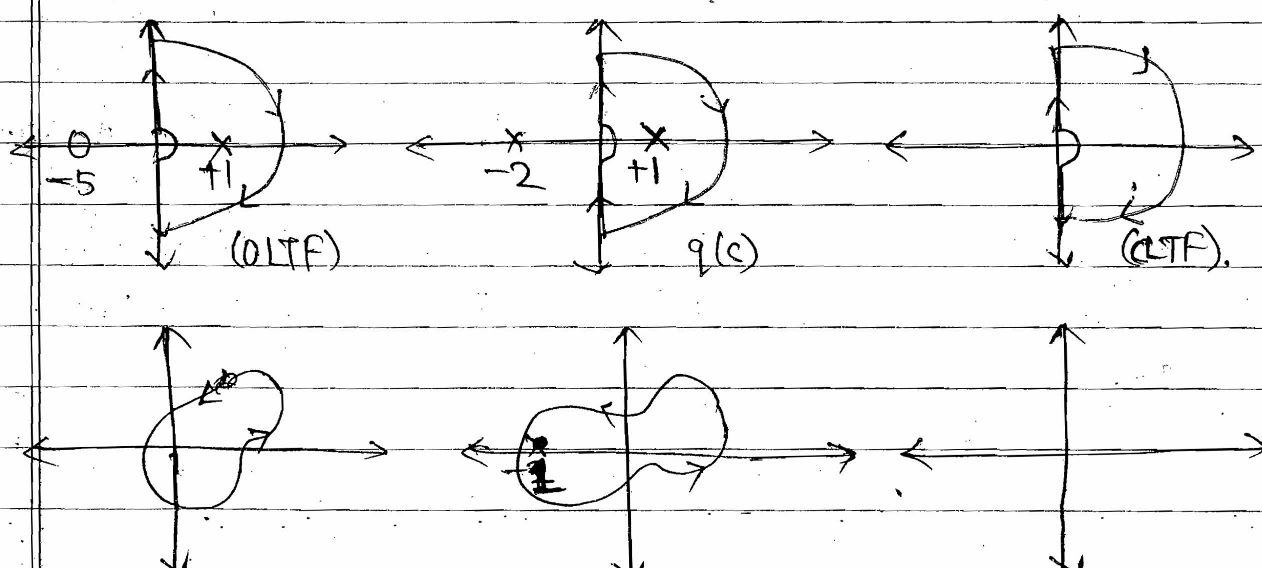




⇒ Poles and zeros encircle origin only as to calculate pole or zero-locations, we put or equate them to 0.



* Example:- $\frac{s+5}{(s-1)} = G(s)H(s)$.



$$\boxed{N = P_+ - Z_+} \quad *$$

where P_+ = Total number of poles in OLTF which is lying in the right-half side of Imaginary axis and this pole itself is pole of characteristics equation.

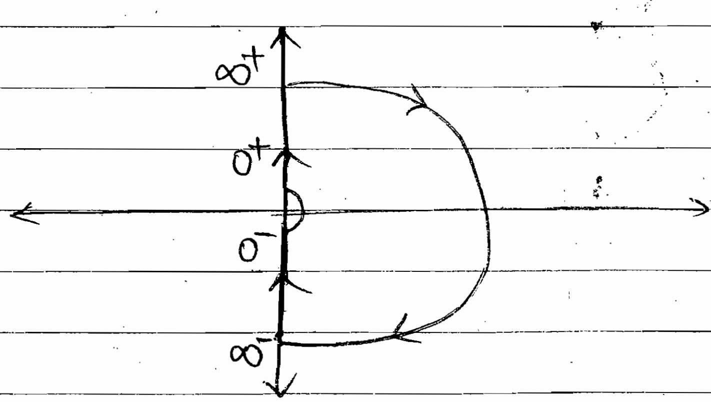
Z_+ = Total number of zeros in characteristics equation which is lying in the right half-side of Imaginary axis and this zero itself is pole of closed Loop TF which is lying in the right-half side of s-plane and for closed loop system to be stable,

N = Total number of Encirclement of (−1,0) point.

* For clockwise encirclement, N should be Negative.
  For Anti-clockwise, encirclement, N will be Positive.

→ **Closing of Nyquist Plot :-**

* Case:-I :- If origin contains n number of poles of Open loop Transfer Function :-



$$(s=0^+) \rightarrow (s=\infty^+) \Rightarrow G(0^+)H(0^+) \rightarrow G(\infty^+)H(\infty^+).$$
$$(s=\infty^+) \rightarrow (s=\infty^-) \Rightarrow G(\infty^+)H(\infty^+) \rightarrow G(\infty^-)H(\infty^-).$$
$$(s=\infty^-) \rightarrow (s=0^-) \Rightarrow G(\infty^-)H(\infty^-) \rightarrow G(0^-)H(0^-).$$
$$(s=0^-) \rightarrow (s=0^+) \Rightarrow G(0^-)H(0^-) \rightarrow G(0^+)H(0^+).$$

- In this case, then $G(\infty^+)H(\infty^+)$ and $G(\infty^-)H(\infty^-)$ will be short-circuited while $G(0^-)H(0^+)$ and $G(0^+)H(0^+)$ will be open-circuited. To close the Nyquist plot from $G(0^-)H(0^-)$ to $G(0^+)H(0^+)$, we will make $(n\pi)$ clockwise encirclement with radius of encirclement $= \infty$ where 'n' is total number of poles in OLTF located at origin.

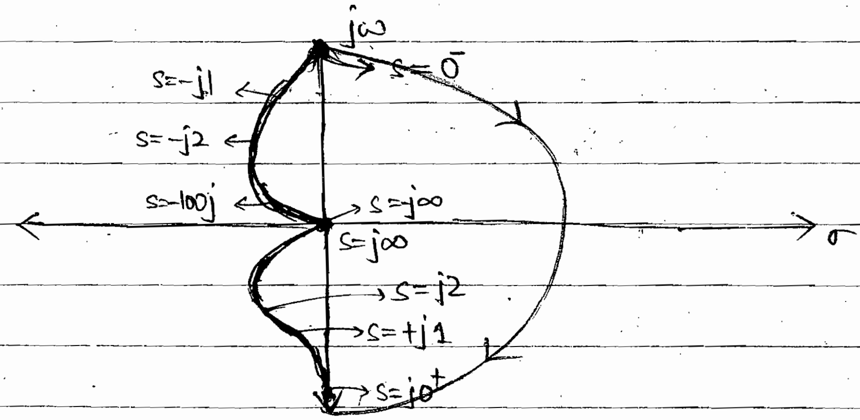
* <u>Example:-</u> $G(s)H(s) = \dfrac{1}{s(s+1)}$

$\Rightarrow$ Put $s = j\omega$, $G(s)H(s)\Big|_{j\omega} = \dfrac{1}{j\omega(j\omega+1)} = \dfrac{1}{\omega\sqrt{1+\omega^2}} \angle -90^\circ - \tan^{-1}(\omega)$

$$G(s)H(s)\Big|_{s=j0} = \frac{1}{0}\angle -90^\circ = \infty \angle -90^\circ$$

$$G(s)H(s)\Big|_{s=j1} = \frac{1}{\sqrt{2}}\angle -135^\circ$$

$$G(s)H(s)\Big|_{s=j2} = \frac{1}{2\sqrt{5}}\angle -153^\circ$$



* <u>Example:-</u> $G(s)H(s) = \dfrac{K}{s^n}$

$\Rightarrow$ Put $s = re^{j\theta}$

$$G(s)H(s)\Big|_{s=re^{j\theta}} = \frac{K}{(re^{j\theta})^n} = \frac{K}{r^n}\cdot e^{-j(n\theta)}$$

- At Initial point, $s = 0e^{-90^\circ}$

$$\therefore G(s)H(s)\Big|_{s=0e^{-90^\circ}} = \frac{K}{(0)^n}\cdot e^{-jn(-90^\circ)} = \infty \cdot e^{jn90^\circ} = \infty \underline{/(90^\circ n)}.$$

- At Final point, $s = 0e^{+90^\circ}$,

$$G(s)H(s)\Big|_{s=0e^{+90^\circ}} = \frac{K}{(0)^n}\cdot e^{-j(n90^\circ)} = \infty \underline{/-(90^\circ)n}.$$

$\therefore$ Change in phase angle of s from Initial point to Final point:
$= \angle S_f - \angle S_I = +90^\circ - (-90^\circ) = 180^\circ.$

or change in phase angle of G(s)H(s) from Initial point to Final point : $\angle G_F - \angle G_I = (-n90^\circ) - (+n90^\circ) = -180^\circ(n).$

Again :- $G(s)H(s) = \dfrac{1}{s(s+1)}$

$$\therefore G(0 \cdot e^{j\theta})\, H(0e^{j\theta}) = \frac{1}{0e^{j\theta}[0e^{j\theta}+1]}$$

$$G(s)H(s)\Big|_{s=0e^{j\theta}} = \frac{1}{0e^{j\theta}} = \infty \underline{/e^{(-j\theta)}} = \infty \underline{/-\theta^\circ}.$$

$\therefore$ When $\theta = -90^\circ$, $G(s)H(s)\Big|_{s=0e^{-j90}} = \infty \underline{/90^\circ}$

and $G(s)H(s)\Big|_{s=0e^{-j45^\circ}} = \infty \underline{/45^\circ}.$

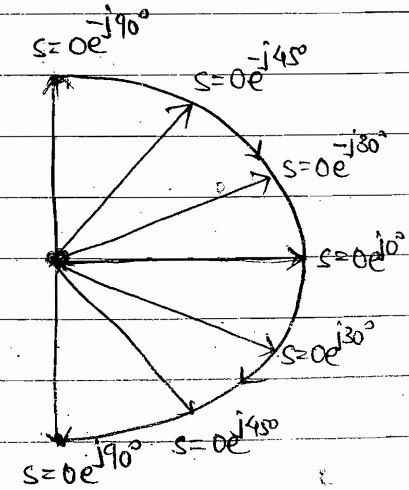
and $G(s)H(s)\Big|_{s=0e^{-j30^\circ}} = \infty \underline{/30^\circ}.$

$G(s)H(s)\Big|_{s=0e^{j0}} = \infty \underline{/0^\circ}.$

$G(s)H(s)\Big|_{s=0e^{j30^\circ}} = \infty \underline{/-30^\circ}$

$G(s)H(s)\Big|_{s=0e^{j45^\circ}} = \infty \underline{/-45^\circ}$

$G(s)H(s)\Big|_{s=0e^{j90^\circ}} = \infty \underline{/-90^\circ}$



Now, put $s = \infty \cdot e^{j\theta}$

$$\therefore G(s)H(s)\Big|_{s=\infty e^{j\theta}} = \frac{1}{(\infty e^{j\theta})(\infty e^{j\theta}+1)} = \frac{1}{\alpha^2} e^{-j2\theta} = 0 \angle -2\theta.$$

$$\therefore G(s)H(s)\Big|_{s=\infty e^{j90^\circ}} = 0 \angle -180^\circ$$

$$G(s)H(s)\Big|_{s=\infty e^{j45^\circ}} = 0 \angle -90^\circ.$$

$$G(s)H(s)\Big|_{s=\infty \cdot e^{-j90^\circ}} = 0 \angle 180^\circ.$$

- These all are just points on origin.

* If Magnitude is ∞ and phase is varying, this will be the Closing condition of $G(s)H(s)\big|_{s=0^-}$ and $G(s)H(s)\big|_{0^+}$.

* If Magnitude is 0 and phase is varying, this will be the shorting condition of $G(s)H(s)\big|_{s=\infty^+}$ and $G(s)H(s)\big|_{s=\infty^-}$.

Case: II :-

Order of Zero is greater than order of Pole :-

- In this case,

$G(s)H(s)\big|_{s=0^-}$ and $G(s)H(s)\big|_{s=0^+}$ will be short-circuit and

$G(\infty^+)H(\infty^+)$ and $G(\infty^-)H(\infty^-)$ will be open-circuited and to close the Nyquist Plot from $G(s)H(s)\big|_{s=\infty^+}$ to $G(s)H(s)\big|_{s=\infty^-}$, we will make $(m-n)\pi$ clockwise encirclements with radius of encirclement $= \infty$, where 'm' is order of Zeros, 'n' is order of Poles.

* Consider :- $G(s)H(s) = \dfrac{K(sT_1+1)^m}{(sT_2+1)^n}$, $m > n$

• Put $s = re^{j\theta}$, $G(s)H(s)\Big|_{s=re^{j\theta}} = \dfrac{K\left[(re^{j\theta})T_1+1\right]^m}{\left[(re^{j\theta})T_2+1\right]^n}$

At Initial point; $s = \infty e^{j90°}$

$$\therefore G(s)H(s)\Big|_{s=\infty e^{j90°}} = \frac{K\left[(\infty e^{j90})T_1+1\right]^m}{\left[(\infty e^{j90})T_2+1\right]^n}$$

$$= K(\infty)^{m-n}\, e^{j90°(m-n)} = \infty \angle (m-n)90°.$$

• At final point: $s = \infty e^{-j90°}$

$$G(s)H(s)\Big|_{s=\infty e^{-j90°}} = \frac{K\left[(\infty e^{-j90°})T_1+1\right]^m}{\left[(\infty e^{-j90°})T_2+1\right]^n}$$

$$= K(\infty)^{m-n} \cdot e^{-j90[m-n]} = \infty \cdot \angle (m-n)[-90°].$$

* Change in Phase Angle of s from Initial to Final point :-

$$= \angle S_f - \angle S_I = -90° - 90° = -180°.$$

* Change in Phase Angle of G(s)H(s) from Initial to Final point :-

$$\angle G_f - \angle G_I = \{-(m-n)90°\} - \{(m-n)90°\}$$

$$= -(m-n)180°.$$

* Example: $G(s)H(s) = \dfrac{s^2}{(s+1)}$

$$\therefore G(s)H(s)\Big|_{s=j\omega} = \frac{-\omega^2}{(j\omega+1)} = \frac{\omega^2}{\sqrt{\omega^2+1}} \angle 180° - \tan^{-1}(\omega).$$

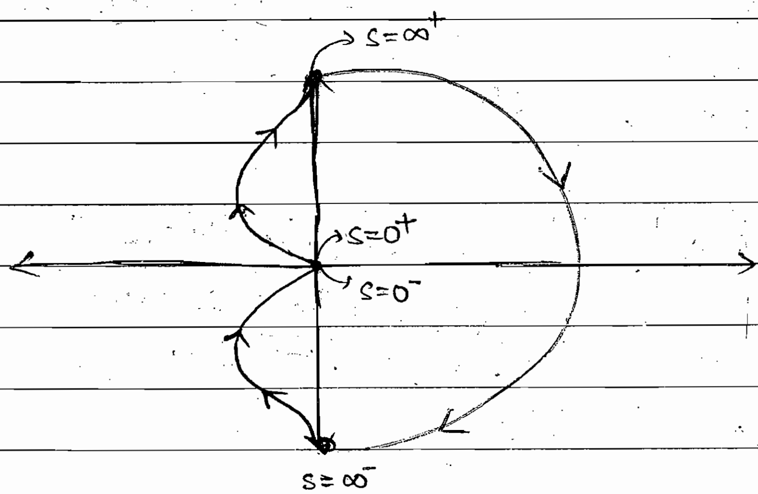
$$G(s)H(s)\Big|_{s=j0} = 0 \angle 180°$$

$$G(s)H(s)\Big|_{s=j1} = \frac{1}{\sqrt{2}} \angle 180° - 45° = \frac{1}{\sqrt{2}} \angle 135°$$

$$G(s)H(s)\Big|_{s=j2} = \frac{4}{\sqrt{5}} \angle 180° - 63° = \frac{4}{\sqrt{5}} \angle 117°$$

$$\frac{\omega^2}{\sqrt{1+\omega^2}} \Rightarrow \frac{\omega^2}{\omega\sqrt{\frac{1}{\omega^2}+1}\sqrt{1}} = \infty$$
$= \infty$

$G(s)H(s)\Big|_{s=j\infty} = \infty \angle +90^\circ$.



Now, put $s = \infty e^{j\theta}$.

$$G(s)H(s)\Big|_{s=\infty e^{j\theta}} = \frac{(\infty e^{j\theta})^2}{(\infty e^{j\theta})^1} = \infty e^{j(2-1)\theta} = \infty \angle \theta.$$

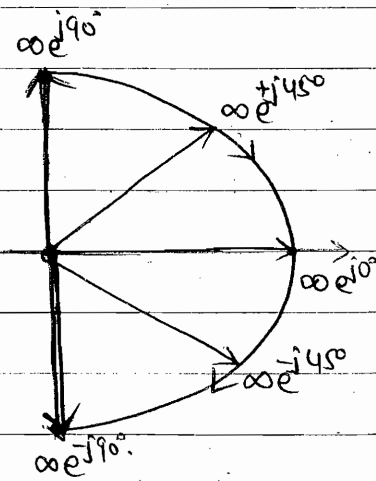
$\therefore G(s)H(s)\Big|_{s=\infty e^{j90^\circ}} = \infty \angle 90^\circ$

$G(s)H(s)\Big|_{s=\infty e^{j45}} = \infty \angle 45^\circ$.

$G(s)H(s)\Big|_{s=\infty e^{j0}} = \infty \angle 0^\circ$

$G(s)H(s)\Big|_{s=\infty e^{-j45^\circ}} = \infty \angle -45^\circ$

$G(s)H(s)\Big|_{s=\infty e^{-j90^\circ}} = \infty \angle -90^\circ$.



(Closing Condition)

For $s = 0e^{j\theta}$, $G(s)H(s) = \frac{(0e^{j\theta})^2}{(0e^{j\theta}+1)} = 0 \angle 2\theta$. → Shorting Condition

* Case:- III: Type of system is 0 and order of pole $\geq$ order of zero:

- In this case, $G(s)H(s)|_{s=0^-,0^+}$ will be short-circuited and $G(s)H(s)|_{\infty^+}$, $G(s)H(s)|_{\infty^-}$ will also be short-circuited.

* Example: $G(s)H(s) = \frac{1}{(s+1)}$.

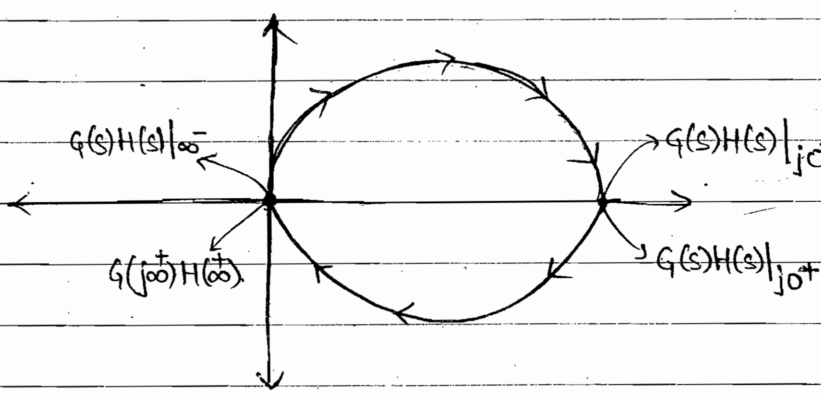
$$\Rightarrow \quad G(s)H(s)|_{s=j\omega} = \frac{1}{j\omega+1} = \frac{1}{\sqrt{\omega^2+1}} \angle -\tan^{-1}(\omega/1).$$

$$G(s)H(s)|_{s=j0^+} = 1\angle 0^\circ.$$

$$G(s)H(s)|_{s=j1} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

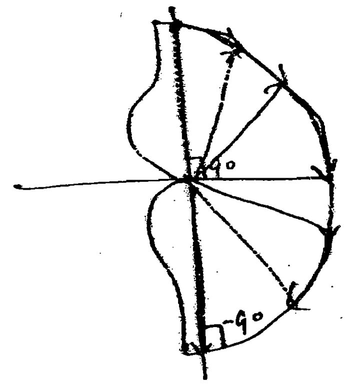
$$G(s)H(s)|_{s=j2} = \frac{1}{\sqrt{5}} \angle -63^\circ$$

$$G(s)H(s)|_{s=j\infty} = 0\angle -90^\circ.$$



For encirclement ∞ radius

$$G(\infty e^{j\theta})H(\infty e^{j\theta}) = \frac{(\infty \cdot e^{j\theta})^2}{(\underbrace{\infty e^{j\theta}+1}_{\infty e^{j\theta}})} = (\infty)^{2-1} \cdot e^{j(2-1)\theta} = \infty\angle 1\theta$$



$$G(\infty \cdot e^{+j90})\cdot H(\infty \cdot e^{j90}) = \infty\angle +90^\circ$$

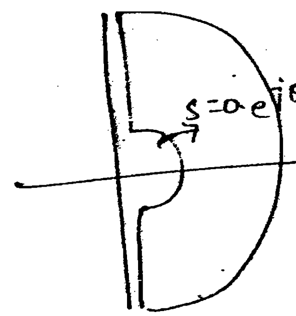
$$G(\infty \cdot e^{j45})\cdot H(\infty \cdot e^{j45}) = \infty\angle +45^\circ$$

$$G(\infty \cdot e^{j0})\cdot H(\infty \cdot e^{j0}) = \infty\angle 0^\circ$$

$$G(\infty \cdot e^{-j45})\cdot H(\infty \cdot e^{-j45}) = \infty\angle -45$$

$$G(\infty e^{-j90})\cdot H(\infty \cdot e^{-j90}) = \infty\angle -90$$

<u>For 0 radius circle</u>



$$G(0 e^{j\theta})\cdot H(0\cdot e^{j\theta}) = \frac{(0\cdot e^{j\theta})^2}{(0\cdot e^{j\theta}+1)}$$

$$= 0^2 \cdot e^{j2\theta} = 0\angle 2\theta$$

Case-3 type of system is 0 and order of pole is either ≥ order of zero

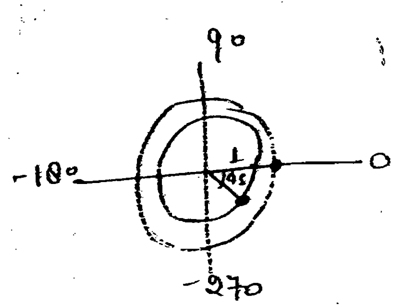
* In case $G(0^-)\cdot H(0^-)$ will be short circuited and $G(0+)H(\infty+)$, $G(\infty-)H(\infty-)$ will also be short circuit

<u>Exa</u> $G(s)\cdot H(s) = \dfrac{1}{s+1}$

For +ve freq line

$$G(j\omega)\cdot H(j\omega) = \frac{1}{j\omega+1} = \frac{1}{\sqrt{1+\omega^2}}\angle -\tan^{-1}\left(\frac{\omega}{1}\right)$$

$$G(j0)\cdot H(j0) = 1\angle -\tan^{-1}0 = 1\angle 0^\circ$$

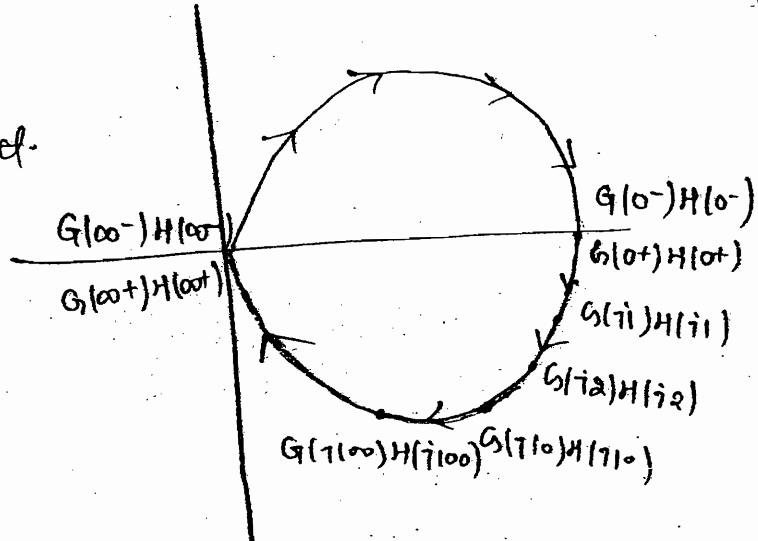


$$G(j1)\cdot H(j1) = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$G(j2)\cdot H(j2) = \frac{1}{\sqrt{5}}\angle -63^\circ$$

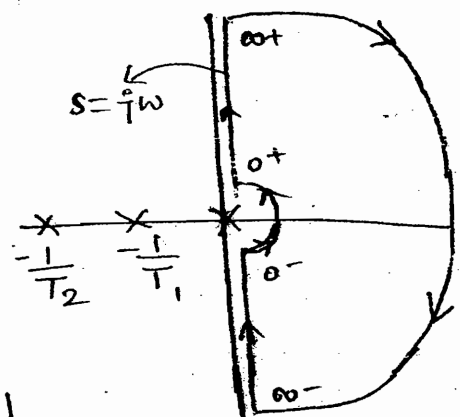
$G(j\infty) \cdot H(j\infty) = 0 \angle -90°$

This Graph already ~~encircl~~ closed



Response of closed loop system with encirclement of (−1,0) point.

$$G(S)H(S) = \frac{K}{S[ST_1+1][ST_2+1]}$$



$$G(j\omega) \cdot H(j\omega) = \frac{K}{(j\omega)[1+j\omega T_1][1+j\omega T_2]}$$

$$|G(j\omega) H(j\omega)| = \frac{1}{\omega\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}}$$

$$\angle G(j\omega) H(j\omega) = \angle -90 - \tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

$\omega = j0$

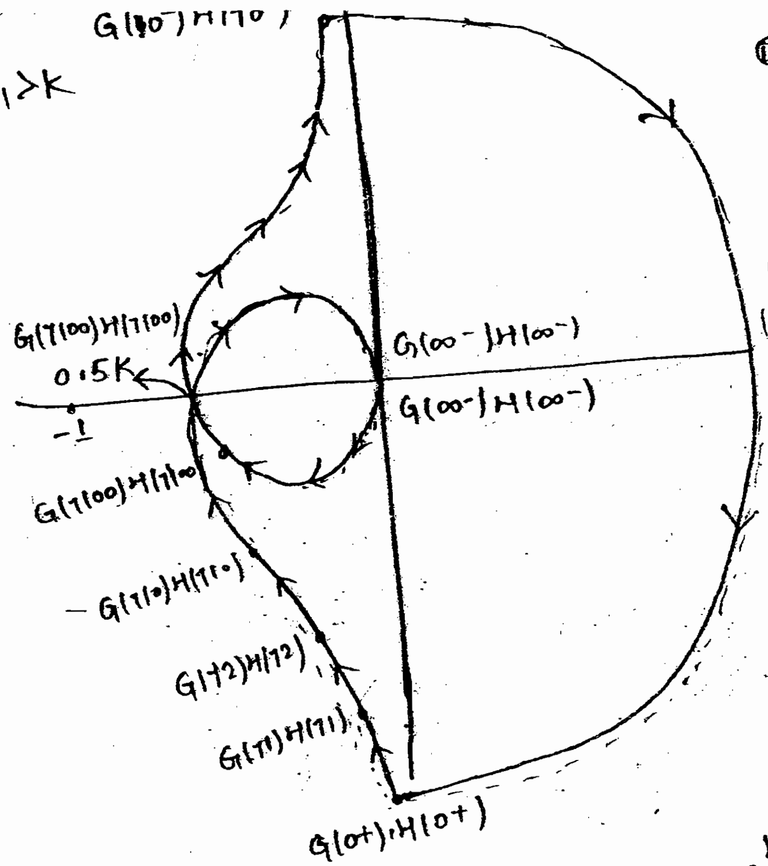
$G(j0) H(j0) = \infty \angle -90$

$G(j\infty) \cdot H(j\infty) = 0 \angle -270°$

$$\frac{C(s)}{R(s)} = \frac{K}{s[sT_1+1][sT_2+1]+K}$$

... plot

$K_3 > K_2 > K_1 > K$



① $0.5K < 1$ — stable

$0.5K < 0.5K_1 < 1$ — stable

$0.5K_1 < 0.5K_2 = 1$ — M.

④ $1 = 0.5K_2 < 0.5K_3$ unstable

① $P_+ = 0$, $N = 0$

④ $P_+ = 0$, $N = -2$

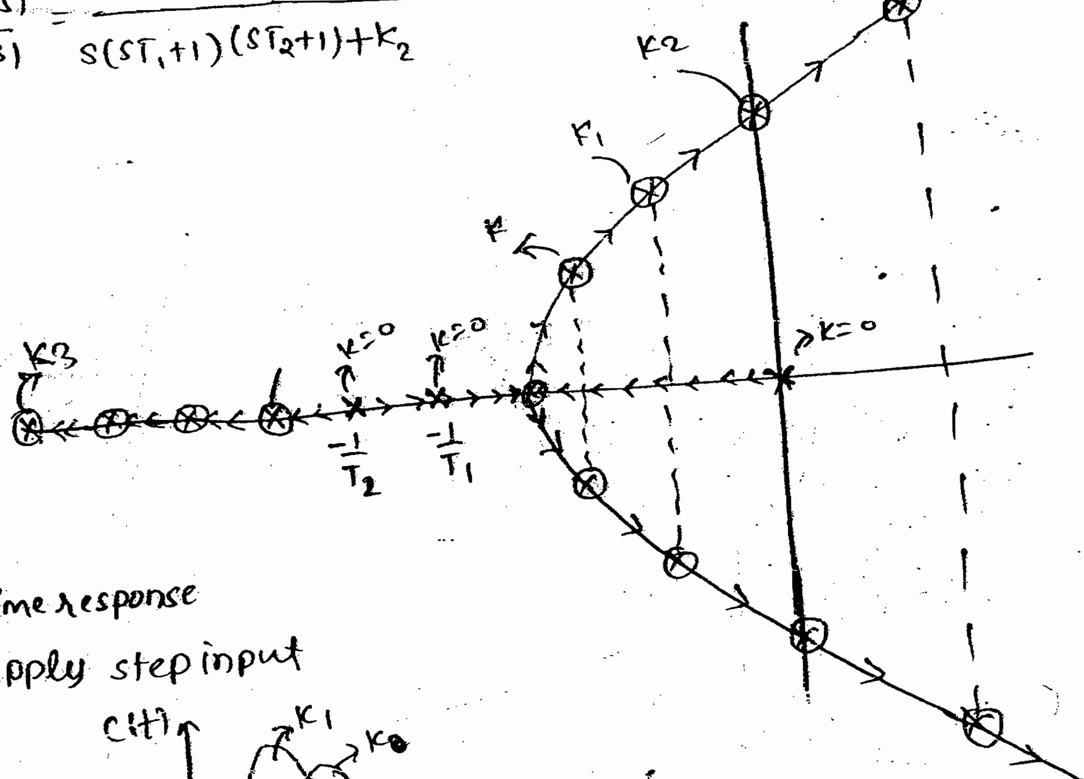
$N = P_+ - Z_+$

$2 = P_+ - Z_+$

$Z_+ = 2 \rightarrow$ unstable

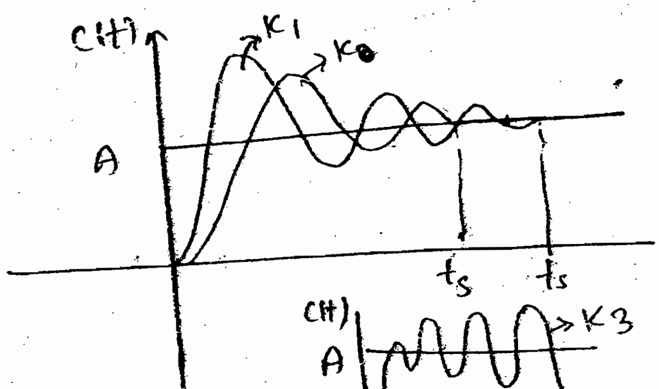
Root locus

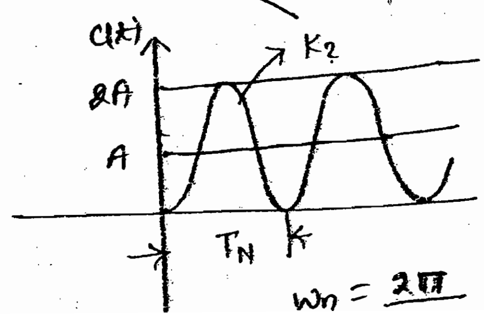
$$\frac{C(s)}{R(s)} = \frac{K}{s(sT_1+1)(sT_2+1)+K_2}$$



time response

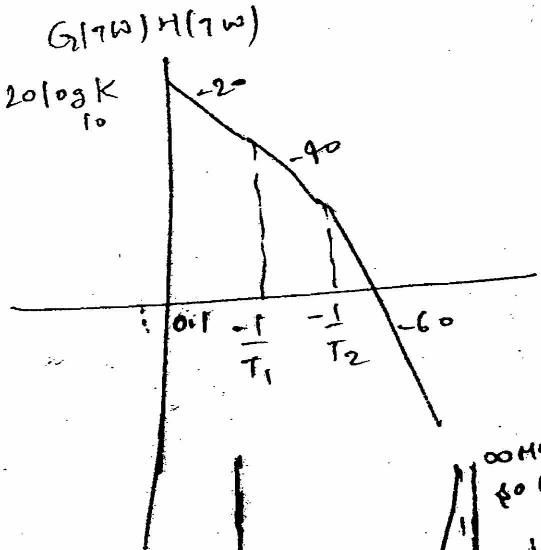
Apply step input



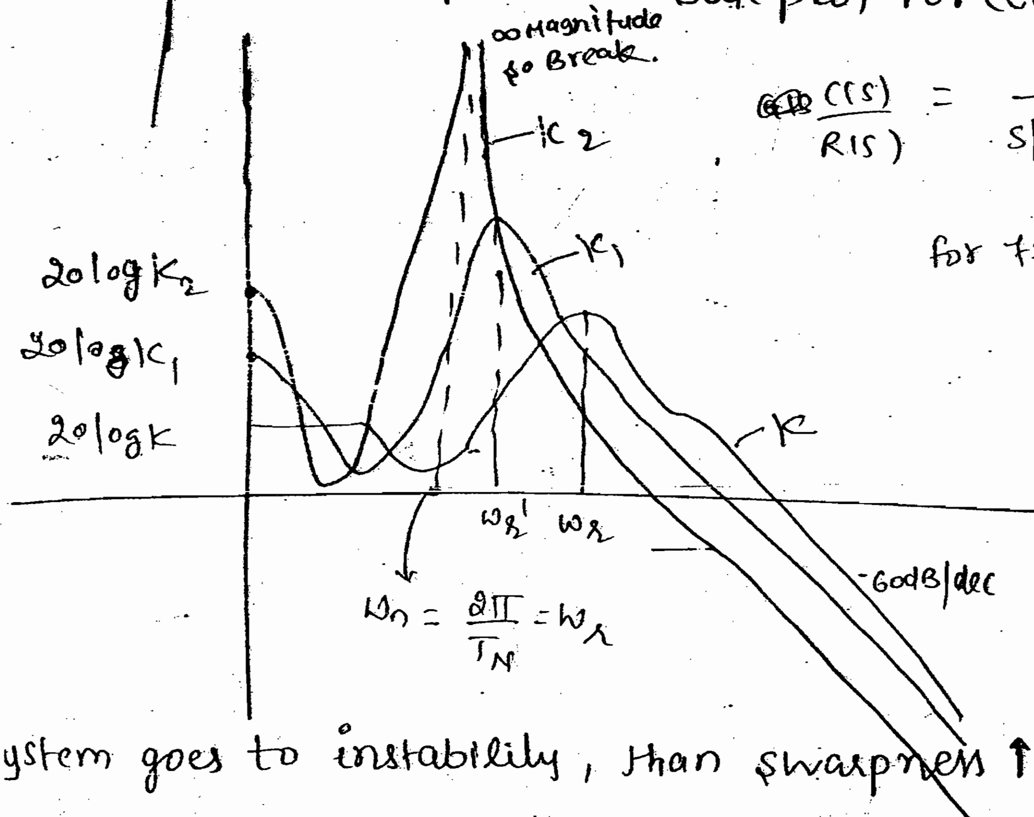


$w_n = 2\pi$

de plot (OLTF)



$$G(s)H(s) = \frac{K}{s[sT_1+1][sT_2+1]}$$



$$\omega_n = \frac{2\pi}{T_N} = \omega_r$$

Bode plot for (CLT/F) for freq^n parameter

$$\frac{C(s)}{R(s)} = \frac{K}{s[sT_1+1][sT_2+1]+K}$$

for freq^n parameter

ystem goes to instability, than sharpness ↑ of Bodeplot.

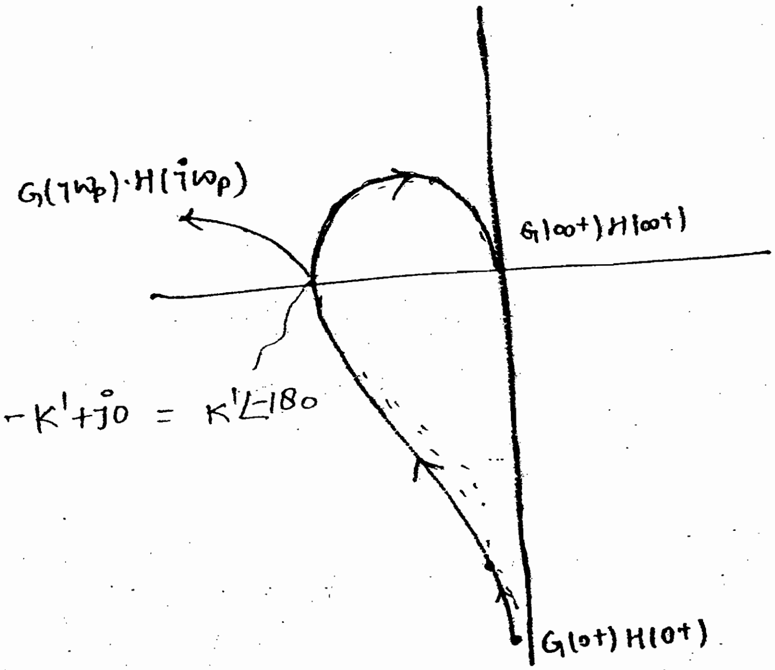
Gain margin and phase margin

$$G(s)\cdot H(s) = \frac{K}{s[sT_1+1][sT_2+1]}$$

$$G(j\omega)\cdot H(j\omega) = \frac{K}{(j\omega)[1+j\omega T_1][1+j\omega T_2]}$$

$$= \frac{K}{\omega\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} \angle -90-\tan^{-1}\left(\frac{\omega T_1}{1}\right)-\tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

$$= |G(j\omega)\cdot H(j\omega)| \angle G(j\omega)\cdot H(j\omega)$$



at $\omega = \omega_p$

$$G(j\omega_p)H(j\omega_p) = \underbrace{|G(j\omega_p)H(j\omega_p)|}_{K'} \underbrace{\angle G(j\omega_p)H(j\omega_p)}_{-180^\circ} = \frac{K}{\omega_p\sqrt{1+(\omega_pT_1)^2}\sqrt{1+(\omega_pT_2)^2}} \angle -18$$

$$-K'+j0 = \sqrt{(-K')^2+(0)^2}\angle \pm \tan^{-1}\frac{0}{-K'} = K'\angle -180 \qquad = K'\angle -180$$

$$\text{Gain Margin} = \frac{1}{|G(j\omega_p)\cdot H(j\omega_p)|} = \frac{1}{|K'|}$$

$$G.M. = -[+20\log|G(j\omega_p)\cdot H(j\omega_p)|] = -[20\log|K'|]$$

when $K' \downarrow \Rightarrow$ GM $\uparrow$

* In polar form
* phase crossover freq^n is defined as freq^n at which phase angle of O.L.T/F become 180° and G.M. is inverse of magnitude at phase cross-over freq^n.

Rectangular form

* In R. form P. crossover frequency is define as freq^n at which imaginary term is zero and real term is Negative and G.M. is inverse of magnitude of real term

n the given OLTF put $s = j\omega$ for positive freq^n line
en by rationalizing numeration and Denominator
remove all imagi term from Denominator and
n of numerator seperate real and imagi term.
then calculate the frequency at which imaginary
term become zero. If real term at that freq^n
is Negative then that freq^n will be phase
cross-over freq^n and inverse of magnitude of
real term will be gain margin.

$$G(s)H(s) = \frac{K}{s[sT_1+1][sT_2+1]}$$

t $s = j\omega$

$$G(j\omega).H(j\omega) = \frac{K \times -j^2 \; [1-j\omega T_1][1-j\omega T_2]}{j\omega(1+j\omega T_1)(1+j\omega T_2)[1-j\omega T_1][1-j\omega T_2]}$$

$$(1+j\omega T_1)(1-j\omega T_2) = (1)^2 - j^2(\omega T_1)^2 = 1+(\omega T_1)^2$$

$$G(j\omega)H(j\omega) = \frac{-jK[(1-\omega^2 T_1 T_2) - j\omega(T_1+T_2)]}{\omega[1+(\omega T_1)^2][1+(\omega T_2)^2]}$$

$$G(j\omega).H(j\omega) = \frac{-K(T_1+T_2)}{[1+(\omega T_1)^2][1+(\omega T_2)^2]} - \frac{jK(1-\omega^2 T_1 T_2)}{\omega[1+(\omega T_1)^2][1+(\omega T_2)^2]}$$

for imagi term zero

$$1-\omega_p^2 T_1 T_2 = 0 \Rightarrow \boxed{\omega_p = \frac{1}{\sqrt{T_1 T_2}}}$$ = phase crossover freq^n.

$$G(j\omega_p)H(j\omega_p) = \frac{-K(T_1+T_2)}{[1+(\omega_p T_1)^2][1+\omega_p^2 T_2^2]} + j0$$

$$= \frac{-K(T_1+T_2)}{\left(1+\frac{T_2}{T_1}\right)\left(1+\frac{T_1}{T_2}\right)} + j0$$

$$G(j\omega_p)\cdot H(j\omega_p) = \frac{-K\cdot(T_1+T_2)\cdot T_1\cdot T_2}{(T_1+T_2)^2} + j0$$

$$G(j\omega_p)\cdot H(j\omega_p) = \underbrace{\frac{-(T_1\cdot T_2)\cdot K}{(T_1+T_2)}}_{-K'+j0} + j0 = -K' + j0$$

$$\Rightarrow \boxed{\text{Gain Margin} = \frac{T_1+T_2}{K\,T_1\cdot T_2}}$$

From Routh Array

$$1+G(s)H(s)=0 \Rightarrow T_1\cdot T_2\, s^3 + (T_1+T_2)\, s^2 + s + K = 0$$

s^3	$T_1\cdot T_2$	1
s^2	T_1+T_2	K
s^1	$\frac{(T_1+T_2)-KT_1T_2}{T_1+T_2}$	0
s^0	K	0

For s/T to be marginally sta

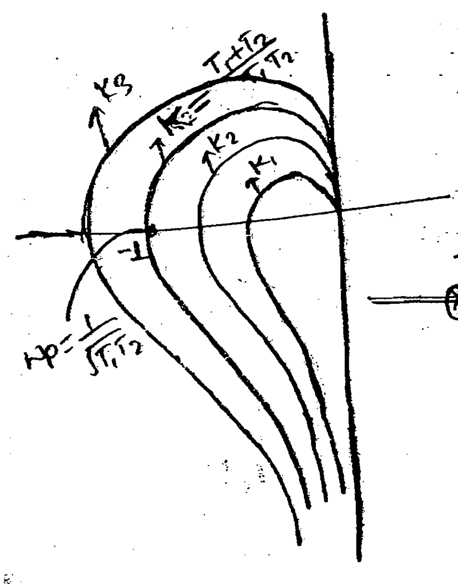
$$\frac{(T_1+T_2)-KT_1T_2}{T_1+T_2} = 0$$

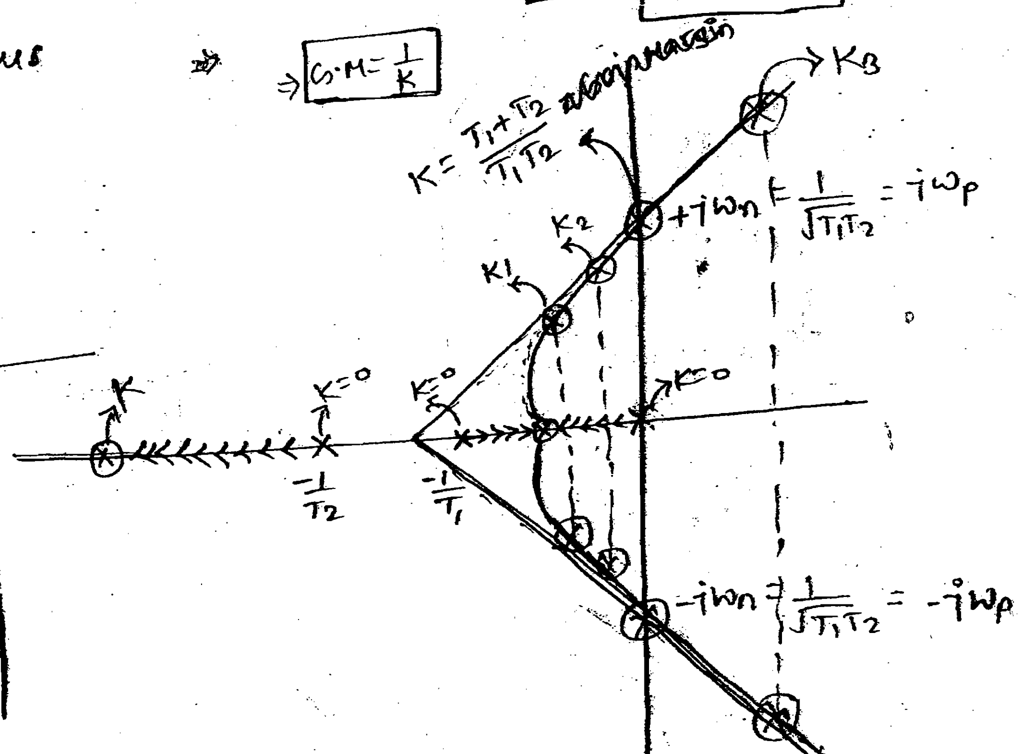
$$K = \frac{T_1+T_2}{T_1\cdot T_2} = \frac{T_1+T_2}{T_1\cdot T_2}$$

Now location of pole $(T_1+T_2)s^2 + \frac{T_1+T_2}{T_1\cdot T_2} = 0$

$$\Rightarrow s = \pm j\left(\frac{1}{\sqrt{T_1T_2}}\right) = \pm j\omega_n \Rightarrow \boxed{\omega_n = \frac{1}{\sqrt{T_1T_2}}}$$

From root locus Correlation $\Rightarrow \boxed{G\cdot M = \frac{1}{K}}$





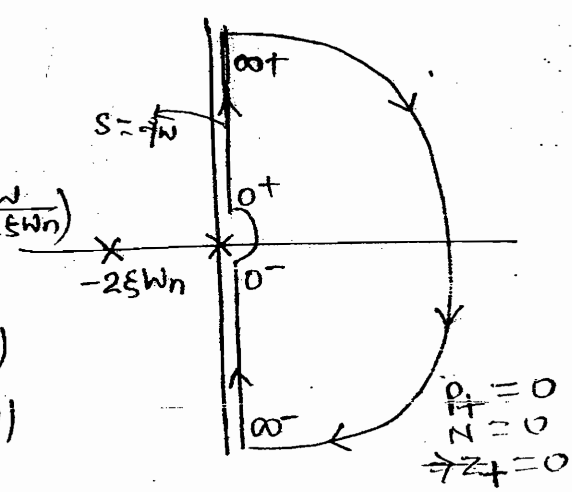
imitation of Gain Margin

If at Any phase cross-over freq^n imagi term is zero and real term is not $-ve$ in that case phase cross over freq^n will not exist and gain margin will be either $+\infty$ or $-\infty$ it will be define by absolute stability of the system.

a

$$G(s)H(s) = \frac{K\omega_n^2}{s(s+2\xi\omega_n)}$$

Specified Region



$$G(j\omega)H(j\omega) = \frac{K\omega_n^2}{j\omega(2\xi\omega_n + j\omega)}$$

$$= \frac{K\cdot\omega_n^2}{\omega\left[\sqrt{(2\xi\omega_n)^2+\omega^2}\right]} \angle -90 - \tan^{-1}\frac{\omega}{(2\xi\omega_n)}$$

$$G(j\omega)H(j\omega) = \frac{K\cdot\omega_n^2 \times -j^2(2\xi\omega_n - j\omega)}{(j\omega)(2\xi\omega_n + j\omega)(2\xi\omega_n - j\omega)}$$

$$= \frac{K\cdot\omega_n^2}{(2\xi\omega_n^2+\omega^2)} - \frac{j(2\xi\omega_n^3)\cdot K}{\omega\left[(2\xi\omega_n)^2+\omega^2\right]} \quad \text{(Rectangular form)}$$

at $\omega = \infty$ (only at $\omega=\infty$ पर imaginary term will be zero)

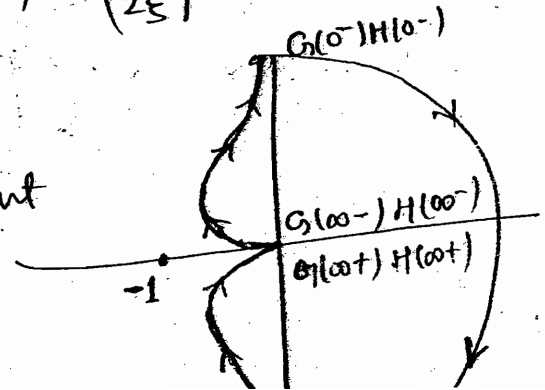
For P. cross-over freq^n imagi $=0$ at this ω real term should be $-ve$)

Here P. crossover freq^n not defined.

$$G(j\infty)H(j\infty) = 0 + j0$$

at $\omega = 0$

$$G(j0)\cdot H(j0) = \left(\frac{K}{2\xi}\right) - j\infty = a\angle -90$$



को any value पर से cut नहीं करेगा। so no encirclement

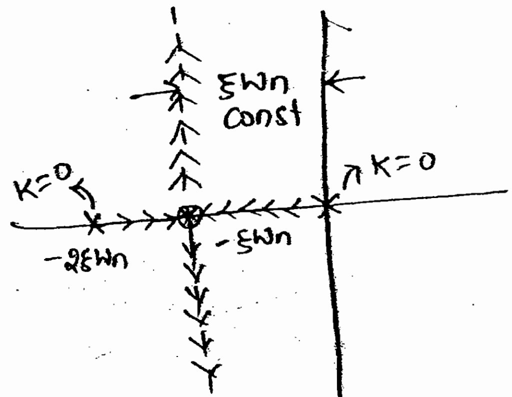
$N = 0$

System Highly stable. so

$G.M. = +\infty$

From root locus.

• . any value of K there is no pole to right side so Highly stable.



$$\frac{C(s)}{R(s)} = \frac{K\cdot\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2 K}$$

$2\xi\omega_n$ → const, $\omega_n^2 K$ → variable

$\omega_n f = \sqrt{K}\,\omega_n$ (variable) $T=\frac{1}{\xi\omega_n}$ (constant)

If this was

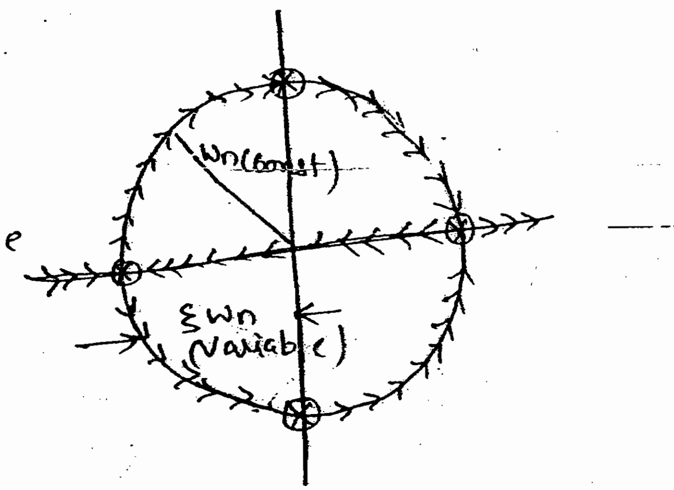
$$G(s)H(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

ω_n^2 → variable, $2\xi\omega_n$ → constant

Here no K term $K=1$ (constant) so we cant draw, Nyquist and Roo only time domain analysis.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

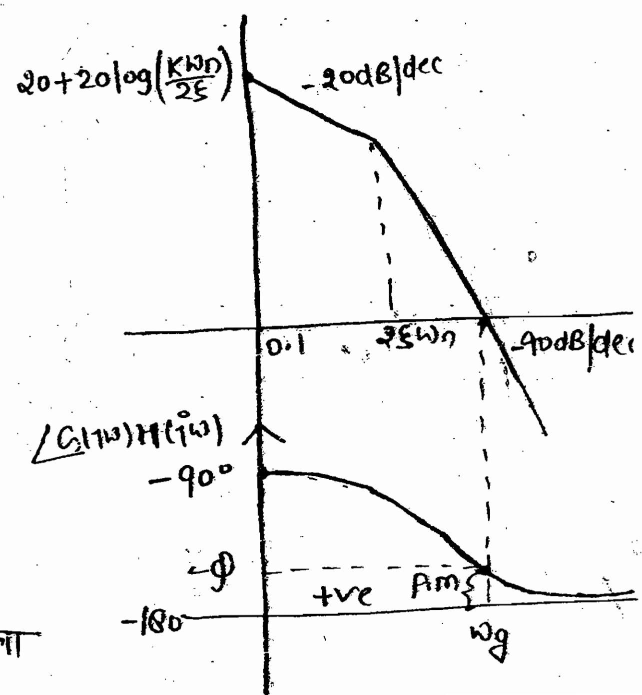
$2\xi\omega_n$ → Variable, ω_n^2 → Constant



So Here it is not always stable
Here stablity decides ξ so
ξ analogous to gain margin.

Now From Bode plot

$$G(s)H(s) = \frac{\left(\frac{K\omega_n}{2\xi}\right)}{s\left[\frac{s}{2\xi\omega_n}+1\right]}$$



It never cut -180 line so phase cross-over freq^n not exists

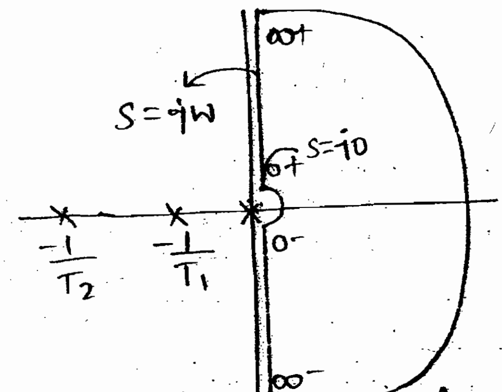
Pm = always +ve so $g.m = +\infty$

$Pm = 180 + \angle G(j\omega_g)\cdot H(j\omega_g)$

अगर P.m –ve होगा तो G.m भी –ve होगा

phase Margin

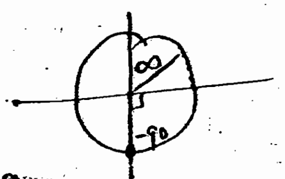
$$G(s)H(s) = \frac{K}{s[sT_1+1][sT_2+1]}$$



$$G(j\omega)H(j\omega) = \frac{K}{j\omega[1+j\omega T_1][1+j\omega T_2]}$$

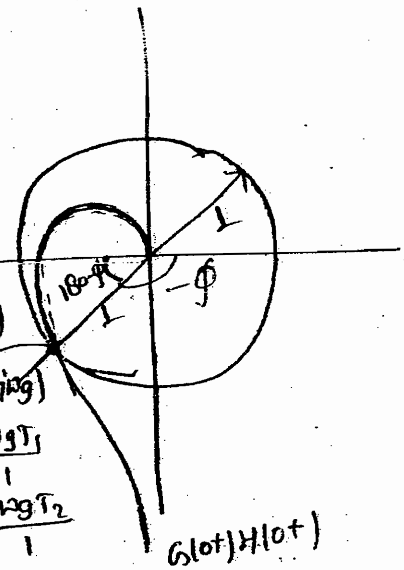
$$= \left\{\frac{K}{\omega\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}}\right\}\angle -90-\tan^{-1}\left(\frac{\omega T_1}{1}\right)-\tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

$$G(j0)H(j0) = \infty\angle -90^\circ$$



- $\omega = \omega_g$

$$G(j\omega_g)H(j\omega_g) = |G(j\omega_g)H(j\omega_g)|\angle G(j\omega_g)H(j\omega_g)$$



$$= \left\{\frac{K}{\omega_g\sqrt{1+(\omega_g T_1)^2}\sqrt{1+(\omega_g T_2)^2}}\right\}\angle -90-\tan^{-1}\frac{\omega_g T_1}{1}-\tan^{-1}\frac{\omega_g T_2}{1}$$

$$= 1\angle -\phi = 1\angle -\phi$$

$$p.m = 180 + \angle G(j\omega_g)\cdot H(j\omega_g) = 180-\phi$$

Gain cross-over Frequency

Gain cross-over freq^n is defined as freq^n at which magnitude of open loop T/F become equal to 1 and p.m. is 180 + phase angle at gain cross-over freq^n.

In given open loop T/F put $s = j\omega$ for +ve positive freq^n line then equate the magnitude of open loop T/F with 1 to calculate

Gain cross-over freq^n - valid G cross-over freq^n should be real and +ve for +ve freq^n line then we will calculate phase angle at gain cross-over freq^n.

$$\boxed{p.m. = 180 + \angle G(j\omega_g)\cdot H(j\omega_g)} = 180 - \phi$$

$$|G(j\omega_g)H(j\omega_g)| = \frac{K}{\omega_g\sqrt{1+(\omega_g T_1)^2}\sqrt{1+(\omega_g T_2)^2}} = 1$$

on solving ω_g = Real and positive. (should be)

$$\angle G(j\omega_g)\cdot H(j\omega_g) = -90 - \tan^{-1}\frac{\omega_g T_1}{1} - \tan^{-1}\frac{\omega_g T_2}{1} = -\phi$$

Limitation of phase margin

* For finite gain cross-over freq^n Nyquist plot should intersect unity circle if any plot is not intersecting unity circle in that case Gain cross-over freq^n will not be real and +ve for +ve freq^n line. It will be either -ve or imagi, or complex which is invalid gain cross-over freq^n as p.m. in that case is either $+\infty$ or $-\infty$ depending on absolute stability of the system

Ex9 $G(s)\cdot H(s) = \dfrac{0.5}{(s+1)}$

$$G(j\omega)H(j\omega) = \frac{0.5}{1+j\omega} = \frac{0.5}{\sqrt{1+\omega^2}}\angle -\tan^{-1}\left(\frac{\omega}{1}\right)$$

$G(j0)H(j0) = 0.5\angle 0^\circ$

$G(j1)H(j1) = \dfrac{0.5}{\sqrt{2}}\angle -45^\circ$

$G(j2)H(j2) = \dfrac{0.5}{\sqrt{2}}\angle -63^\circ$

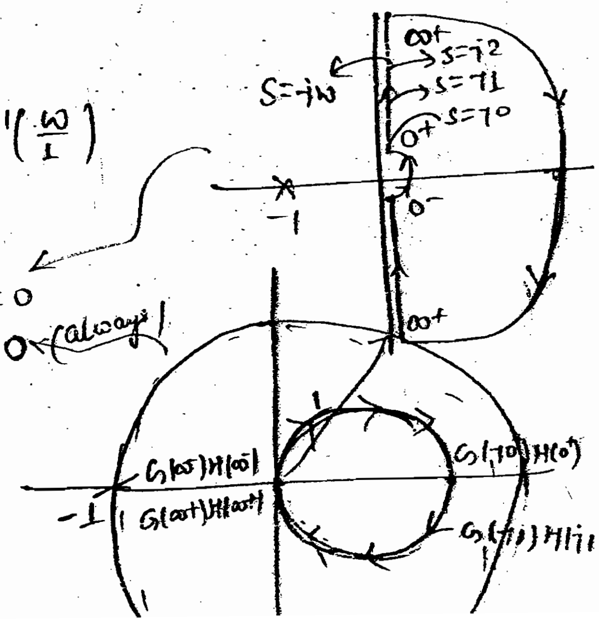
$G(j\infty)H(j\infty) = 0\angle -90^\circ$ Highly stable

p.m. = $+\infty$

$P_+ = 0$

$N = 0$ (always)

$\Rightarrow Z_+ = 0$



* never cut unit circuit

$|G(j\omega_g)H(j\omega_g)| = 1 \Rightarrow \frac{0.5}{\sqrt{1+\omega_g^2}} = 1 \Rightarrow \omega_g = \pm j\sqrt{0.75}$

$\downarrow$ invalid

Gain margin for this

$$G(j\omega)H(j\omega) = \frac{0.5(1-j\omega)}{(1+j\omega)(1-j\omega)} = \frac{0.5}{1+\omega^2} - j\frac{0.5\omega}{1+\omega^2}$$

$\omega = 0$

$$G(j0)H(j0) = +0.5 + j0 \quad (\text{real term not -ve})$$

so not ω_p

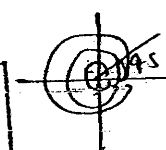
$\omega = \infty$

$$G(j\infty)H(j\infty) = \frac{0.5}{1+\omega^2} - \frac{j0.5}{\frac{1}{\omega}+\omega} = \frac{0.5}{1+\infty} - \frac{j0.5}{\frac{1}{\infty}+\infty}$$

$$= +0 + j0 \quad \{\text{Here also real term not -ve}\}$$

System always stable so gain margin also $= +\infty$

plot को कितना भी rotate करते रहे (∞ times) कभी भी $(-1, 0)$ को cut नहीं करेगा $\Rightarrow$ $P.M = +\infty$



$45 + 1\times 360$
$45 + 2\times 360$
$45 + 3\times 360$
$45 + n\times 360$
$45 + \infty \times 360$
$= \infty$

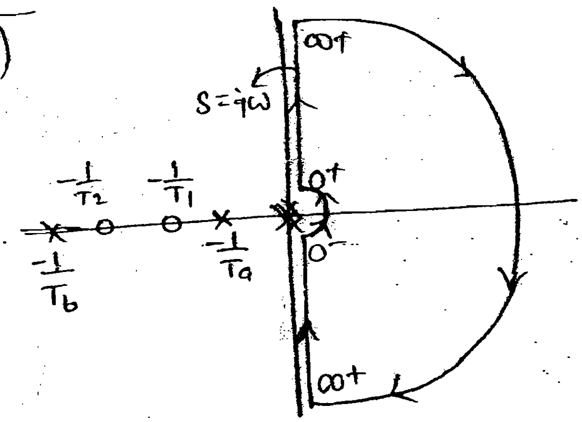
Conditionally stable system

If any plot contain more than one phase cross-over frequency in that case relative stability will be indefined and its stability will be define by absolute stability and for system to be stable Z_+ should be $= 0$, Z_+ is zero of charac^n eq^n lying in the right side of the imaginary axis which itself is pole of closed loop system lying in the right half side of imagi axis

In most cases no of phase cross-over freq^n will be equal to no of zero in open loop T/F

$$G(s)H(s) = \frac{K(sT_1+1)(sT_2+1)}{s^3(sT_a+1)(sT_b+1)}$$

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega T_1)(1+j\omega T_2)}{(j\omega)^3(1+j\omega T_a)(1+j\omega T_b)}$$



$$|G(j\omega)H(j\omega)| \angle G(j\omega)H(j\omega)$$

$$= \frac{K\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}}{\omega^3\sqrt{1+(\omega T_a)^2}\sqrt{1+(\omega T_b)^2}} \angle -270 + \tan^{-1}\frac{\omega T_1}{1} + \tan^{-1}\frac{\omega T_1}{1} - \tan^{-1}\frac{\omega T_a}{1} - \tan^{-1}\frac{\omega T_b}{1}$$

$$G(j0)\cdot H(j0) = \infty \angle -270^\circ \; (+90)$$

$$G(j\infty)H(j\infty) = 0 \angle -270^\circ \; (+90)$$

If $(-1,0)$ lie here

$P_+ = 0$

$N = -1+0+0-1$

$N = -2$

$N = P_+ - Z_+ \Rightarrow \boxed{Z_+ = +2}$

$\Rightarrow$ unstable CLS/T

If $(-1,0)$ lie there

$N = 0+1+0-1 = 0$

$P_+ = 0$

$N = P_+ - Z_+ \Rightarrow \boxed{Z_+ = 0}$

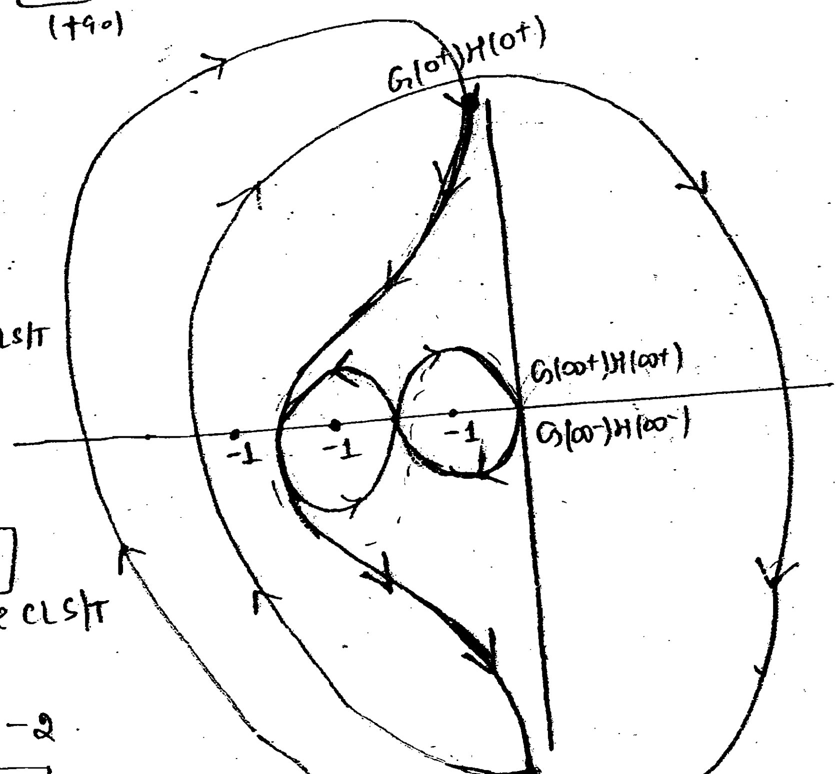
$\Rightarrow$ stable CLS/T

$P_+ = 0$

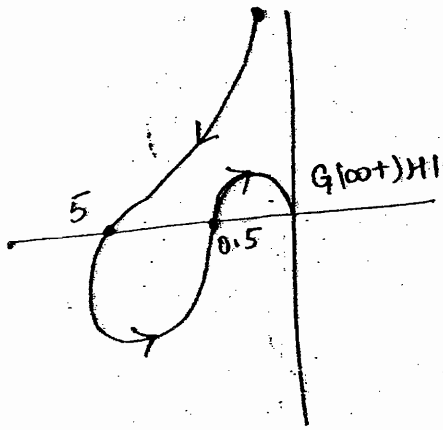
$N = 0+0+1-1 = -2$

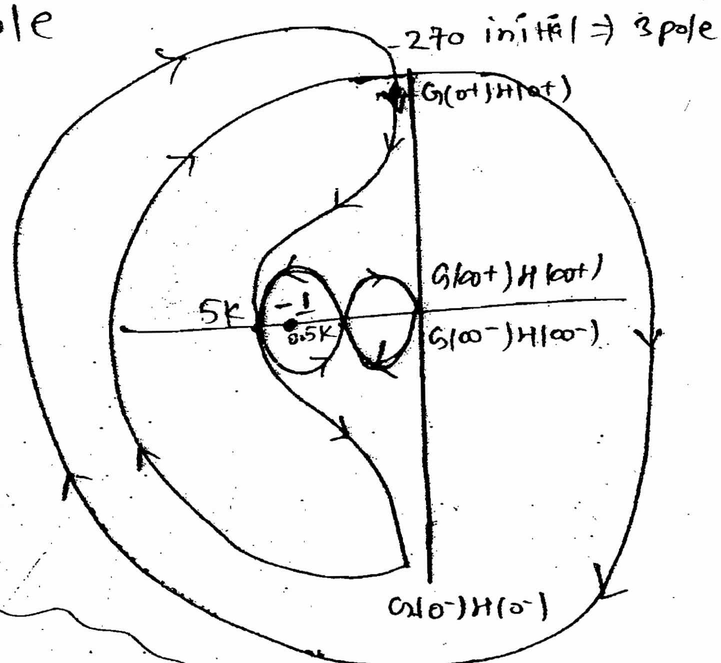
$N = P_+ - Z_+ \Rightarrow \boxed{Z_+ = 2}$

$\Rightarrow$ CLS/T unstable



- Below is plot for stable open loop S/T having unity gain find the range of gain for closed loop system to be stable





If $G(s)H(s) = \frac{K}{s+1}$

$$G(j\omega)\cdot H(j\omega) = \frac{1\times K}{1+j\omega} = \frac{1\times K}{\sqrt{1+\omega^2}}\angle -\tan^{-1}\frac{\omega}{1}$$

$G(j0)H(j0) = 1\times K\angle 0$

$G(j1)H(j1) = \frac{1\times K}{\sqrt{2}}\angle -45°$

$G(j2)H(j2) = \frac{1\times K}{\sqrt{5}}\angle -63°$

$G(j\infty)H(j\infty) = 0\times K\angle -90°$

at
initially start from -270
> 3 pole at origin
⇒ 3 encircles.

for stable

$0.5K < 1$

$5K > 1$

$$\boxed{\frac{1}{5} < K < 2}$$

∵ Given S/T OLS/T stable ⇒ $P_+ = 0$

$Z_+ = 0$

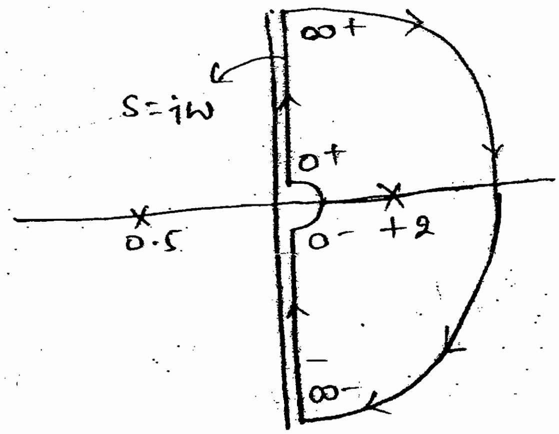
$N = P_+ - Z_+ ⇒ N = 0$

for stable N should be 0

Q. The open loop T/F of a s/T is $G(s)\cdot H(s) = \frac{K(s+1)}{(s+0.5)(s-2)}$ Draw its Nyquist plot and determine range of K for closed loop s/T to be stable

Sol^n

specified region.



$$G(s)H(s) = \frac{K(s+1)}{(s+0.5)(s-2)}$$

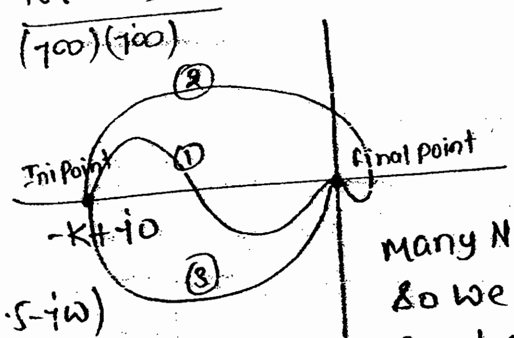
$$G(j\omega)H(j\omega) = \frac{K(1+j\omega)}{(0.5+j\omega)(-2+j\omega)}$$

$$G(j0)H(j0) = \frac{K[1+j0]}{(0.5+j0)(-2+j0)}$$

$$= \frac{K[1]}{0.5(-2)} = \frac{K}{-1} = \frac{K}{j^2} = K\angle -180 = -K + j0$$

$$G(j\infty)H(j\infty) = \frac{K[1+j\infty]}{(0.5+j\infty)(-2+j\infty)} = \frac{K[j\infty]}{(j\infty)(j\infty)}$$

$$= \frac{K}{j\infty} = 0\angle -90$$



Many No of ways so we require Exact analysis

$$G(j\omega)H(j\omega) = \frac{K[1+j\omega]\,(-2-j\omega)(0.5-j\omega)}{(0.5+j\omega)(-2+j\omega)(-2-j\omega)(0.5-j\omega)}$$

$$= \frac{-K[1+2.5\omega^2]}{(0.25+\omega^2)(4+\omega^2)} - \frac{jK\omega(\omega^2-0.5)}{(0.25+\omega^2)(4+\omega^2)}$$

For all possible value of ω real term remain −ve.
For imag term

→ $\omega^2 < 0.25 \Rightarrow \omega < \sqrt{0.25} = 0.707$

Imaginary term will be +ve

$w^2 = 0.5 \Rightarrow w = \sqrt{0.5}$

Ima term $= 0$ & real term $= -ve$

So $w_p = \pm\sqrt{0.5}$

$$G(i\sqrt{0.5}).H(i\sqrt{0.5}) = \frac{-K[1+2.5\times 0.5]}{(0.25+0.5](4+0.5)} \mp j0$$

$$= -\frac{2}{3}K + i0$$

$w^2 > 0.5 \Rightarrow w > \sqrt{0.5}$

Imagi term $= -ve$

$0 < w < \sqrt{0.5}$

Re $= -ve$, Imag $= +ve$

$\Rightarrow$ 2 Quadrant का plot

$\sqrt{0.5} < w < \infty$

Re $\rightarrow -ve$, Imag $= -ve$

3rd Quadrant

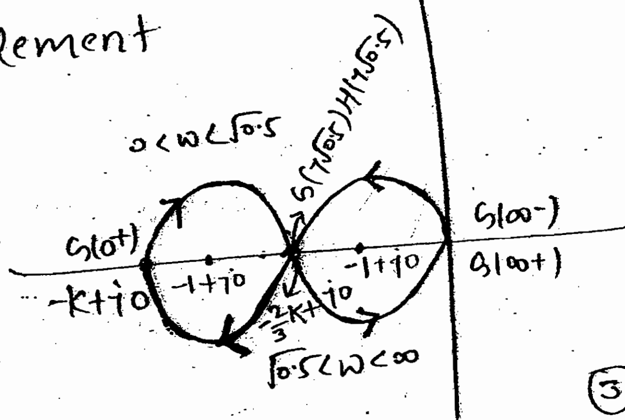
$w = \sqrt{0.5}$ Re $= -\frac{2}{3}$, IM $= 0$

~~$w > \sqrt{0.5} < \infty$~~

plot will be

is is lies under III rd case

o no require encirclement



$K < 1$

$P_+ = 1$

$N = 0$

$N = P_+ - Z_+ \Rightarrow \boxed{Z_+ = 1}$

unstable

(2)

$\boxed{1 < K < 1.5}$

$K > 1$

$\frac{2}{3}K < 1$

$K < \frac{3}{2}$

$P_+ = 1$

$N = -1$

$N = P_+ - Z_+$

$-1 = 1 - Z_+ \Rightarrow \boxed{Z_+ = 2}$

(3)

$K > 1$

$\frac{2}{3}K > 1$

$K > \frac{3}{2}$

$\boxed{K > 1.5}$

$P_+ = 1$

$N = ~~0~~ +1$

$N = P_+ - Z_+$

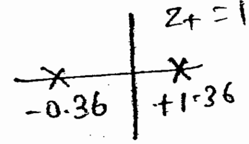
$\boxed{Z_+ = 0}$ of char eqn

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K(s+1)}{s^2+(K-1.5)s+(K+1)}$$

$$1+G(s)H(s) = s^2+(K-1.5)s+(K+1) = 0$$

$\boxed{K=0.5}$ in this one pole in right side.

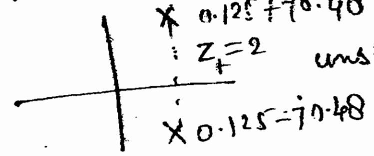
$$s^2-1s+0.5=0 \;,\; s=+1.36, -0.36$$



unstable

$\boxed{K=1.25}$

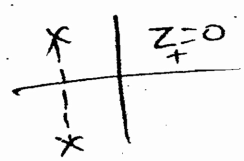
$$s^2-0.25s+0.25=0 \;,\; s=+0.125 \pm j0.48$$



unstable

3) $\boxed{K=2}$

$$s^2+0.5s+1=0 \Rightarrow s=-0.25 \pm j0.96$$

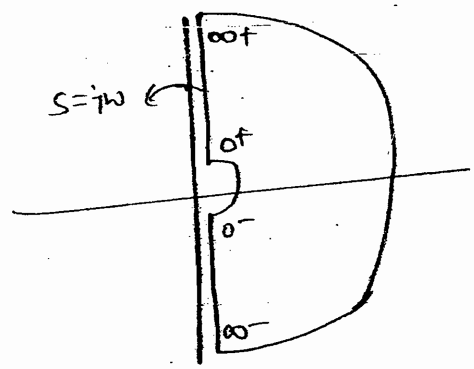


stable

Q The open loop T/F of a S/T is $G(s)H(s) = \dfrac{(s+0.25)}{s^2(s+0.5)(s+1)}$

Determine both absolute stability as well as relative stability

Sol^n



$$G(s)H(s) = \frac{s+0.25}{s^2(s+0.5)(s+1)}$$

$$G(j\omega)H(j\omega) = \frac{(0.25+j\omega)}{(j\omega)^2(0.5+j\omega)(1+j\omega)}$$

$$G(j0)H(j0) = \frac{(0.25+j0)}{(j0)^2(0.5+j0)(1+j0)}$$ No phase inform

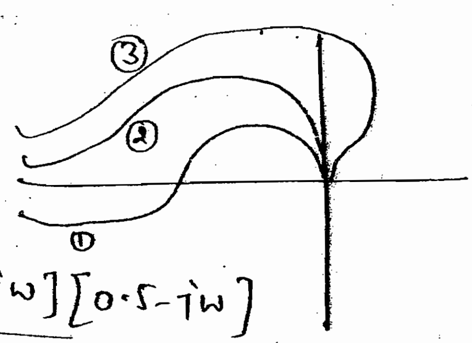
$$= \frac{(0.25)}{(j0)^2(0.5)(1)} = \frac{1}{j^2 0} = \infty \angle -180^\circ$$

$$G(j\infty)\cdot H(j\infty) = \frac{(0.25+j\infty)}{(j\infty)^2(0.5+j\infty)(1+j\infty)} = \frac{\cancel{0.25}\, j\infty}{(j\infty)^2(j\infty)j\infty}$$

Neglect

$$= \frac{1}{j^3\infty} = 0 \angle -270 \; (+90)$$

can be many plot to find Exact plot we use Rationalization Method



$$G(j\omega)H(j\omega) = \frac{-(0.25+j\omega)[1-j\omega][0.5-j\omega]}{\omega^2[0.5+j\omega][1+j\omega][0.5-j\omega][1-j\omega]}$$

on solving

$$= -\frac{[0.125+1.25\omega^2]}{\omega^2(1+\omega^2)(0.25+\omega^2)} + \frac{j[\omega^2-0.125]}{\omega[1+\omega^2)(0.25+\omega^2)}$$

For all +ve values of ω Real term will always remain -ve

For Imag term

$\omega^2 < 0.125$, $\omega < \sqrt{0.125}$

$\Rightarrow$ Ima term = -ve for this freq^n range.

$\omega^2 = 0.125$, $\omega = \pm\sqrt{0.125}$

$\omega_p = \pm\sqrt{0.125}$

$$G(j\sqrt{0.125})H(j\sqrt{0.125}) = \frac{-(0.125+1.25\times0.125)}{(0.125)(1+0.125)(0.25+0.125)} + j0$$

$$= -5.33 + j0$$

$\omega^2 > 0.125$, $\omega > \sqrt{0.125}$

Imag term = +ve

$0 < \omega < \sqrt{0.125}$ $\rightarrow$ Real term $\rightarrow$ -ve, Imag term $\rightarrow$ -ve $\Rightarrow$ plot originate from 3rd Quadra

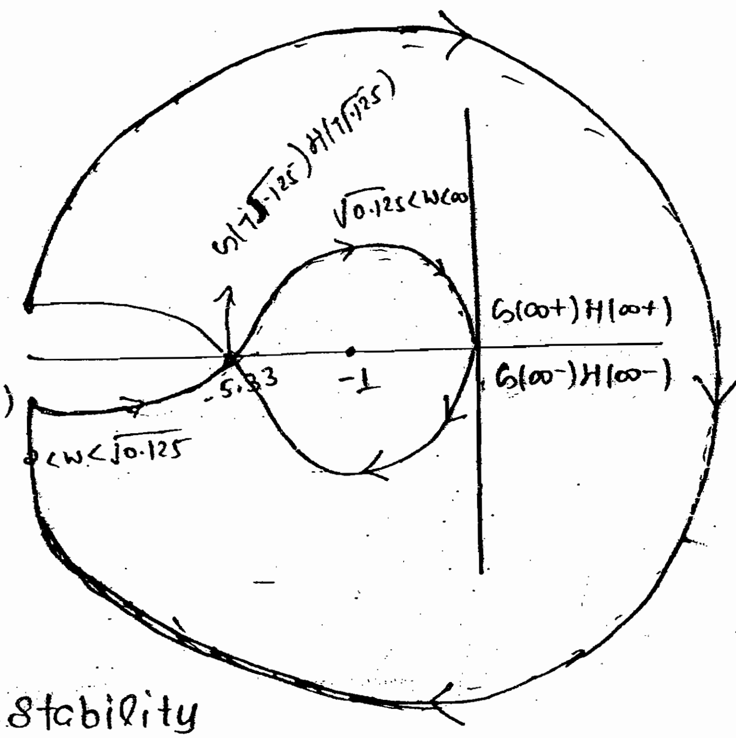
$\omega = \sqrt{0.125}$ Re = -5.33 (Intersect -ve Real Axis)

Ima = 0

$\sqrt{0.125} < \omega < \infty$ $\underbrace{\text{Re +ve, Im +ve}}$

$\Rightarrow$ terminate 2nd Quadrant

$\Rightarrow$ Exact plot is 1



$P_+ = 0$

$N = -2$

$N = P_+ - Z_+$

$\boxed{Z_+ = 2} \Rightarrow$ unsta

This stability define interms of Pole so absolute stability

Relative stability

$$G.M. = \frac{1}{|G(j\omega_p) \cdot H(j\omega_p)|} = \frac{1}{|5.33|} = 0.187 < 1 \text{ (unstable)}$$

$$G.M|_{dB} = 20\log\frac{1}{|5.33|} = -14.56 \quad (-ve\ G.M) \Rightarrow \text{s/t unstabl}$$

$$\frac{C(s)}{R(s)} = \frac{(s + 0.25)}{s^4 + 1.5s^3 + 0.5s^2 + s + 0.25} \quad \text{(closed loop T/F function)}$$

$$1 + G(s)H(s) = s^4 + 1.5s^3 + 0.5s^2 + s + 0.25$$

s^4	1	0.5	0.25
s^3	1.5	1	0
s^2	-0.16	0.25	0
s^1	+3.2	0	0
s^0	+0.25	0	0

$\Rightarrow$ 2 sign change so unstable.

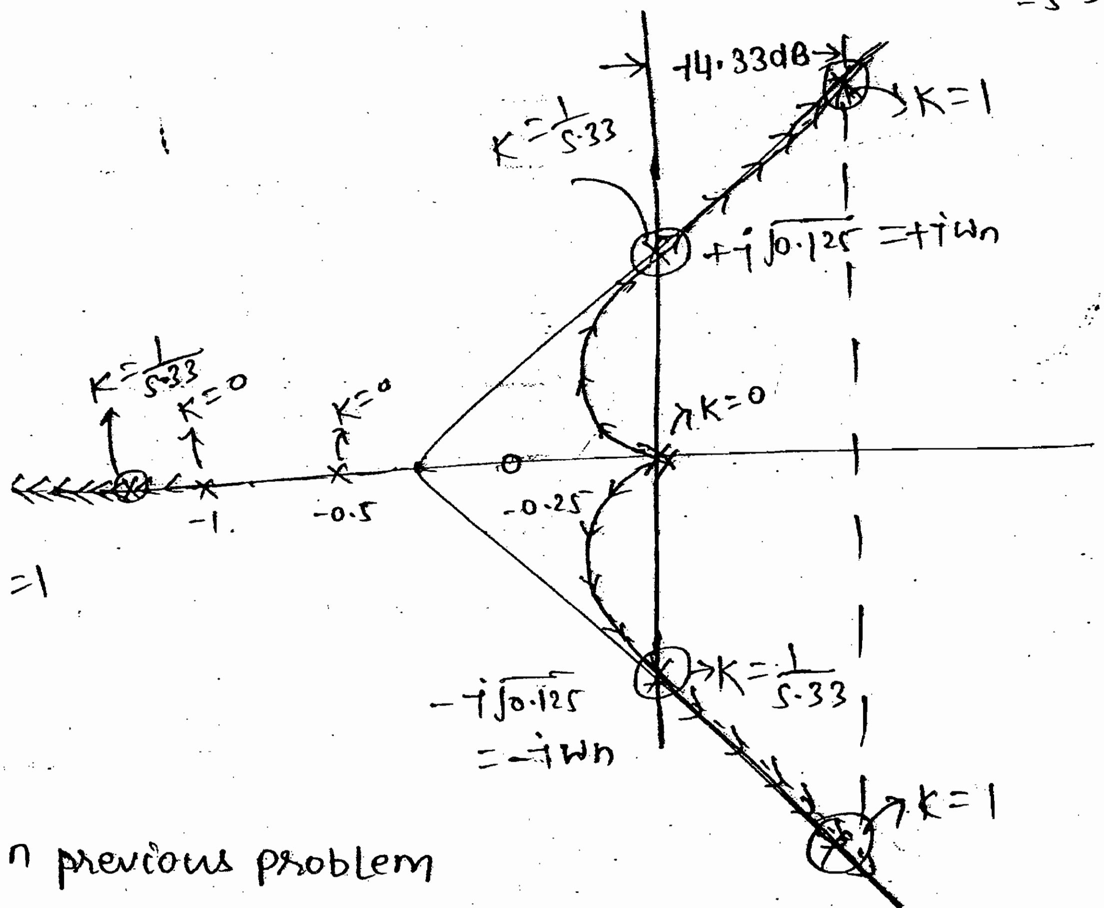
ω From root locus

$$G(s)H(s) = \frac{K(s+0.25)}{s^2(s+0.5)(s+1)}$$

$G(j\sqrt{0.125})H(j\sqrt{0.125})$
$= -5.33K + j0$

$-5.33K + j0 = -1$
$\Rightarrow K = \frac{1}{5.33}$

$0 < K < \frac{1}{5.33}$

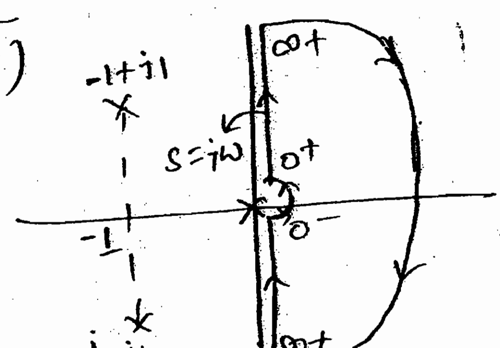


n previous problem
=1 था so at K=1 पर 2 pole on right side so
system unstable at K=1

The open loop T/F of a system is $G(s)H(s) = \frac{2.2}{s(s+1)(s^2+2s+2)}$
Draw its Nyquist plot and determine both absolute and relative stability also determine the range of gain for closed loop S/T to be stable

Sol^n

$$G(s)H(s) = \frac{2.2}{s(s+1)(s^2+2s+2)}$$



$$G(j\omega)\cdot H(j\omega) = \frac{2.2}{(j\omega)[1+j\omega][(j\omega)^2+2(j\omega)+2]}$$

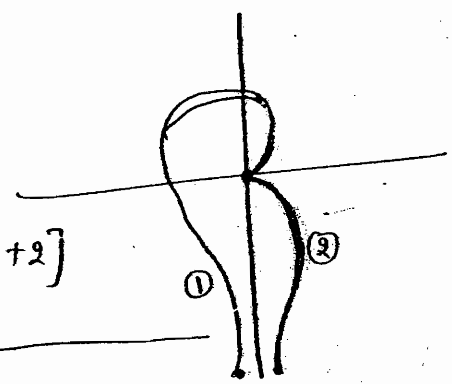
$$G(j0)\cdot H(j0) = \frac{2.2}{(j0)[\underbrace{1+j0}_{\downarrow 0}][\underbrace{(j0)^2+2(j0)}_{\downarrow 0}+2]} = \frac{2.2}{j0\times 2}$$

$$= \frac{1}{j0} = \infty\angle -90^\circ$$

$$G(j\infty)H(j\infty) = \frac{2.2}{(j\infty)[\underbrace{1}_{\text{neglect}}+j\infty][(j\infty)^2+\underbrace{2(j\infty)+2}_{\text{neglect}}]}$$

$$= \frac{2.2}{j^4\infty} = 0\angle -360^\circ$$

Exact Analysis



$$G(j\omega)\cdot H(j\omega) = \frac{2.2}{j\omega(1+j\omega)[(j\omega)^2+2(j\omega)+2]}$$

$$= \frac{-j^2\, 2.2\,[1-j\omega][(2-\omega^2)-j2\omega]}{(j\omega)(1+j\omega)[(2-\omega^2)+j2\omega][(2-\omega^2)-j2\omega][1-j\omega]}$$

$$= \frac{-2.2[4-\omega^2]}{(1+\omega^2)[(2-\omega^2)^2+4\omega^2]} - \frac{j\,2.2(2-3\omega^2)}{\omega[1+\omega^2][(2-\omega^2)+4\omega^2]}$$

→ for Imaginary term.

→ $3\omega^2 < 2 \Rightarrow \omega < \sqrt{\frac{2}{3}} = 0.816$

Imag term = −ve

→ $3\omega^2 = 2 \Rightarrow \omega = \pm\sqrt{\frac{2}{3}} = 0.816$

$$G\left(j\sqrt{\tfrac{2}{3}}\right)\cdot H\left(j\sqrt{\tfrac{2}{3}}\right) = -\frac{2.2\left[4-\frac{2}{3}\right]}{\left(1+\frac{2}{3}\right)\left[\left(2-\frac{2}{3}\right)^2+4\times\frac{2}{3}\right]} + j0$$

$$= -0.99 + j0$$

$3w^2 > 2 \Rightarrow w > \sqrt{\frac{2}{3}} \Rightarrow$ Im term = +ve

For Real term.

$w^2 < 4 \Rightarrow w < 2$, Re term −ve

$w < 2$

$w^2 = 4 \Rightarrow w = \pm 2$

$$G(j2)\cdot H(j2) = 0 - \frac{j2\cdot 2\,(2-12)}{2(1+4)[(2-4)^2+4\times 4]} = 0 + j0.11$$

$\Rightarrow$ Real term $= 0$, Im $= +j0.11$

$w^2 > 4 \Rightarrow w > 2 \Rightarrow$ Real term +ve

$0 < w < \sqrt{\frac{2}{3}}$ Im = −ve & Real −ve ($\Rightarrow$ originate 3rd quadr)

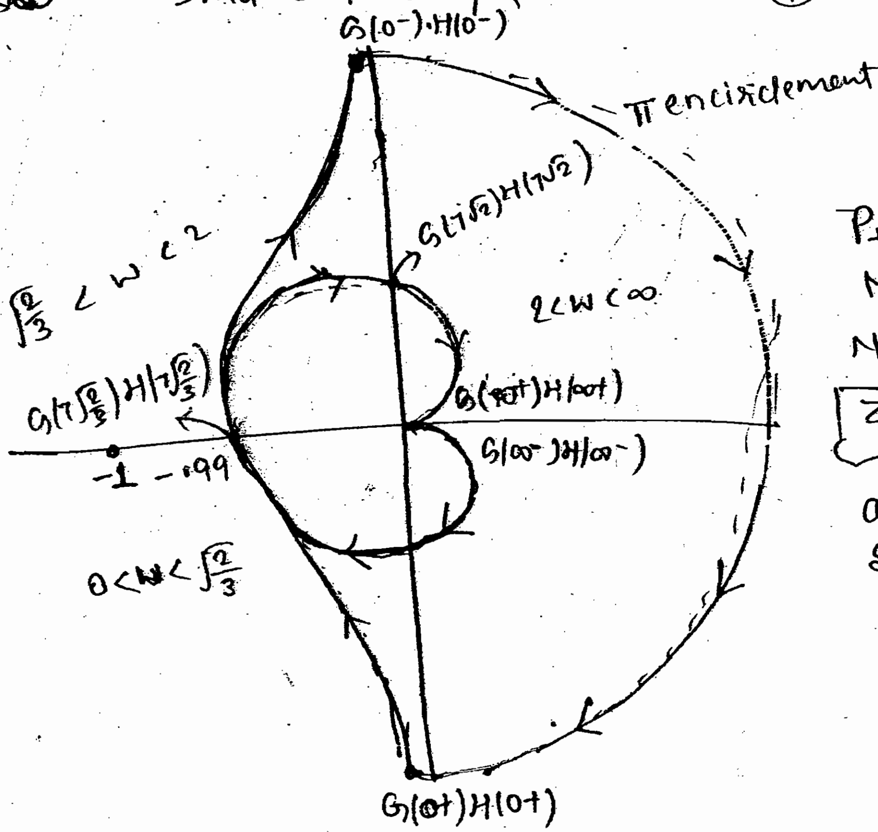
$w = \sqrt{\frac{2}{3}}$ intersect at Negative real Axis

$\sqrt{\frac{2}{3}} < w < 2$ Imag = +ve & Real = −ve ②

$w = 2$ Imag = $j0.11$, Re = 0

$2 < w < \infty$ Ima = +ve Re = +ve ①

Case I



$P_+ = 0$

$N = 0$

$N = P_+ - Z_+$

$\boxed{Z_+ = 0}$

absolute Stable s/m

Relative stability

$G(j\omega_p)H(j\omega_p) = -0.99 + j0$

$$G.M. = \frac{1}{|G(j\omega_p)H(j\omega_p)|} = \frac{1}{|0.99|} = 1.01 > 1$$

$G.M = 20\log 1.01 = +0.08\,dB \Rightarrow$ CL s/m will be stable

$$G(s)\cdot H(s) = \frac{\left\{2.2 \times \frac{1}{2.2}\right\}K}{s(s+1)(s^2+2s+2)}$$

$$G(j\omega_p)\cdot H(j\omega_p) = \left\{0.99 \times \frac{1}{2.2}\right\}K + j0$$

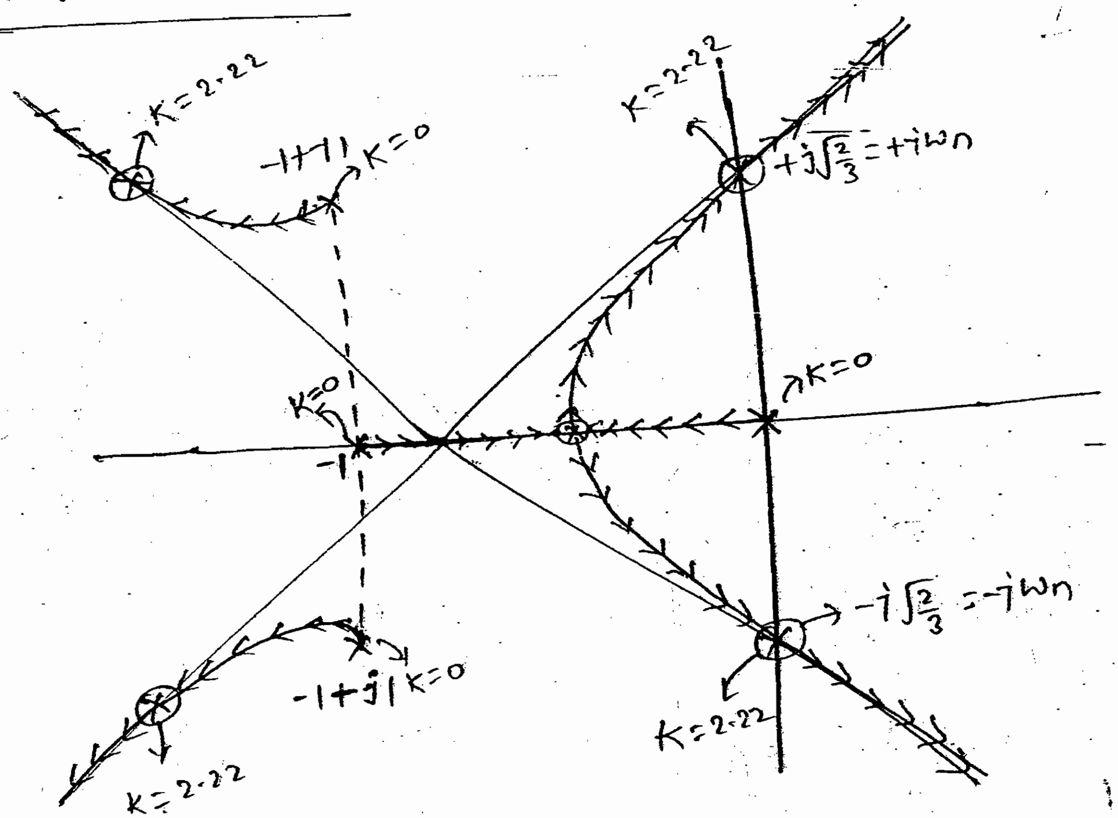
$$K_{MS} = \frac{2.2}{0.9} = 2.22$$

$$\omega_n = \omega_p = \sqrt{\frac{2}{3}}$$

$$\boxed{0 < K < 2.22}$$

Root locus का gain, gain होता है। not dc gain ⇒ G(s)H(s) standard form हो या न हो

From Root locus



The open loop T/F of a system is $G(S)\cdot H(S) = \dfrac{K}{S^3[TS+1]}$

Determine gain margin of the system

$1+G(S)H(S) = 0 \Rightarrow TS^4 + S^3 + K = 0$ – unstable system

minimum phase S/T so

$GM = -\infty$

From root locus

$S_P = 0, 0, 0, -\frac{1}{T}$

$1+G(S)H(S) = 0$

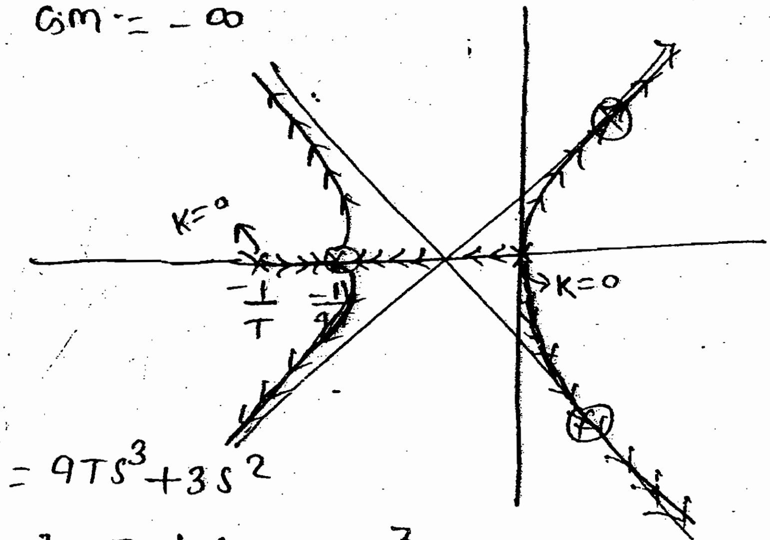
$1+\dfrac{K}{S^3(TS+1)} = 0$

$-K = TS^4 + S^3 \Rightarrow -\dfrac{dK}{dS} = 4TS^3 + 3S^2$

$\dfrac{dK}{dS} = 0 \Rightarrow S^2[4TS+3] = 0 \Rightarrow S = 0, 0, -\dfrac{3}{4T}$

centroid $x = \dfrac{\Sigma P - \Sigma Z}{P-Z} = -\dfrac{1}{4T}$

$\theta_0 = 45°, \theta_1 = 135, \theta_2 = 225, \theta_3 = 315°$



⇒ 2 poles always right side so S/T unstable,

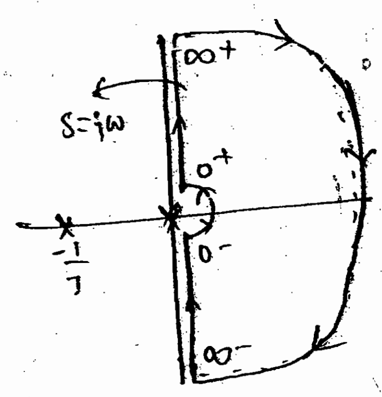
S^4	T	0	K
S^3	1	0	0
S^2	0	K	0
S^1	$-\infty$	0	0
S^0	+K	0	0

For any value of K

any odd row ≠ 0

rom Nyquist plot

$G(j\omega)H(j\omega) = \dfrac{K}{(j\omega)^3(1+j\omega T)}$



$G(j0)H(j0) = \dfrac{K}{(j0)^3(1+j0)}$ ↓ no infor^n

$= \dfrac{K}{j^3 0} = \infty \angle -270 (+90)$

$G(j\infty)H(j\infty) = \frac{K}{(j\infty)^3(1+j\infty)} = \frac{K}{(j\infty)^4} = 0\angle{-360}\,(0°)$

$P_+ = 0$

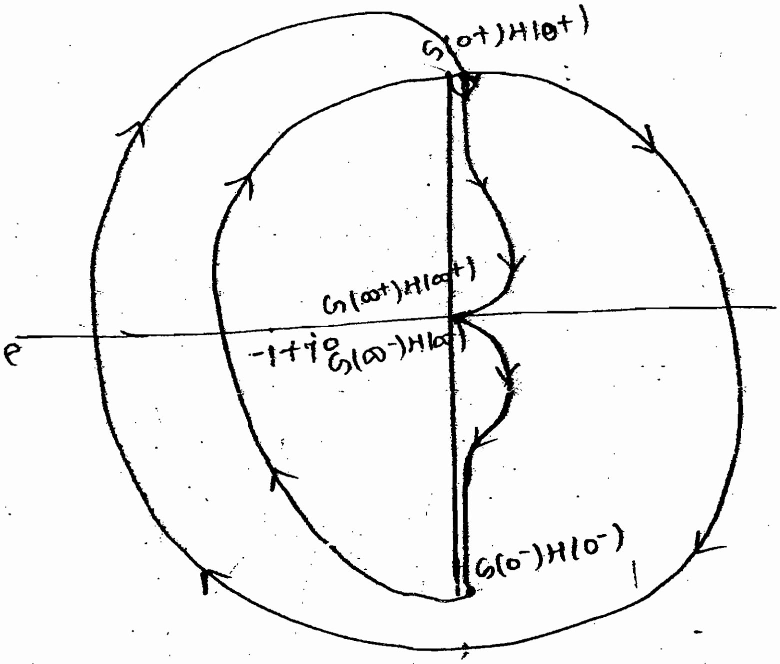
$N = -2$

$N = P_+ - Z_+$

$\boxed{Z_+ = 2}$

always unstable

$\Rightarrow G.M = -\infty$



From exact analysis

$$G(j\omega)H(j\omega) = \frac{j^4 K\,[1-j\omega T]}{(j\omega)^3\,[1+j\omega T]\,[1-j\omega T]} = \frac{KT}{\omega^2[1+(\omega T)^2]} + j\frac{K}{\omega^3[1+(\omega T)^2]}$$

for all values of ω (0 to $+\infty$) Real and Imagi term = +ve

$G(j\infty)H(j\infty) = 0 + j0$ No phase cross-over freq^n exists so G.M $< \begin{matrix} +\infty \\ -\infty \end{matrix}$

G.M $< \begin{matrix} +\infty \\ -\infty \end{matrix}$ } depends on absolute stability.

Q $G(s)H(s) = \frac{Ks^3}{(s+1)(s+2)}$ Draw its N. plot and Determine range of K for closed Loop s/T to be stable.

Sol^n

$$G(j\omega)H(j\omega) = \frac{K(j\omega)^3}{(1+j\omega)(2+j\omega)} =$$

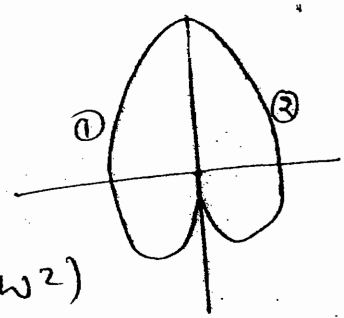
$$G(j0)H(j0) = \frac{K(j0)^3}{(1+j0)(2+j0)} = \frac{Kj^3 0^3}{\cancel{10\times1}\ 1\times2}$$

$$= 0\angle{+270}\,(-90°)$$

$$G(j\infty)H(j\infty) = \frac{K(j\infty)^3}{(1+j\infty)(2+j\infty)} = \frac{K(j\infty)^3}{j\infty \times j\infty}$$

$$= \infty \angle \pm 90^\circ$$

$$G(j\omega)H(j\omega) = \frac{-jK(\omega)^3(1-j\omega)(2-j\omega)}{(1+j\omega)(2+j\omega)(1-j\omega)(2-j\omega)}$$



$$= \frac{-3K\omega^4}{(1+\omega^2)(4+\omega^2)} - \frac{jK\omega^3(2-\omega^2)}{(1+\omega^2)(4+\omega^2)}$$

For all positive values of ω real term will remain $-ve$

For Imagi term

$\omega^2 < 2 \Rightarrow \omega < \sqrt{2}$ Ima term $= -ve$

$\omega^2 = 2 \Rightarrow \omega_p = \pm\sqrt{2}$ $G(j\sqrt{2})H(j\sqrt{2}) = \dfrac{-3K \times (2)^2}{(1+2)(4+2)} + j0$

Real term $= -\frac{2}{3}K$, ima term $= 0$ $= -\frac{2}{3}K + j0$

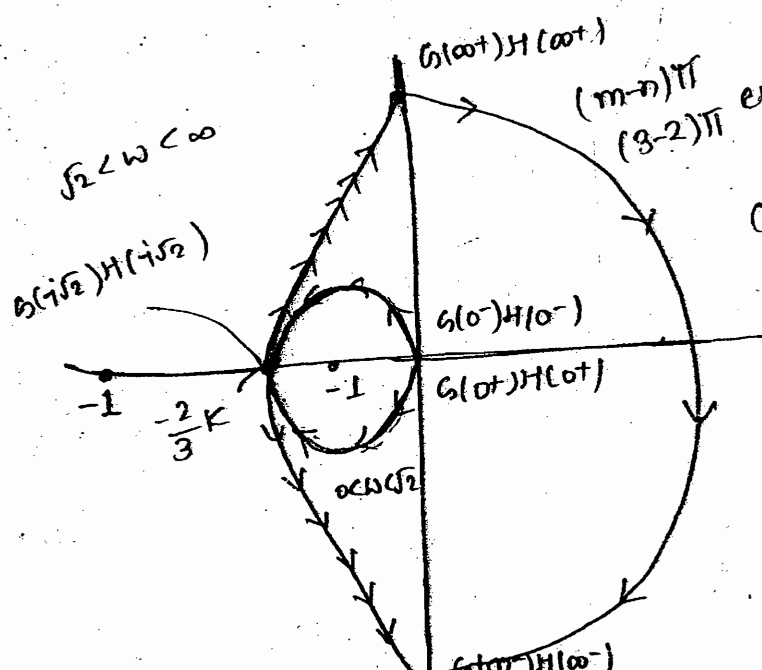
$\omega^2 > 2 \Rightarrow \omega > \sqrt{2}$

ima term $= +ve$

$0 < \omega < \sqrt{2}$ Real term $-ve$, Imag term $-ve$ } Plot will be 3rd Quadrant

$\omega = \sqrt{2}$ intersects

$\sqrt{2} < \omega < \infty$ Real term $\rightarrow -ve$, ima $+ve$



$(m-n)\pi$ encirclement

$(3-2)\pi$

Come under second case

$\boxed{0 < K < 1.5}$ $\qquad \frac{2}{3}K < 1, \ K < \frac{3}{2}$

$P_+ = 0$

$N = 0$ $\Rightarrow N = P_+ - Z_+ \Rightarrow \boxed{Z_+ = 0}$ c/s/T stable for ranse of K.

$\boxed{K > 1.5}$ $\qquad \frac{2}{3}K > 1, \ K > \frac{3}{2}$

$P_+ = 0$

$N = -2$ $\qquad N = P_+ - Z_+ \Rightarrow \boxed{Z_+ = 2}$ For this ranse of K in this case this is unstable.

Relative stability

$$G.m. = \frac{1}{|G(j\omega_p)H(j\omega_p)|} = \frac{1}{|\frac{2}{3}K|}$$

$$|G.m|_{dB} = -\left\{20\log\left|\frac{2}{3}K\right|\right\} \qquad \frac{C(s)}{R(s)} = \frac{Ks^3}{Ks^3 + s^2 + 3s + 2}$$

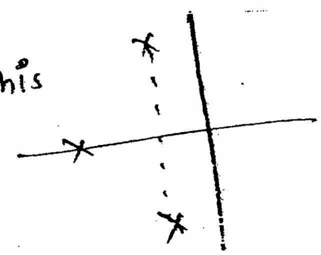
$$1 + G(s)H(s) = Ks^3 + s^2 + 3s + 2$$

let $\boxed{K=1}$ $\quad s^3 + s^2 + 3s + 2 = 0 \Rightarrow s = -0.715, \ -0.14 \pm j1.66$

$Gm = -20\log\left(\frac{2}{3}\times 1\right) = +3.6\ dB$ $\quad$ stable for this

For this (+ve G.m)

$\Rightarrow$ stable s/T



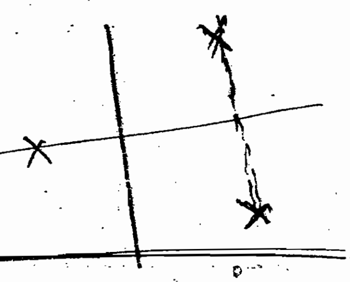
let $\boxed{K=2}$ $\quad 2s^3 + s^2 + 3s + 2 = 0 \Rightarrow s = -0.63, +0.06 \pm j1.25$

unstable s/T $\Leftarrow$

For this range of K

$G.m. = -20\log\left(\frac{2}{3}\times 2\right)$

$G.m. = -2.5\ dB$ (-ve G.m) $\Rightarrow$ unstable s/T



$\because G(j\omega_p)\cdot H(j\omega_p) = -\frac{2}{3}K + j0$

$\Rightarrow$ In Root locus imaginary पर root locus $-\frac{2}{3}K = -1 \Rightarrow K = \frac{3}{2}$ पर cut करेगा ।

From root locus

$s_z = 0, 0, 0, \quad s_p = -1, -2$

$1 + G(s)H(s) = 0 \Rightarrow -K = \frac{s^2+3s+2}{s^3}$

$-\frac{dK}{ds} = \frac{s^3(2s+3) - (s^2+3s+2) \times 3s^2}{(s^3)^2} \Rightarrow \frac{dK}{ds} = 0$

$s^2[-s^3 - 6s - 6] = 0 \Rightarrow s = 0, 0, \cancel{-4.73}, \cancel{-1.26}$

$x = \frac{\Sigma p - \Sigma z}{P - Z} = \frac{-3-0}{2-3} = +3$

$\theta_K = \frac{(2K+1)180}{P-Z} \Rightarrow \theta_0 = 180^\circ$

m.s. $1 + G(s) \cdot H(s) = Ks^3 + s^2 + 3s + 2 = 0$

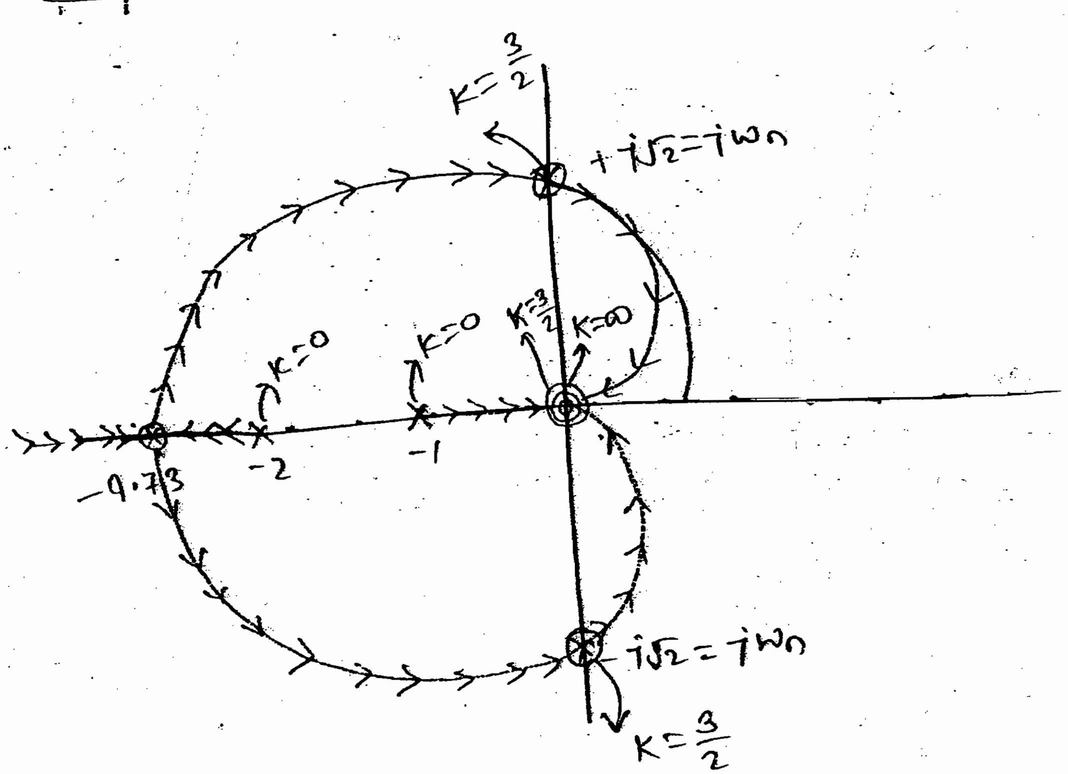
s^3	K	3
s^2	1	2
s^1	$3-2K$	0
s^0	2	

For marginally stable odd row should be $= 0 \Rightarrow 3 - 2K = 0$

$\boxed{K = \frac{3}{2}}$

$s^2 + 2 = 0 \Rightarrow s = \pm j\sqrt{2} = \pm j\omega_n$

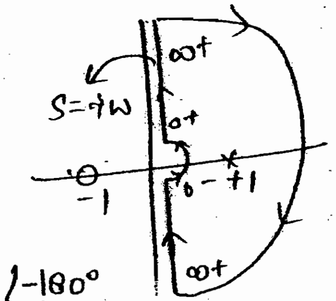
$\boxed{\omega_n = \sqrt{2}}$



Q. The OLTF of s/T is $G(s)\cdot H(s) = \frac{K(s+1)}{(1-s)}$ DRAW. its Nyquist plot and determine range of K for CL s/T to be stable

Solⁿ * when pole and zero locate at mirror image about imaginary axis that s/T is known as all pass s/T and Nyquist plot of all pass s/T will always be circular in shape

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega)}{(1-j\omega)}$$

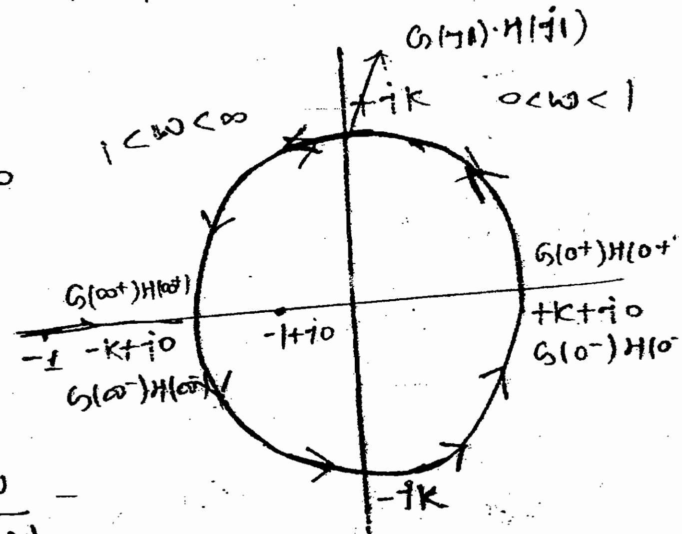


$$G(j0)H(j0) = K\angle 0^\circ$$

$$G(j\infty)H(j\infty) = \frac{K(1+j\infty)}{(1-j\infty)} = \frac{K}{-1} = \frac{1}{j^2} = K\angle -180^\circ$$

$$G(j0)H(j0) = +K+j0 = K\angle 0^\circ$$

$$G(j\infty)H(j\infty) = -K+j0 = K\angle -180$$



$$G(j\omega)H(j\omega) = \frac{K[1+j\omega][1+j\omega]}{[1-j\omega][1+j\omega]}$$

$$= \frac{K[1-\omega^2]}{(1+\omega^2)} + j\frac{2K\omega}{(1+\omega^2)}$$

→ for all +ve values of ω real term remain +ve

→ $\omega < 1$ real term +ve

→ $\omega^2 = 1 \Rightarrow \omega = \pm 1$ Real term = 0, Imag term = K

$G(j1)H(j1) = 0 + j\frac{2K \times 1}{2} = 0 + jK$

→ $\omega^2 > 1 \Rightarrow \omega > 1$ Real term → -ve

$0 < \omega < 1$ ima term = +ve & Real term = +ve (1)

$\omega = 1$ ima term = +jK, real term = 0

$1 < \omega < \infty$ ima term = +ve, real term = -ve (2)

$$G.m. = \frac{1}{|G(j\omega_p)\cdot H(j\omega_p)|} = \frac{1}{|K|} \quad \& \; |G.m|_{dB} = -[20\log|K|]$$

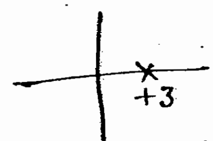
$\boxed{<1}$ $P_+ = 1,\ N = 0 \Rightarrow N = P_+ - Z_+ \Rightarrow \boxed{Z_+ = 1}$ zero of c.eqⁿ = pole of CLS/T
CLS/T unstable

$\boxed{>1}$ $P_+ = 1,\ N = 1 \Rightarrow \boxed{Z_+ = 0} \Rightarrow$ stable closed loop system

$$\frac{C(s)}{R(s)} \Rightarrow \frac{G(s)}{1+G(s)H(s)} = \frac{K(s+1)}{-(1-K)s+(1+K)}$$

$1+G(s)H(s) = 0 \Rightarrow -(1-K)s+(1+K) = 0$

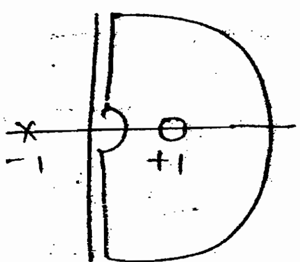
If $\boxed{K=+0.5}$ $\quad -0.5s+1.5=0 \Rightarrow s=+3$



$G.m. = -[20\log(0.5)] = +6.02\,dB$ (unstable)

If $\boxed{K=2}$ $\quad s+3=0 \Rightarrow s=-3$

$G.m. = -[20\log|2|] = -6.02\,dB$ (Non mini p. S/T stable ⇒ G.m = −ve)

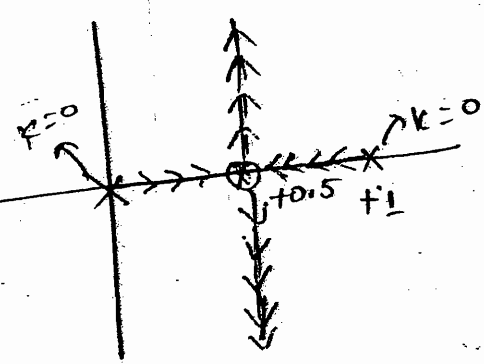


If this case
this is No m.p.S/T but in right side no pole
so this is stable
so. for S/T S/T G.m = +ve not −ve
because I dont bother about zero on right side.

$$G(s)H(s) = \frac{K(1-s)}{(1+s)}$$

$$G(s)H(s) = \frac{5}{s(1-s)} \quad \text{If} = \frac{K}{s(1-s)}$$

Gain margin
$= +\infty$
unstable S/T for all values of K

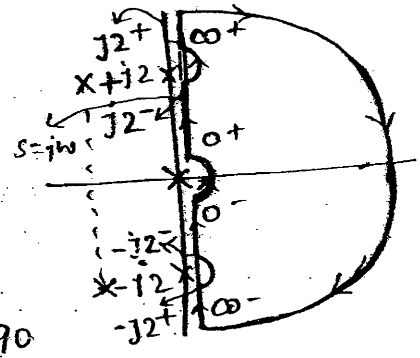


The OLTF of a sH is $G(s)H(s) = \frac{100}{s(s^2+4)}$ Draw its Nyquist plot and Determine G.M. of the system.

Solⁿ

$2^+ = 2+0.00\ldots1$

$2^- = 2-0.00\ldots1$



$$G(j\omega)H(j\omega) = \frac{100}{j\omega(4+(j\omega)^2)}$$

$$G(j0)H(j0) = \frac{100}{j0[(j0)^2+4]} = \frac{1}{j0} = \infty\angle-90$$

$G(j0)H(j0) = \infty\angle-90^\circ$

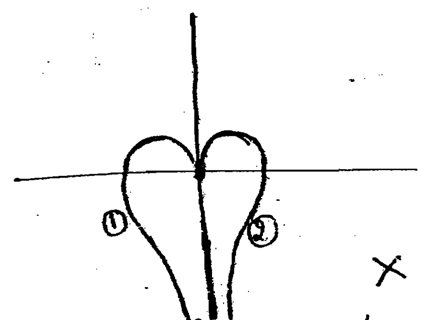
$$G(j\infty)H(j\infty) = \frac{100}{j\infty[(j\infty)^2+4]} = \frac{1}{j^3\infty} = 0\angle-270(+90)$$

$G(j\infty)H(j\infty) = 0\angle-270$

$$G(j\omega)H(j\omega) = \frac{-j^2\,100}{}$$

$$G(j\omega)H(j\omega) = 0 - j\frac{100}{(\omega)[4-\omega^2]}$$

⇒ real term will always remain Zero only imaginary term exists.



× bother not due to imag pole

$$G(j0)H(j0) = 0 - \frac{j\,100}{0[4-0]} = \infty\angle-90 = 0-j\infty$$

$$G(j0.5)H(j0.5) = 0 - \frac{j\,100}{0.5[4-(0.5)^2]} = 0-j53.36$$

$$G(j1)H(j1) = 0 - \frac{j100}{1[4-1]} = 0-j33.33$$

$$G(j1.25)H(j1.25) = 0 - \frac{j\,100}{1.25[4-(1.25)^2]} = 0-j32.82$$

$$G(j1.5)H(j1.5) = 0 - \frac{j\,100}{1.5[4-(2.25)]} = 0-j38.09$$

$$G(j1.75)H(j1.75) = 0 - \frac{j\,100}{1.75[4-(1.75)^2]} = 0-j60.95$$

$$G(j2^-)H(j2^-) = 0 - \frac{j100}{(1.999\ldots)[4-3.99\ldots4]} = 0 - \frac{j}{(2^-)[0.00\ldots1]}$$

$$(2^-)^2 = 4 - 3.99\ldots9 = 0.00\ldots1$$
$$(2^+)^2 = 4 - 4.000\ldots1 = -0.00\ldots1$$

$$= 0 - j\infty$$

$$G(j2^+)H(j2^+) = 0 - j\frac{100}{(2+0.00\ldots1)\cdot[4-4.000\ldots1]}$$

$$= 0 + \frac{j}{(2^+)[+0.00\ldots1]} = 0 + j\infty$$

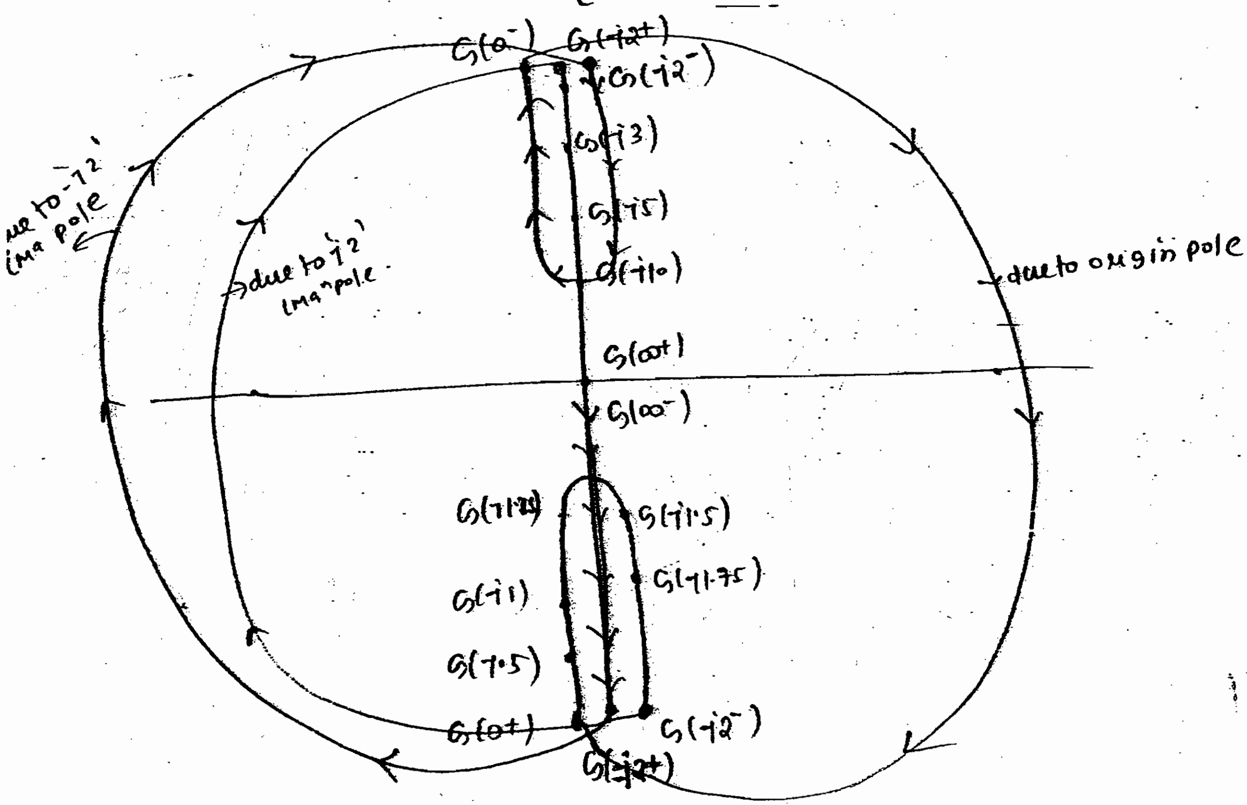
$$G(j3)H(j3) = 0 - \frac{j100}{3[4-9]} = 0 - \frac{j100}{3\times(-5)} = 0 + \frac{j100}{3\times(+5)}$$

$$= 0 + j6.66$$

$$G(j5)H(j5) = 0 - \frac{j100}{5[-19]} = 0 + j0.95$$

$$G(j10)H(j10) = 0 - \frac{j100}{10[4-100]} = 0 + j0.104$$

$$G(j\infty)H(j\infty) = 0 - \frac{j100}{\infty[4-\infty]} = 0 + j0$$



given OLTF Marginally stable

These three line are single line just for understanding

Exact plot

$P_+ = 0$

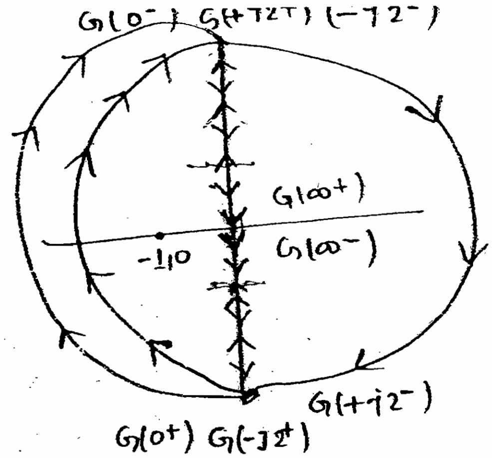
$N = -2$ (Always)

$N = P_+ - Z_+$

$-2 = 0 - Z_+$

$\boxed{Z_+ = 2}$ — always ⇒ always unstable so

$G.M = -\infty$



From Root locus:

$$G(s)H(s) = \frac{\overset{\nearrow K \text{ for } 100}{100}}{s(s^2+4)} = \frac{K}{s(s^2+4)}$$

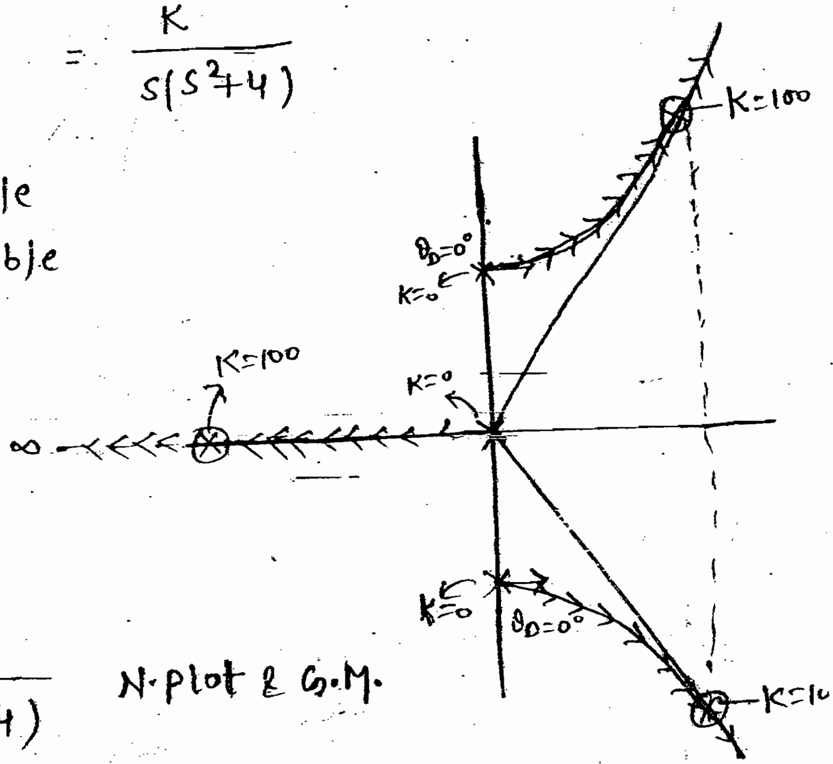
K=0 पर Marginally stable

K=100 पर Highly unstable

$G.M = -\infty$ (M.P.S.I.T)

No Breakpoint

Centroid = origin



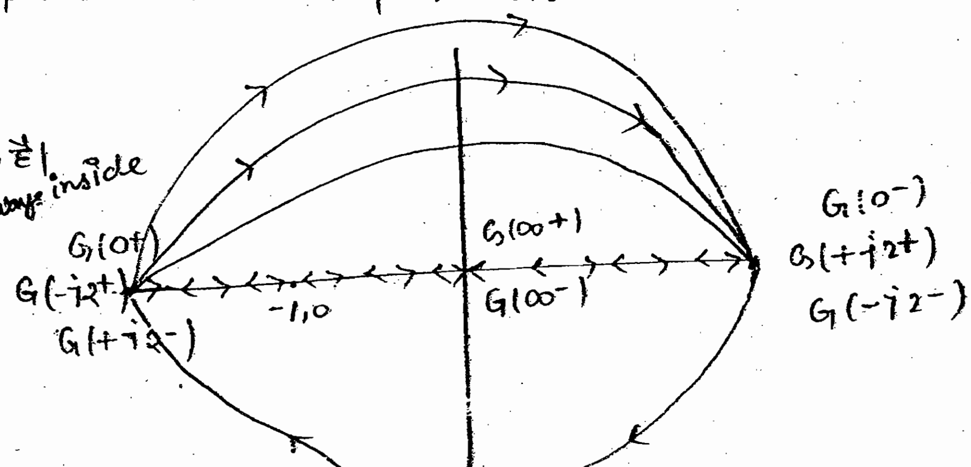
Q $G(s)H(s) = \dfrac{100}{s^2(s^2+4)}$ N.plot & G.M.

$$G(j\omega)H(j\omega) = 0 + \frac{100}{(j\omega)^2[4-\omega^2]} = -\frac{100}{\omega^2[4-\omega^2]} + j0$$

real axis   imaginary

⇒ All points exists on real axis.

Radius ∞ है। always inside → −1,0



$P_+ = 0$

$N = -2$

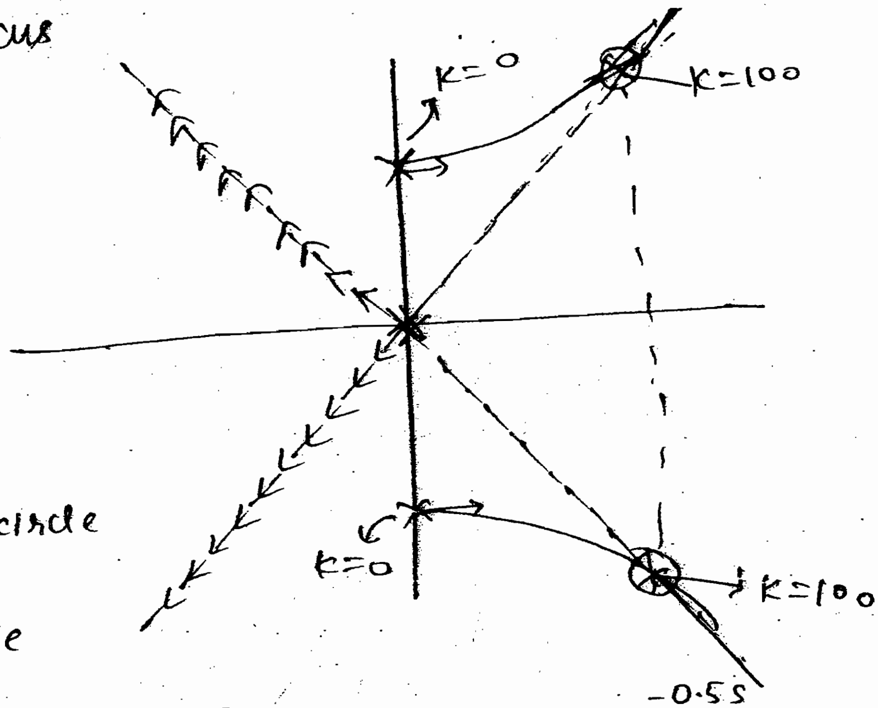
$\boxed{Z_+ = 2}$

↓

unstable

⇒ $G.M = -\infty$

?rom Root locus

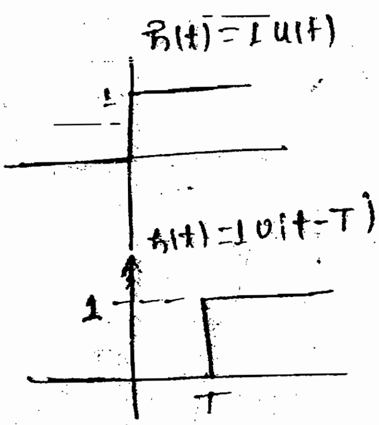


poles always
ight side
nstable
$G.m = -\infty$

$G(s)H(s) = \frac{1}{s}$ ← N.P is semicircle

$G(s)H(s) = \frac{1}{s^2}$ ← N.P is circle

The OLTF of a s/T is $G(s)H(s) = \frac{K \cdot e^{-0.5s}}{s+1}$ Draw its N. plot and Determine the range of K for close loop system to be stable.



$H(s) = \frac{1}{s} \Rightarrow H(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega}\angle -90^\circ$

$H(s) = \frac{e^{-sT}}{s} \Rightarrow H(j\omega) = \frac{e^{-j\omega T}}{(j\omega)}$

$\because 1\, e^{-j\omega T} = 1\cos(\omega T) - j\sin(\omega T)$

$= \sqrt{\cos^2(\omega T) + \sin^2(\omega T)}\ \angle \tan^{-1}\left\{\frac{-\sin\omega T}{\cos\omega T}\right\}$

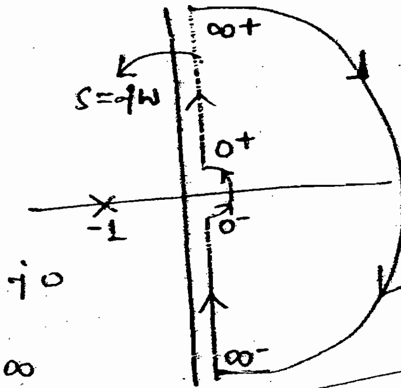
$= 1 \angle -\omega T$

$H(j\omega) = \frac{e^{-j(\omega T)}}{(j\omega)} = \frac{1}{\omega}\angle -90 - (\omega T)$

The Transportation lag s/T is non minimum phase s/T

NMPS ← If pole Right side then NMPS → unstable
If zero " " " NMPS → not unstable.

$$G(s)H(s) = \frac{K \cdot e^{-0.5s}}{(s+1)}$$



$$G(j\omega)H(j\omega) = \frac{K \cdot e^{-0.5j\omega}}{1+j\omega}$$

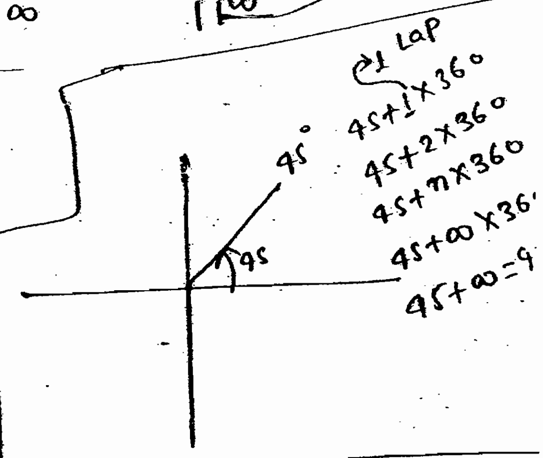
$$G(j0)H(j0) = \frac{K \cdot e^{-j0}}{1+j0} = K\angle 0^\circ = K + j0$$

$$G(j\infty)H(j\infty) = \frac{K \cdot e^{-j\infty}}{1+j\infty} = \left(\frac{K}{\infty}\right)\cdot\frac{e^{-j\infty}}{j}$$

$$= 0\angle -90-\infty$$

$$= 0\angle -90-\infty\times 360$$

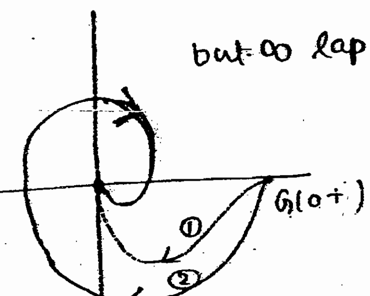
$$= 0\angle -90^\circ$$



$$G(j\omega)H(j\omega) = \frac{K \cdot e^{-0.5(j\omega)}(1-j\omega)}{(1+j\omega)(1-j\omega)}$$

but ∞ lap लेगा

$$= \frac{K\left[\cos(0.5\omega) - j\sin(0.5\omega)\right](1-j\omega)}{(1+\omega^2)}$$



$$= \frac{K\left[\cos(0.5\omega) - \omega\sin(0.5\omega)\right]}{(1+\omega^2)} - j\frac{\left[\omega\cos(0.5\omega) + \sin(0.5\omega)\right]}{(1+\omega^2)}$$

for Imaginary term to be zero (for ω_{pc})

$$\frac{\omega\cos(0.5\omega) + \sin(0.5\omega)}{1+\omega^2} = 0 \Rightarrow \tan^{-1}(-\omega) = 0.5\omega$$

$$\boxed{\begin{aligned} \because \tan^{-1}(\omega) &= \omega - \frac{\omega^3}{3} + \frac{\omega^5}{5} - \frac{\omega^7}{7} + \cdots \\ \tan^{-1}(-\omega) &= -\omega + \frac{\omega^3}{3} - \frac{\omega^5}{5} + \frac{\omega^7}{7} + \cdots \end{aligned}}$$ — Neglect High degree term

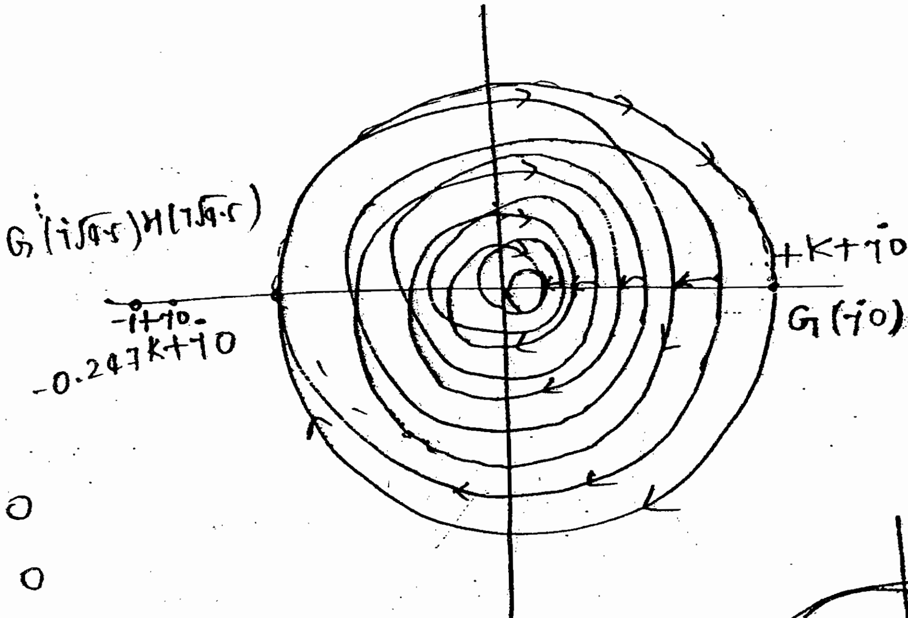
$$-\omega + \frac{\omega^3}{3} = 0.5\omega \Rightarrow \omega\left[\omega^2 - 4.5\right] = 0$$

$$\omega = 0, \pm 2.12$$

$$G(j0)H(j0) = K + j0$$

$K[\cos(0.5\times\sqrt{4.5}) - \sqrt{4.5}\,\sin($

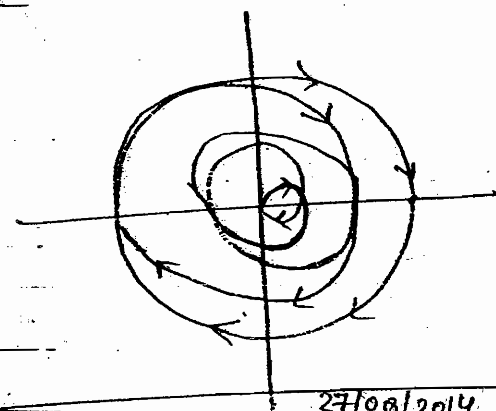
$= -0.2475 + j0$



$P_+ = 0$

$Z_+ = 0$

$N = P_+ - Z_+ = 0 - 0 = 0$



For stability

$0.247K < 1 \Rightarrow K < \frac{1}{0.247}$

27/08/2014

M & N Circle

M circle give the information of Magnitude of closed loop system

N-circle gives information of phase Angle of closed loop S/T and when M circle is superimposed on N-circle that chart is known as Nichols chart thus Nichols chart is used to calculate magnitude ~~of closed loop~~ and phase angle of closed loop S/T from its open loop plot

For Nichols chart implementation its is essential that close loop S/T should be of unity feedback.

M - Circle (Magnitude circle)

given $G(s)H(s)$ = something and By Rationalizing

$$G(j\omega)H(j\omega) = x + jy = G(j\omega) \quad \{\because H(j\omega) = 1\}$$

$$T(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)\cdot H(j\omega)} = \frac{x+jy}{(1+x)+jy}$$

$$|T(j\omega)| = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}} = M \text{ (magnitude of CLTF)}$$

$$M^2 = \frac{x^2+y^2}{(1+x)^2+y^2} \Rightarrow M^2[x^2+2x+1+y^2] = x^2+y^2$$

$$x^2[1-M^2] - 2\cdot x M^2 + y^2[1-M^2] = M^2 \quad \text{--- (1)}$$

$$x^2 - 2\cdot x \frac{M^2}{1-M^2} + \left\{\frac{M^2}{1-M^2}\right\}^2 + y^2\cdot 1 = \frac{M^2}{1-M^2} + \left\{\frac{M^2}{1-M^2}\right\}^2 = \frac{M^2(1-M^2)+M^4}{(1-M^2)^2}$$

$$= \frac{M^2 - M^4 + M^4}{(1-M^2)^2} = \left(\frac{M}{1-M^2}\right)^2$$

$$\boxed{\left[x - \left(\frac{M^2}{1-M^2}\right)\right]^2 + (y-0)^2 = \left(\frac{M}{1-M^2}\right)^2}$$ Equation of circle

Centre $\left\{\frac{M^2}{1-m^2}, 0\right\} = \left\{\frac{M^2}{1-m^2}, 0\right\}$ radius $= \left[\frac{M}{1-M^2}\right]$

$\boxed{\text{For } M=1}$ eqn (1) $-2x = 1 \Rightarrow x = -\frac{1}{2}$ (straight line)

for $M=1$ eqn of circle will be converted into eqn of line

$\boxed{M=0}$ centre $[0,0]$, radius $=0$ $\quad \therefore \left[\frac{1}{\frac{1}{M^2}-1}, 0\right]$

$\boxed{M=\infty}$ centre $[-1,0]$, radius $=0$

$$\Rightarrow \begin{array}{l} 0 < M < 1 \\ 1 < M < \infty \end{array}$$

se 1. $0 < M < 1$ * In this case with ↑ M centre of
ircle will shift toward ∞ at +ve x-axis and its
corresponding radius will also increase so all the
circle lie in right side M = 1 line lying this region
$0 < M < 1$

ase 2 $1 < M < \infty$ * In this case with ↑ the value of
M centre of centre will shift toward (-1, 0) point
nd its corresponding radius will ↓ all circle
ie in the left half side of M = 1 line lying in
this region

N-circle

$$T(i\omega) = \frac{x + iy}{(1+x) + iy} \Rightarrow \angle T(i\omega) = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x} = \theta$$

$$= \tan^{-1}\frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y}{x} \times \frac{y}{1+x}} = \tan^{-1}\frac{y + xy - xy}{x^2 + x + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x^2 + x + y^2} \Rightarrow \underset{N}{\underset{\parallel}{\tan\theta}} = \frac{y}{x^2 + x + y^2}$$

$$x^2 + x + y^2 = \frac{y}{N} \Rightarrow x^2 + x + y^2 - \frac{y}{N} = 0$$

$$x^2 + 2 \cdot x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + y^2 - 2 \times y \times \frac{1}{2N} + \left(\frac{1}{2N}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2N}\right)^2$$

$$\boxed{\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \left(\sqrt{\frac{1}{4} + \frac{1}{4N^2}}\right)^2}$$

Centre $\left[+\frac{1}{2}, \frac{1}{2N}\right]$, Radius $= \sqrt{\frac{1}{4} + \frac{1}{4N^2}}$

$= 90^\circ$
$= +\infty$ Centre $\left[-\frac{1}{2}, 0\right], \left[\frac{1}{2}\right]$

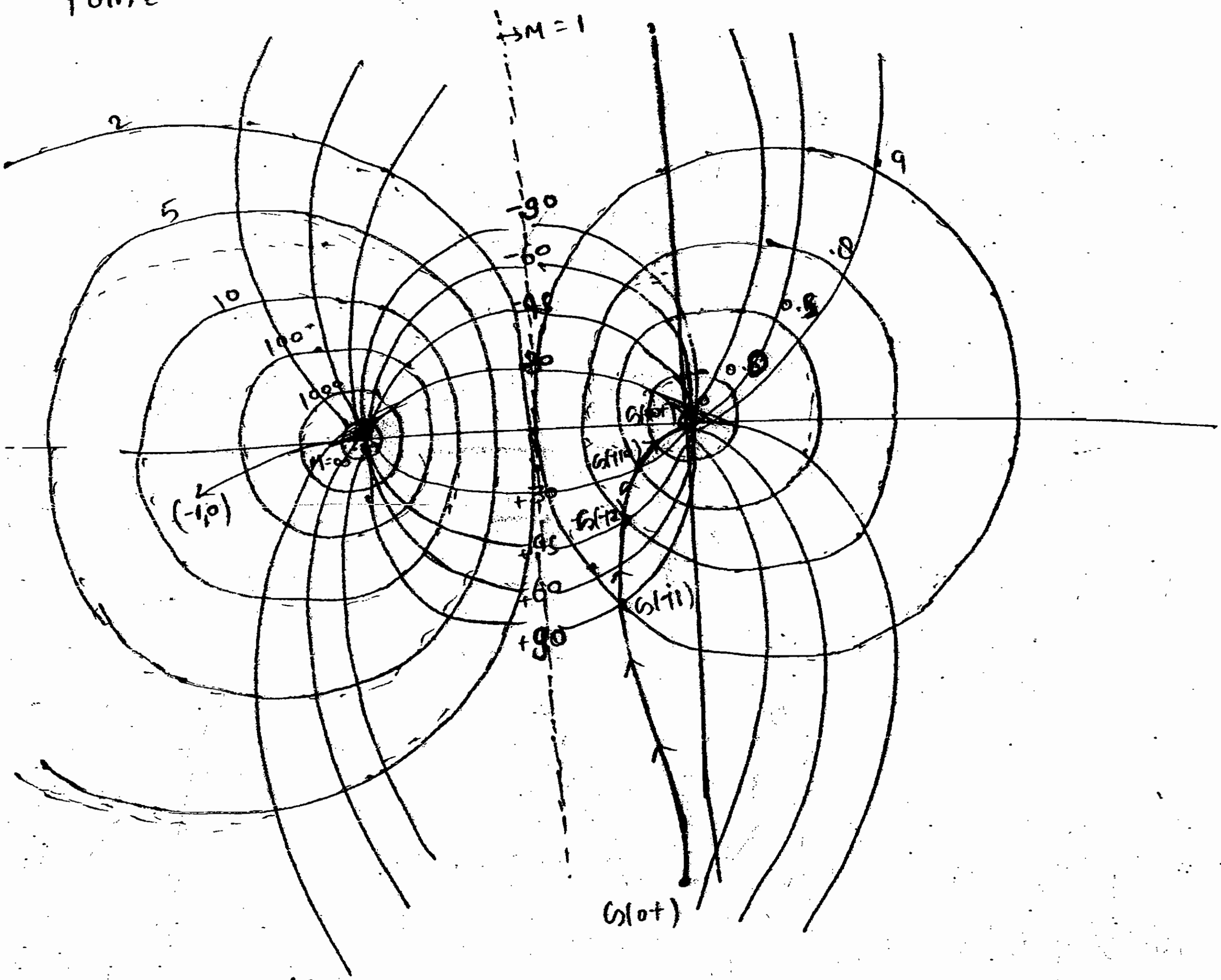
$= -90$
Centre $\left[-\frac{1}{2}, 0\right], \left[\frac{1}{2}\right]$
$= -\infty$

$\phi = 0^\circ, N = 0^\circ$ Centre $(-\frac{1}{2}, \infty)$, radius $= \infty \rightarrow$ x-axis.

$\phi = +ve, N = +ve$ centre $(-\frac{1}{2}, +ve\, y)$

$\phi = -ve, N = -ve$ centre $(-\frac{1}{2}, -ve\, y)$

* The centre of N-circle will locate at M = 1 line and all N-circle will pass through origin and (-1,0) point.



$$G(s)H(s) = \frac{K}{s(s+1)}$$

$$G(j\omega) = \frac{K}{j\omega[1+j\omega]} = \frac{K}{\omega\sqrt{1+\omega^2}} \angle -90 - \tan^{-1}\frac{\omega}{1}$$

$$G(j0)H(j0) = \infty \angle -90^\circ$$

$$G(j1)H(j1) = \frac{K}{\sqrt{2}} \angle -135^\circ$$

$$G(j2)H(j2) = \frac{K}{2\sqrt{5}} = \angle -153^\circ$$

$$G(j\infty)H(j\infty) = 0 \angle -180^\circ$$

$$T(j\omega) = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}$$

$T(j1) = 0.9\angle{+90^\circ}$, $T(j2) = 0.8\angle{+45^\circ}$

$T(j10) = 0.5\angle{30^\circ}$

STATE SPACE Analysis

If any S/T has zero initial condition in that case overall o/p response will be zero state response and both ~~space~~ state space Analysis and T/F will give same o/p response Hence for convinience of calculaⁿ we will follow T/F Method to calculate o/p response.

If any S/T has finite Initial condition in that case overall o/p response will be some of zero state response and zero input response while T/F Method will give only zero input response which is incorrect so we will follow state space Analysis to calculate its o/p response

State eqⁿ is a differential eqⁿ which contain only state variable and input. Holding property will exists only with state variable while input will have instantaneous effect capacitor Voltage is a state variable while capacitor current is input for charging or discharging of state variable

Inductor current is a state variable while inductors voltage is input for charging or discharging of inductors current

State variable has non unique solution while output o/p has unique solution also eigen value which is location of pole has unique identity.

State Matrix is in the form of

$$[\dot{x}_i] = [A]_{n\times n}[x] + [B]_{n\times l}[u]$$

$$[y] = [C]_{m\times n}[x] + [D]_{m\times l}[u]$$

where x is state variable, u = input, y = output

n = is total no of state variable,

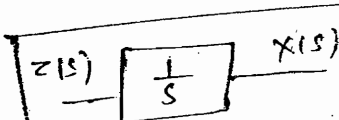
l = is total no of input

m = is total no of output

① DIRECT DECOMPOSITION METHOD

$$\frac{Y(s)}{U(s)} = \frac{a_0 s^3 + a_1 s^2 + a_2 s + a_3}{s^3 + b_1 s^2 + b_2 s + b_3}$$

$$= \frac{a_0 + \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{s^3}}{1 - \left[-\frac{b_1}{s} - \frac{b_2}{s^2} - \frac{b_3}{s^3}\right]}$$



$$\frac{x(s)}{z(s)} = \frac{1}{s}$$

$$z(s) = s \cdot x(s)$$

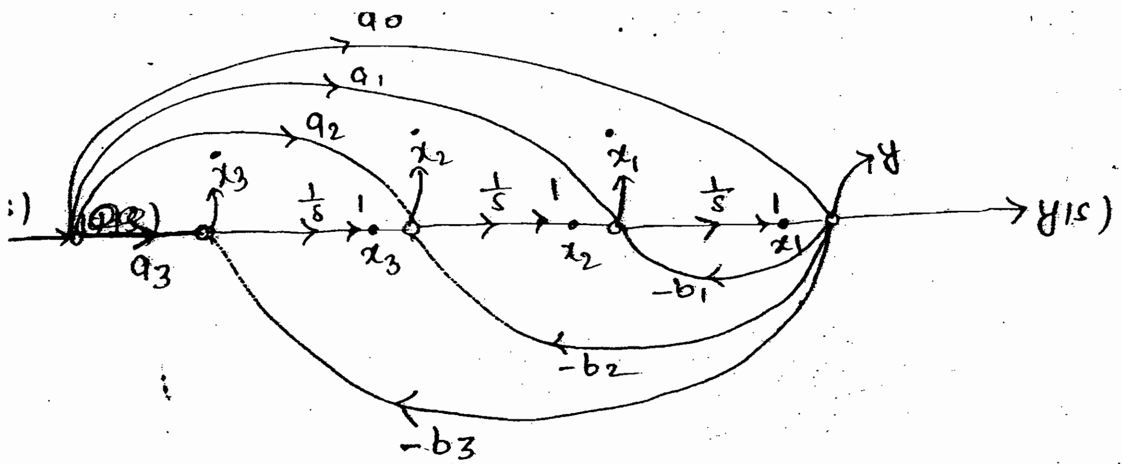
$$z(t) = \frac{d}{dt}x(t) = \dot{x}$$

Mason Gain formulae $= \dfrac{\Sigma P_K \Delta_K}{\Delta}$

$\Delta_K = 1 - [$Sum of all loop not touching k^{th} f.path$] + [$sum of product of two n.t. loop not touch k^{th} f.path$] + \dots$

$\Delta = 1 - [$sum of all loop$] + [$S.O.P of two non touch loop$] - [$S.O.P of three non touching loop$]$

$$= \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{1 - [\text{Sum of all loop}]} = \frac{P_1 + P_2 + P_3 + P_4}{1 - [L_1 + L_2 + L_3]}$$



quations from this SFG

$y = 1\cdot x_1 + a_0 u \quad , \quad \dot{x}_1 = 1x_2 + a_1 \cdot u - b_1 y$

$\dot{x}_1 = 1x_2 + a_1 u - b_1[x_1 + a_0 u] \Rightarrow \boxed{\dot{x}_1 = -b_1 x_1 + 1x_2 + (a_1 - b_1 a_0)u}$

$\dot{x}_2 = 1\cdot x_3 + a_2 u - b_2 \cdot y \Rightarrow \dot{x}_2 = 1\cdot x_3 + a_2 u - b_2[1x_1 + a_0 u]$

$\boxed{\dot{x}_2 = -b_2 x_1 + 1x_3 + (a_2 - b_2 a_0)\cdot u}$

$\dot{x}_3 = a_3 \cdot u - b_3 \cdot y \Rightarrow \dot{x}_3 = a_3 u - b_3[1x_1 + a_0 u]$

$\Rightarrow \boxed{\dot{x}_3 = -b_3 x_1 + (a_3 - b_3 a_0) u}$

Statematrix and output Matrix

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -b_1 & 1 & 0 \\ -b_2 & 0 & 1 \\ -b_3 & 0 & 0 \end{bmatrix}}_{\substack{n \times n \\ 3 \times 3}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} a_1 - b_1 a_0 \\ a_2 - b_2 a_0 \\ a_3 - b_3 a_0 \end{bmatrix}}_{3 \times 1} [u]$$

$$[y] = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{1 \times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{[a_0]}_{1 \times 1} [u]$$

Phase-variable form :

In phase variable form all element of either first column or last row exists and element just above the diagonal element will be one while rest of the other element will be zero and by using direct decomposition method state ~~variable~~ Matrix will always be in ~~state-V~~ $\xrightarrow{\text{statematrix}}$ phase variable form.

$$[\dot{x}] = [A]_{n\times n}[x] + [B]_{n\times l}[U]$$

$$[y] = [C]_{m\times n}[x] + [D]_{m\times l}[U]$$

* The rank of statematrix A will be equal to the total no of state variable.

Parallel decomposition method

$$\frac{Y(s)}{U(s)} = \frac{1}{(s+2)(s+3)(s+4)} = \frac{a_0}{s+2} + \frac{a_1}{s+3} + \frac{a_2}{s+4}$$

$$= \frac{(1/2)}{s+2} + \frac{(-1)}{s+3} + \frac{(1/2)}{s+4} = \cancel{X_1(s) + X_2(s) + X_3(s)}$$

$$\cancel{X_1(s)} = \cancel{\frac{1/2}{s+2}}$$

$$Y(s) = \frac{1/2}{s+2}U(s) + \left(\frac{-1}{s+3}\right)U(s) + \frac{(1/2)}{s+4}U(s) = X_1(s) + X_2(s) + X$$

$$X_1(s) = \frac{1/2}{s+2}U(s) \Rightarrow (s+2)X_1(s) = \frac{1}{2}U(s)$$

$$sX_1(s) + 2X_1(s) = \frac{1}{2}U(s) \Rightarrow sX_1(s) = -2X_1(s) + \frac{1}{2}U(s)$$

$$\boxed{\dot{x}_1 = -2x_1 + \frac{1}{2}U}$$ first state eqn.

$$X_2(s) = \frac{-1}{s+3}U(s) \Rightarrow sX_2(s) + 3X_2(s) = -1\,U(s)$$

$$sX_2(s) = -3X_2(s) + (-1)U(s) \Rightarrow \boxed{\dot{x}_2 = -3x_2 + (-1)u}$$ 2nd.

$$X_3(s) = \frac{1/2}{s+4} U(s) \Rightarrow sX_3(s) + 4X_3(s) = \frac{1}{2}U(s)$$

$$sX_3(s) = -4X_3(s) + \frac{1}{2}U(s) \Rightarrow \boxed{\dot{x}_3 = -4x_3 + \frac{1}{2}u} \quad 3^{rd}$$

$$\boxed{y = x_1 + x_2 + x_3}$$

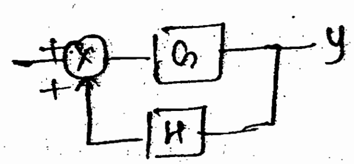
State matrix from these state equation -

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\overbrace{\begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}}^{A}}_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\overbrace{\begin{bmatrix} (1/2) \\ (-1) \\ (1/2) \end{bmatrix}}^{B}}_{3\times1} [u]$$

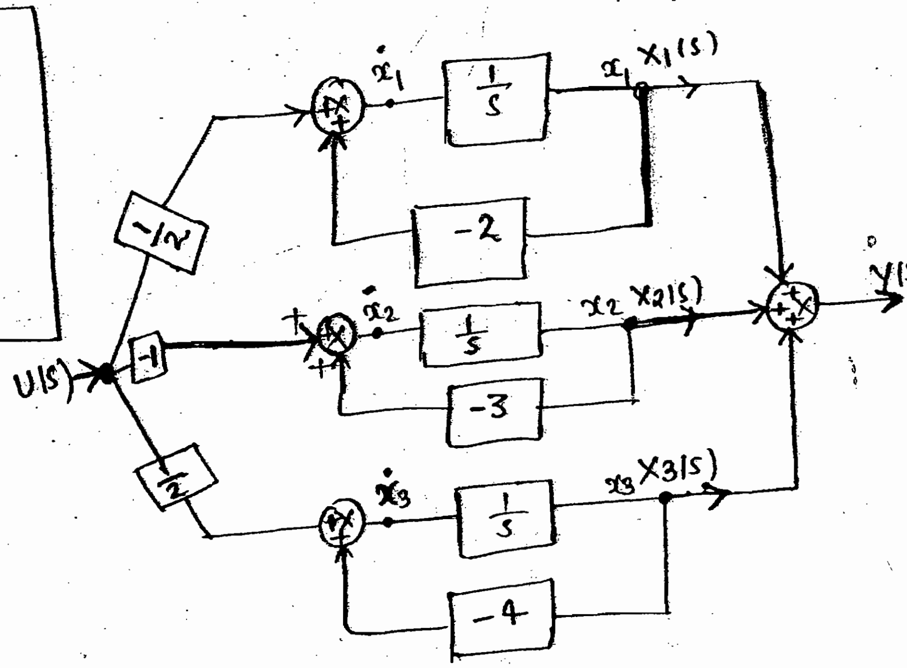
$$[y] = \underbrace{\underbrace{[1 \;\; 1 \;\; 1]}_{1\times3}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\underbrace{[0]}_{1\times1}}_{D} [u]$$

By using parallel decomposition method state matrix A will always be a diagonal matrix and its diagonal element represents eigen value which is location of pole.

$$\frac{X_1(s)}{U(s)} = \frac{1/2}{s+2} = \frac{1}{2} \times \left\{ \frac{\left(\frac{1}{s}\right)}{1 - \left[\left(\frac{1}{s}\right)\cdot(-2)\right]} \right\}$$



$$\frac{Y}{X} = \frac{G}{1 - \{GH\}}$$



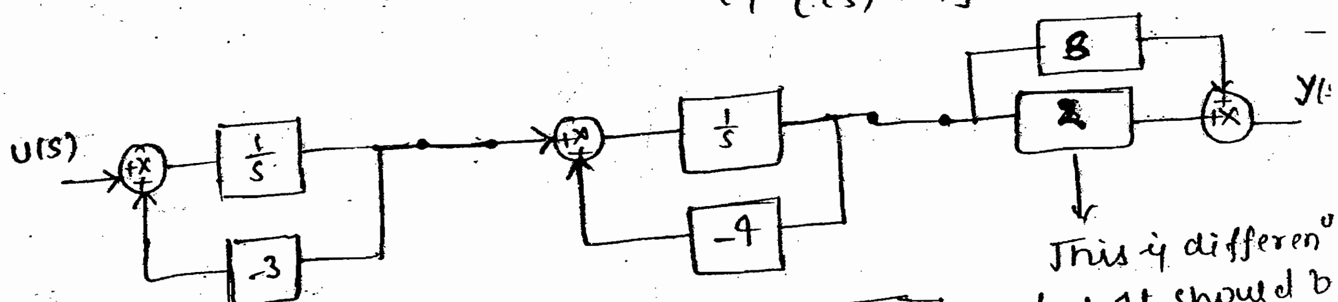
$$\frac{X_2(s)}{U(s)} = \frac{-1}{s+3} = (-1) \times \left\{ \frac{\frac{1}{s}}{1-\left[\left(\frac{1}{s}\right)\times(-3)\right]} \right\}$$

$$\frac{X_3(s)}{U(s)} = \frac{1/2}{s+4} = \left(\frac{1}{2}\right) \times \left\{ \frac{(1/s)}{1-\left[\left(\frac{1}{s}\right)\times(-4)\right]} \right\}$$

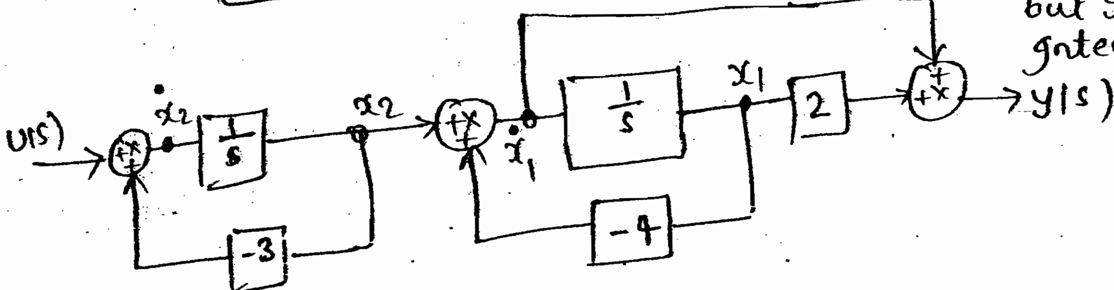
Cascade decomposition Method

$$\frac{Y(s)}{U(s)} = \frac{(s+2)}{(s+3)(s+4)} = (s+2) \times \frac{1}{(s+3)} \times \frac{1}{(s+4)}$$

$$= (s+2) \times \left\{ \frac{\frac{1}{s}}{1-\left[\left(\frac{1}{s}\right)\times(-3)\right]} \right\} \times \left\{ \frac{\frac{1}{s}}{1-\left[\left(\frac{1}{s}\right)\times(-4)\right]} \right\}$$



This is different but it should be integrator



$y = 2x_1 + \dot{x}_1$, $\boxed{\dot{x}_1 = -4x_1 + 1 \cdot x_2}$

$y = 2x_1 - 4x_1 + 1 \cdot x_2 \Rightarrow \boxed{y = -2x_1 + 1x_2}$

$\boxed{\dot{x}_2 = -3x_2 + U \cdot 1}$

total No. of pole = to No of state variab
= rank of state matri.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -4 & 1 \\ 0 & -3 \end{bmatrix}}_{2\times 2}^{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{2\times 1} + \overbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}^{B} \begin{bmatrix} U \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 1 \end{bmatrix}}_{C,\ 1\times 2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D,\ 1\times 1} \begin{bmatrix} U \end{bmatrix}$$

Rank of state matrix [A] = 2 = to No of pole = total No of state variable

By using cascade decom'' Method state Matrix [A] will always be U.T.M and its Diagonal element represents eigen value which is location of pole.

Solution of state Matrix

$[\dot{x}] = [A][x] + [B][U]$ Laplace

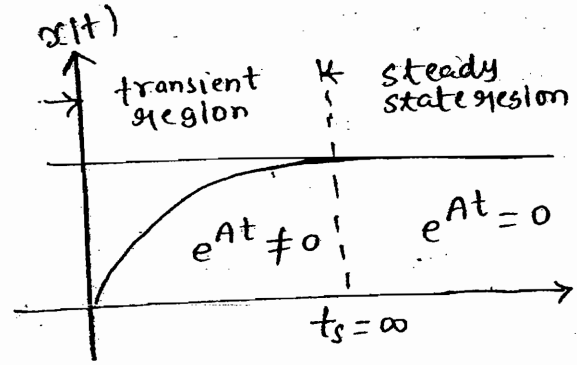
$[SX(s) - x(0)] = [A]X(s) + [B]\cdot U(s)$

$[SI-A]\, X(s) = x(0) + [B]U(s) \Rightarrow X(s) = \frac{1}{[SI-A]} x(0) + \frac{[B]}{[SI-A]} U(s)$

$X(s) = [SI-A]^{-1} x(0) + [SI-A]^{-1}[B]U(s)$

inverse laplace

$x(t) = \underbrace{e^{+At} x(0) + B e^{At}}_{\text{transient region}} U$



$[SI-A]^{-1} \rightarrow$ transient element in L.D.

$e^{At} \rightarrow$ transient element in T.D

$[SI-A]^{-1} \rightarrow$ state transition Matrix.

$[I]_{n \times n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$[I]_{1 \times 1} = 1$

$X(s) = \underbrace{[SI-A]^{-1} x(0)}_{\text{Zero input response}} + \underbrace{[SI-A]^{-1}[B]U(s)}_{\text{zero state response}}$

zero state response → Transfer funⁿ Method

$\boxed{\text{overall output response} = ZIR + ZSR}$

- If any s/T has zero initial condition then overall o/p response will be zero state response and T/F Method also gave zero state response so in that case we will follow T/F Method

Q. calculate o/p response. If any S/T has finite initial condn in that case overall o/p response will be sum of zero state response and zero input respon while T/F method will give only ZSR which will be less than overall o/p response while ZIR

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u] \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$[y] = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0][u]$$

on the given state matrix obtain time dependent relation of state variable and o/p if input is unit step

Soln first method $\dot{x}_1 = 1x_1 + 1\overset{\text{input}}{u(t)}$, $\dot{x}_2 = 1x_1 + 1x_2 + 1u(t)$

Apply L.T. $[sX_1(s) - x_1(0)] = 1X_1(s) + \left(\frac{1}{s}\right)$

$(s-1)X_1(s) = x_1(0) + \frac{1}{s} = 1 + \frac{1}{s} = \frac{(s+1)}{s} \Rightarrow X_1(s) = \frac{s+1}{s(s-1)}$

$X_1(s) = \frac{s+1}{s(s-1)} = \frac{a_0}{s} + \frac{a_1}{s-1} = \frac{-1}{s} + \frac{2}{(s-1)}$

$\boxed{x_1(t) = -1 + 2e^{t}}$

$[sX_2(s) - x_2(0)] = X_1(s) + X_2(s) + \frac{1}{s}$

$(s-1)X_2(s) = X_1(s) + x_2(0) + \frac{1}{s} = \frac{s+1}{s(s-1)} + 0 + \frac{1}{s}$

$(s-1)X_2(s) = \frac{s+1+s-1}{s(s-1)} = \frac{2}{s-1} \Rightarrow X_2(s) = \frac{2}{(s-1)^2}$

$\boxed{x_2(t) = 2te^{t}}$

o/p $y = 1x_1 + 0x_2 + 0u = x_1 \Rightarrow \boxed{y = x_1(t) = -1 + 2e^{+t}}$

output

Diagonal form है तब use. करना है

Second method $[X(s)] = [sI - A]^{-1}\cdot[x(0)] + [sI-A]^{-1}[B][U(s)]$

$$[I] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow s[I] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} (s-1) & 0 \\ -1 & (s-1) \end{bmatrix}$$

$$\text{Co-factor} = \begin{bmatrix} (-1)^{1+1}(s-1) & (-1)^{1+2}(-1) \\ (-1)^{2+1}\cdot 0 & (-1)^{2+2}(s-1) \end{bmatrix} = \begin{bmatrix} (s-1) & +1 \\ 0 & (s-1) \end{bmatrix}$$

$$\text{Adj} = \begin{bmatrix} (s-1) & 0 \\ +1 & (s-1) \end{bmatrix} \Rightarrow [sI - A]^{-1} = \frac{\text{Adj}}{|\text{Det}|} = \frac{\begin{bmatrix} (s-1) & 0 \\ 1 & (s-1) \end{bmatrix}}{(s-1)^2}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}}{(s-1)^2} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

ut

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \frac{\begin{bmatrix} (s-1) & 0 \\ 1 & (s-1) \end{bmatrix}}{(s-1)^2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{\begin{bmatrix} (s-1) & 0 \\ 1 & (s-1) \end{bmatrix}}{(s-1)^2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\left(\frac{1}{s}\right)$$

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \frac{\begin{bmatrix} (s-1) \\ 1 \end{bmatrix}}{(s-1)^2} + \frac{\begin{bmatrix} (s-1) \\ 1+s-1 \end{bmatrix}}{s(s-1)^2} = \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{(s-1)^2} \end{bmatrix} + \begin{bmatrix} \frac{1}{s(s-1)} \\ \frac{1}{\cancel{s}(s-1)^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-1} + \frac{1}{s(s-1)} \\ \frac{2}{(s-1)^2} \end{bmatrix} = \begin{bmatrix} \frac{s+1}{s(s-1)} \\ \frac{2}{(s-1)^2} \end{bmatrix}$$

$$\Rightarrow X_1(s) = \frac{s+1}{s(s-1)}, \quad X_2(s) = \frac{2}{(s-1)^2}$$

$$X_1(s) = \frac{a_0}{s} + \frac{a_1}{s-1} = \frac{-1}{s} + \frac{2}{s-1}$$

$$\boxed{x_1(t) = -1 + 2e^{+1t}} \qquad \boxed{x_2(t) = 2\cdot t\, e^{+1t}}$$

If I used T/F Method then initial condition $= x(0) = 0$.

$$x_1(s) = \frac{1}{s(s-1)} = \frac{-1}{s} + \frac{1}{s-1}, \quad x_2(s) = \frac{1}{(s-1)^2}$$

$$\boxed{x_1(t) = -1 + 1 \cdot e^{+1t}} \qquad \boxed{x_2(t) = t e^{+1t}}$$

Diagonalizing process

* If given state Matrix [A] is not in diagonal form then by using diagonalization process we will convert state Matrix [A] into diagonal M. and its diagonal element represents its eigenvalue which is location of pole

Case1 If state Matrix [A] is not in phase variable for

In this case we use $[\lambda I - A] = 0$ to calculate eigen value if we replace λ by s then this will be characteristics eqn of given T/F or system then we will use this relation $[\lambda I - A][x] = 0$ to calculate eigen vector and from that eigen vector we will construct model Matrix [M] then By variable transformation we will transform state variable X into new state variable z

$$[x] = [M][z], \quad [\dot{x}] = [M][\dot{z}], \quad z(0) = [M]^{-1} x(0)$$

If

$$[\dot{x}] = [\overset{\text{Non diagonal Matrix}}{A}][x] + [B][u]$$

$$[y] = [c][x] + [D][u]$$

Now

$$[M][\dot{z}] = [A][M][z] + [B][u]$$

$$[y] = [c][M][z] + [D][u]$$

$$[\dot{z}] = \{\underbrace{[M]^{-1}[A][M]}_{A}\}[z] + \{\underbrace{[M]^{-1}[B]}_{B}\}[u] \longrightarrow (A)$$

Case 2 If given state Matrix [A] is in phase variable form in this case we use $[\lambda I - A] = 0$ to calculate eigen value and this eigen value itself will be location of pole then from this eigen value we will construct Vandermonde Matrix

$$[V] = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix}$$ where λ_1 and λ_2 are eigen value

then by variable transformation we will transform state variable x into new state variable z

$$[X] = [V][Z] \quad , \quad [\dot{X}] = [V][\dot{Z}]$$

$$Z(0) = [V]^{-1} x(0)$$

$$[\dot{X}] = [A][X] + [B][U]$$

$$[Y] = [C][X] + [D][U]$$

$$[\dot{Z}] = \{\underbrace{[V]^{-1}[A][V]}_{\to A}\}[Z] + \{\underbrace{[V]^{-1}[B]}_{\to B}\}[U]$$

$$[Y] = \{\underbrace{[C][V]}_{C}\}[Z] + \underbrace{[D]}_{D}[U]$$

For the given state matrix obtain time dependent relation of state variable and output if input is unit step input

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix}}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[U] \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$[Y] = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0][U]$$

This matrix is not in diagonal form & not in p.v.form

& this is case 1

$[\lambda I - A] = \begin{bmatrix} \lambda-1 & -4 \\ 2 & \lambda+5 \end{bmatrix} \Rightarrow |\lambda I - A| = (\lambda-1)(\lambda+5)+8$

$(\lambda+1)(\lambda+3) = 0 \Rightarrow \lambda = -1, -3$ ($\Rightarrow$ location of pole)

$[\lambda I - A][x] = 0 \Rightarrow \begin{bmatrix} \lambda-1 & -4 \\ 2 & \lambda+5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

For $\lambda = -1$

$\begin{bmatrix} -2 & -4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow -2x_1 - 4x_2 = 0 \Rightarrow \boxed{x_1 + 2x_2 =}$

$2x_1 + 4x_2 = 0 \Rightarrow \boxed{x_1 + 2x_2 = 0}$ If $x_1 = 1$ Then $x_2 = -0.5$

For $\lambda = -3$

$\begin{bmatrix} -4 & -4 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$ $\quad -4x_1 - 4x_2 = 0 \Rightarrow \boxed{x_1 + x_2 = 0}$ If $x_1 = 1$ Then $x_2 = -1$

$2x_1 + 2x_2 = 0 \Rightarrow \boxed{x_1 + x_2 = 0}$

$[M] = \begin{bmatrix} 1 & 1 \\ -0.5 & -1 \end{bmatrix} \Rightarrow M^{-1} = \begin{bmatrix} 2 & 2 \\ -1 & -2 \end{bmatrix}$

Now

$[M]^{-1}[A]\cdot[M] = \begin{bmatrix} 2 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -0.5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$

$[M]^{-1}[B] = \begin{bmatrix} 2 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$[C][M] = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -0.5 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$\begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} = [M]^{-1}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Now substitute these all in eqn (A)

$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \end{bmatrix}[U]$

$[y] = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + [0][U]$

Solution $\dot{z}_1 = -1 z_1 + 2u$, $\dot{z}_2 = -3 z_2 - 2u$

King L.T. $s Z_1(s) - z_1(0) = -1 Z_1(s) + \frac{2}{s} \Rightarrow (s+1) Z_1(s) = \underset{2}{z_1(0)} + \frac{2}{s}$

$\Rightarrow Z_1(s) = \frac{2}{s}$, $\boxed{z_1(t) = 2u(t)}$

$Z_2(s) - z_2(0) = -3 Z_2(s) - \frac{2}{s} \Rightarrow (s+3) Z_2(s) = \overset{-1}{z_2(0)} - \frac{2}{s}$

$Z_2(s) = -\frac{(s+2)}{s(s+3)} \Rightarrow Z_2(s) = \frac{a_0}{s} + \frac{a_1}{s+3} = \frac{-2/3}{s} - \frac{1/3}{s+3}$

$$\boxed{z_2(t) = -\frac{2}{3}\,u(t) - \frac{1}{3} e^{-3t}}$$

$[x] = [M][z]$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -0.5 & -1 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \Rightarrow \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -0.5 & -1 \end{bmatrix} \begin{bmatrix} 2u(t) \\ -\frac{2}{3} - \frac{1}{3} e^{-3t} \end{bmatrix}$$

$$= \begin{bmatrix} 2 - \frac{2}{3} - \frac{1}{3} \cdot e^{-3t} \\ -1 + \frac{2}{3} + \frac{1}{3} \cdot e^{-3t} \end{bmatrix} = \begin{bmatrix} \frac{4}{3} - \frac{1}{3} e^{-3t} \\ -\frac{1}{3} + \frac{1}{3} e^{-3t} \end{bmatrix}$$

$$\boxed{x_1(t) = \frac{4}{3} - \frac{1}{3} e^{-3t}} , \boxed{x_2(t) = -\frac{1}{3} + \frac{1}{3} e^{-3t} = -\frac{1}{3} + \frac{1}{3} e^{-3t}}$$

$\| \; y(t)$

For the given state matrix obtain its solu^n by using diagonaliza^n Method if input is unit step.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$[y] = [6 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0][u]$$

Sol^n This is case 2

$[\lambda I - A] = \begin{bmatrix} \lambda & -1 \\ 6 & \lambda+5 \end{bmatrix}$, $|\lambda I - A| = \lambda^2 + 5\lambda + 6 = 0$

$\Rightarrow \lambda = -2, -3$ $\qquad V = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} \ \end{bmatrix} [u]$$

$$[y] = [4, 3] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + [0][u]$$

$\downarrow \phi 01^{n}$

TRANSFER FUNCTION FROM STATE MATRIX

$$\frac{Y(s)}{U(s)} = [C][sI-A]^{-1}[B] + [D] \longrightarrow \text{overall T/F}$$

$\longrightarrow$ unique solution

$$\frac{X(s)}{U(s)} = [sI-A]^{-1}[B] \longrightarrow \text{T/F of state variable.}$$

$\longrightarrow$ Non unique solution.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ +4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} [u]$$

obtain its overall T/F and T/F of state variable

$$[y] = [1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0][u]$$

$$[sI-A] = \begin{bmatrix} (s-1) & +2 \\ -4 & (s+5) \end{bmatrix}, \quad \text{cofactor} = \begin{bmatrix} s+5 & -2 \\ +4 & s-1 \end{bmatrix}$$

$$\frac{\text{Adj}}{|\det|} = \frac{\begin{bmatrix} (s+5) & -2 \\ +4 & s-1 \end{bmatrix}}{(s^2+4s+3)} \Rightarrow [sI-A]^{-1} = \frac{\begin{bmatrix} (s+5) & -2 \\ 4 & (s-1) \end{bmatrix}}{(s+1)(s+3)}$$

chara eq^n we can find pole from here

$$\frac{Y(s)}{U(s)} = [1 \quad 1] \frac{\begin{bmatrix} (s+5) & -2 \\ 4 & (s-1) \end{bmatrix}}{(s+1)(s+3)} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + [0]$$

$$\frac{Y(s)}{U(s)} = [1 \quad 1] \frac{\begin{bmatrix} 2s+10-2 \\ 8+s-1 \end{bmatrix}}{s^2+4s+3} = [1 \quad 1] \frac{\begin{bmatrix} 2s+8 \\ s+7 \end{bmatrix}}{s^2+4s+3}$$

$$\frac{Y(s)}{U(s)} = \frac{2s+8+s+7}{s^2+4s+3} = \frac{3s+15}{(s+1)(s+3)} \Rightarrow \boxed{\frac{Y(s)}{U(s)} = \frac{3(s+5)}{(s+1)(s+3)}}$$

$$\begin{bmatrix} \dfrac{X_1(s)}{U(s)} \\ \dfrac{X_2(s)}{U(s)} \end{bmatrix} = \frac{\begin{bmatrix} (s+5) & -2 \\ 4 & (s-1) \end{bmatrix}}{(s+1)(s+3)} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{\begin{bmatrix} 2s+10-2 \\ 8+s-1 \end{bmatrix}}{(s+1)(s+3)} = \frac{\begin{bmatrix} 2s+8 \\ s+7 \end{bmatrix}}{(s+1)(s+3)}$$

$$\begin{bmatrix} \dfrac{X_1(s)}{U(s)} \\ \vdots \end{bmatrix} = \begin{bmatrix} 2s+8 / (s+1)(s+3) \\ s+7 / (s+1)(s+3) \end{bmatrix} \Rightarrow \boxed{\frac{X_1(s)}{U(s)} = \frac{2s+8}{(s+1)(s+3)}}$$

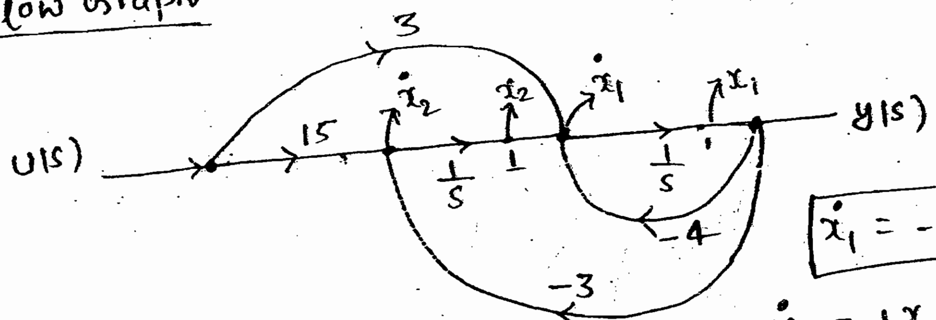
$$\frac{Y(s)}{U(s)} = \frac{X_1(s)+X_2(s)}{U(s)} \Rightarrow \frac{Y(s)}{U(s)} = \frac{X_1(s)}{U(s)} + \frac{X_2(s)}{U(s)}$$

$$\frac{Y(s)}{U(s)} = \frac{2s+8}{(s+1)(s+3)} + \frac{s+7}{(s+1)(s+3)} = \frac{3s+15}{(s+1)(s+3)} = \frac{3(s+5)}{(s+1)(s+3)}$$

Now

$$\frac{Y(s)}{U(s)} = \frac{3s+15}{(s+1)(s+3)} = \frac{\frac{3}{s}+\frac{15}{s^2}}{1-\left[-\frac{4}{s}-\frac{3}{s^2}\right]}$$ 2 integrator

S. Flow Graph



Asign statevariai

$$\boxed{\dot{x}_1 = -4x_1 + 1x_2 + 3U}$$

$$\boxed{y = 1x_1} \qquad \dot{x}_1 = 1x_2 - 4y + 3U \Rightarrow \dot{x}_1 = 1x_2 - 4x_1 + 3U$$

$$\dot{x}_2 = -3y + 15U = -3x_1 + 15U \Rightarrow \boxed{\dot{x}_2 = -3x_1 + 15U}$$

statematrix

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}-4 & 1\\ -3 & 0\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}3\\ 15\end{bmatrix}}_{B}\begin{bmatrix}U\end{bmatrix}$$ ⇒ different state matrix come.

$$\begin{bmatrix}y\end{bmatrix} = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\end{bmatrix}}_{D}\begin{bmatrix}U\end{bmatrix}$$

Now T/F from this

$$\frac{Y(s)}{U(s)} = [C][sI-A]^{-1}[B]+[D] = \frac{3s+15}{(s+1)(s+3)}$$ ⇒ तो same but

$$\frac{X_1(s)}{U(s)} = \frac{3s+15}{(s+1)(s+3)}, \quad \frac{X_2(s)}{U(s)} = \frac{(\quad)}{(s+1)(s+3)}$$

⇒ If SFG change The state variai change so T/F of state var change.

and SFG can be different -2

Controllability and observability

Controllability define that all state variables are input dependent and by applying appropriate input we can obtain any desirable output of state variable. controlla'. is cheack by controllabi test Matrix Qc

$$\boxed{[Q_c] = [B : AB : A^2B : --- A^{n-1}B]}$$

where n = total No of state variable which itself is rank of state Matrix [A]

nd For system to be controllable the essential requirement is that determinant of $[Q_c]$ should not be '0'

ntrollable $\boxed{|Q_c| \neq 0}$ ⇒ {Rank of $[Q_c]$ = Rank of S.M. $[A] = n$}

⇒ All n no of state variable are input dependent

but if $\boxed{|Q_c| = 0}$ ⇒ { Rank of $[Q_c]$ < Rank of S.M. $[A] = n$ some outpf n no of state variable is independent of input }

uncontrolable

observability

* observability define that output is dependent on all n no. of state variable. observaly is cheack by obserbility test Matrix

$$\boxed{[Q_0] = [C^T : A^TC^T : (A^T)^2C^T : ----- (A^T)^{n-1}C^T]}$$

n = total no of state variable = Rank of stateMatrix [A]

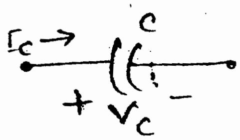
or S/T to be observable, ~~else~~ |determinant of obser $[Q_0]$ should not be ~~=~~ 0 → observable

$\boxed{[Q_0] \neq 0}$ { Rank of $[Q_0]$ = Rank of $[A] = n$ ⇒ all n no of state variable ~~the~~ output depends on

$[Q_0]$ ... { Rank of $[Q_0]$ < Rank of $[A] = n$ ⇒ output will be

State space representation of Electrical N/W

- capacitor: * In case of capacitor its voltage is a state variable while current accross capacitor is input for charging or discharging of state variable.

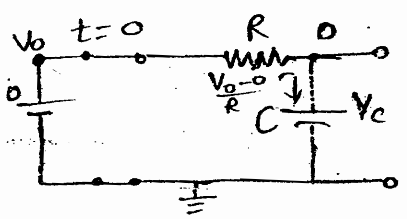


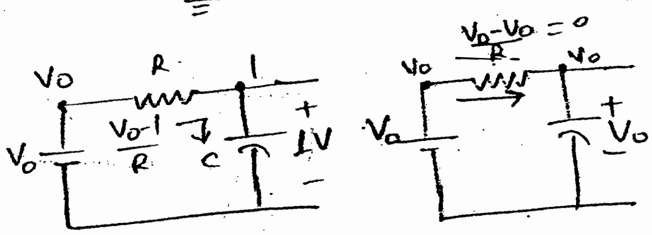
$$\left[\frac{dx}{dt}\right] = [A][x] + [B][u] \Rightarrow$$

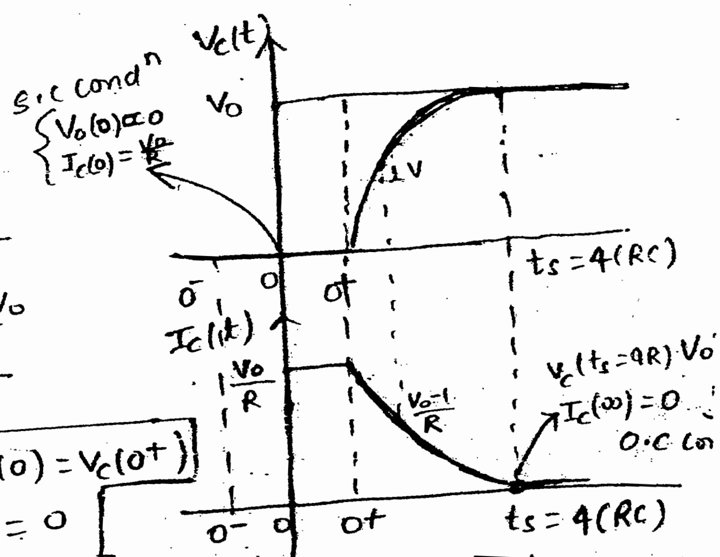
$$C\frac{dV_c}{dt} = I_c \Rightarrow \frac{dV_c}{dt} = \frac{I_c}{C} \Rightarrow \underbrace{\left[\frac{dV_c}{dt}\right]}_{\text{state variable}} = [0][V_c] + \left[\frac{1}{C}\right]\underbrace{[I_c]}_{\text{input}}$$

$\boxed{V_c(t^-) = V_c(t) = V_c(t^+)}$ $\boxed{I_c(t^-) \neq I_c(t^+)}$

at $t=0$ $\boxed{V_c(0^-) = V_c(0) = V_c(0+)}$, $\boxed{I_c(0^-) \neq I_c(0+)}$



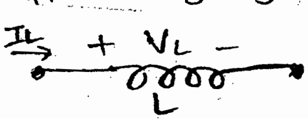




from figure

$$\boxed{\begin{aligned} &V_c(0^-) = V_c(0) = V_c(0^+) \\ &I_c(0^-) = 0 \\ &I_c(0^+) = \frac{V_0}{R} \Rightarrow I_c(0^-) \neq I_c(0^+) \end{aligned}}$$

2 Inductor * In case of Inductor its current is a state variable while voltage is input for charging (or) discharging of state variable.

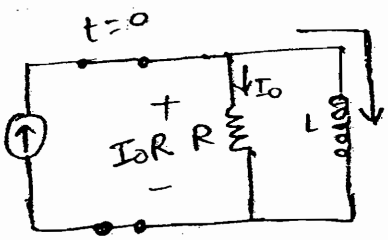


$$V_L = L\frac{dI_L}{dt} \Rightarrow \left[\frac{dI_L}{dt}\right] = [0]\underbrace{[I_L]}_{\text{state variable}} + \left[\frac{1}{L}\right]\underbrace{[V_L]}_{\text{inp}}$$

$\boxed{I_L(t^-) = I_L(t) = I_L(t^+)} \Rightarrow$ while $\boxed{V_L(t^-) \neq V_L(t^+)}$

at If $t=0$

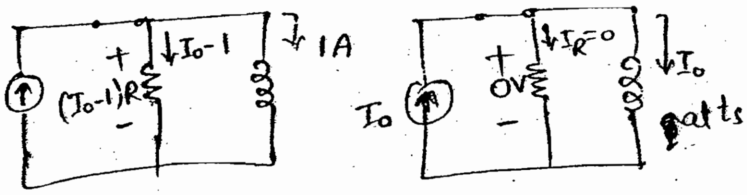
$\boxed{I_L(0^-) = I_L(0) = I_L(0+)}$ and $\boxed{V_L(0^-) \neq V_L(0+)}$

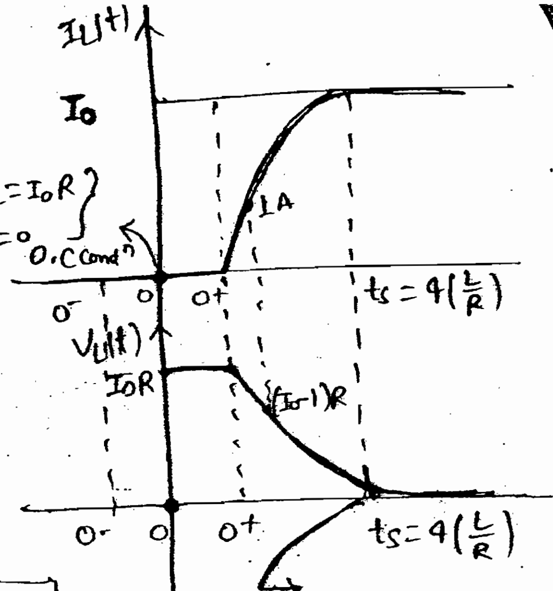


$I_L = 0$

$V_L(0) = I_0 R$

$I_L(0) = 0$



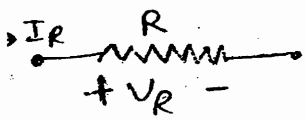


$I_L(0^-) = I_L(0) = I_L(0^+)$

$V_L(0^-) = 0$, $V_L(0^+) = I_0 R \Rightarrow V_L(0^-) \neq V_L(0^+)$

S.C Cond^n $\left\{ \begin{array}{l} I_L(t_s = 4\frac{L}{R} = \infty) = I_0 \\ V_L(t_s = 4\frac{L}{R} = \infty) = 0 \end{array} \right\}$

) Resistor

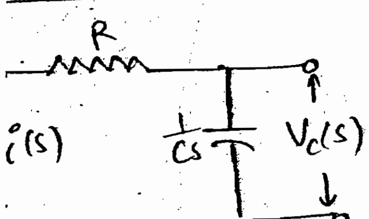


$[V_R] = [R][I_R]$, $[I_R] = [\frac{1}{R}][V_R]$

No any eq^n in differential form so no state variable.

Block representation of Electrical Network

R-C circuit

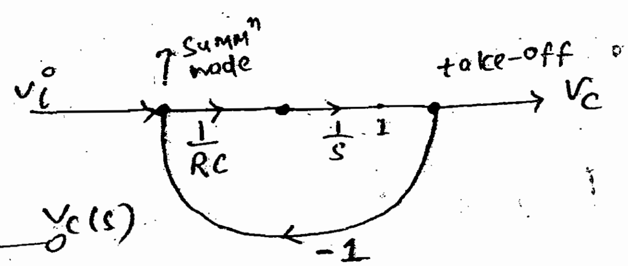


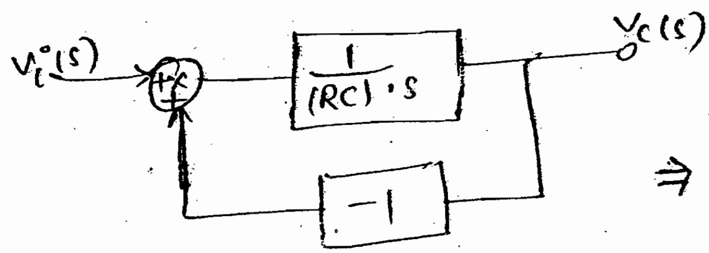
$$\frac{V_c(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

$\frac{+K}{s(s-5)} \Rightarrow$

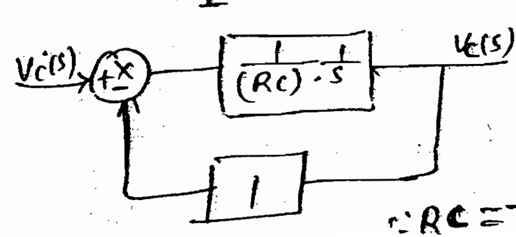
↳ $s-5$ की वजह से जो K आयेगा उसी से F/B decide

$$\frac{V_c(s)}{V_i(s)} = \frac{(1/RC)\cdot\frac{1}{s}}{1 - \left[-\frac{1}{RC}\times\frac{1}{s}\right]} \Rightarrow$$





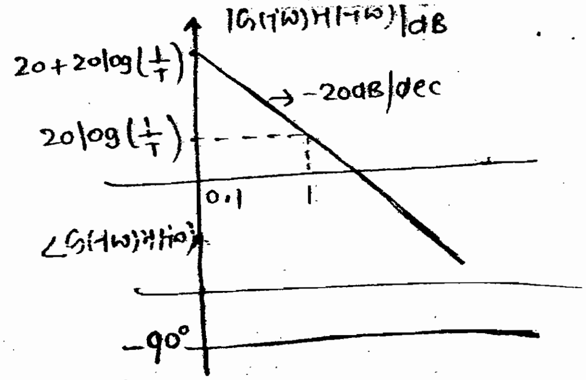
$\Rightarrow$



$\because RC = T$

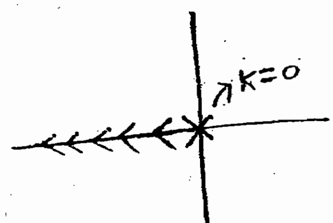
Now Bode plot from Block diagram

$$G(s)H(s) = \frac{1}{Ts} = \frac{1/T}{s}$$



* Now Root locus

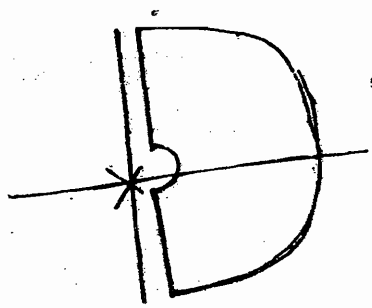
$$G(s)H(s) = \frac{1}{Ts} = \frac{(1/T)}{s}$$



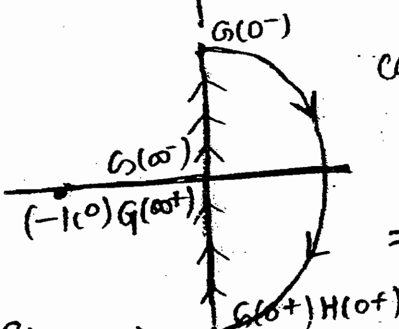
SIT is High! Stable so $G \cdot M = +\infty$

* Now Nyquist plot

$$G(s)H(s) = \frac{1}{Ts} = \frac{(\frac{1}{T})}{s}$$



$P_+ = 0$



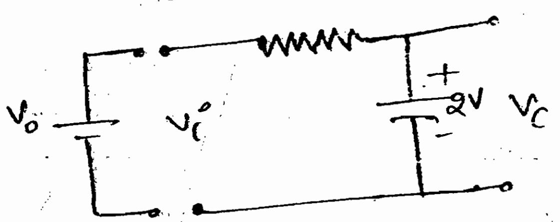
case 1

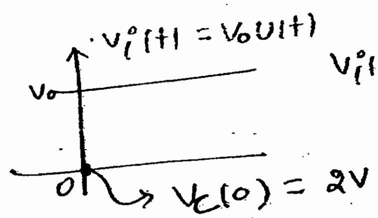
$P_+ = 0$

$N = 0$

$\Rightarrow \boxed{Z_+ = 0}$

Now Initial Voltage accross Capacitor given





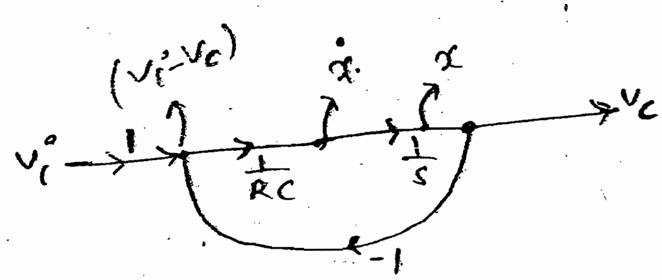
$V_i(s) = \frac{V_0}{s}$

$$\frac{V_c(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} = \frac{1/RC}{s+\frac{1}{RC}} \Rightarrow V_c(s) = \frac{\frac{1}{RC}}{s(s+\frac{1}{RC})} = \frac{a_0}{s} + \frac{a_1}{s+_{1}}$$

$$\boxed{V_c(t) = V_0[1-e^{-t/RC}]}$$

at $t=0$ $V_c(0) = 0 \to$ which wrong so Now we use state sp

$$\frac{V_c(s)}{V_i(s)} = \frac{\frac{1}{RC}\times\frac{1}{s}}{1-[-\frac{1}{RC}\times\frac{1}{s}]}$$



$\boxed{=x}$

$(0) = x(0) = 2V$

$$\dot{x} = \frac{1}{RC}[V_i - V_c] = \frac{1}{RC}[V_i - x]$$

$$[\dot{x}] = \left[-\frac{1}{RC}\right][x] + \left[\frac{1}{RC}\right][V_i] \quad \left\{[\dot{X}] = [A][X] + [B][U]\right\}$$

solution of this

$$X] = [sI - A]^{-1}[X(0)] + [sI - A]^{-1}[B][U] \qquad \{I = 1\}$$

$$X] = [s - A]^{-1}x(0) + (s - A)^{-1}B \cdot [U_i(s)]$$

$$X] = \frac{1}{(s-A)}x(0) + \frac{1}{(s-A)} \cdot B \cdot [U_i(s)] \qquad \left\{ A = -\frac{1}{RC},\ B = \frac{1}{RC} \right\}$$

$$x] = \frac{x(0)}{(s+\frac{1}{RC})} + \frac{(\frac{1}{RC})[\frac{V_0}{s}]}{[s+\frac{1}{RC}]} \Rightarrow [X(s)] = \frac{x(0)}{(s+\frac{1}{RC})} + \frac{\frac{V_0}{RC}}{(s(s+\frac{1}{RC}))}$$

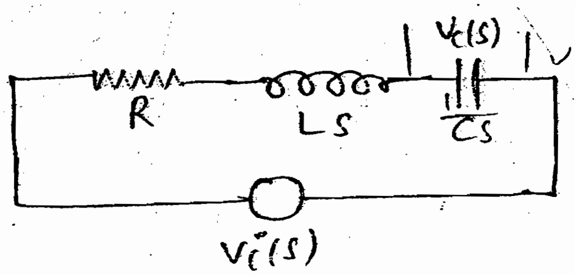
$$X(s)] = \frac{x(0)}{s+\frac{1}{RC}} + \left\{\frac{V_0}{s} - \frac{V_0}{(s+\frac{1}{RC})}\right\} \Rightarrow x(t) = x(0)e^{-t/RC} + \{V_0 - V_0 e^{-t/RC}\}$$

$$\boxed{x(t) = V_c(t) = [2e^{-t/RC}] + [V_0\{1 - e^{-t/RC}\}]}$$

ow at $t = 0$ $\boxed{V_c(0) = 2V}$ this is write

ZIR ZSR

Block representation of series RLC circuit

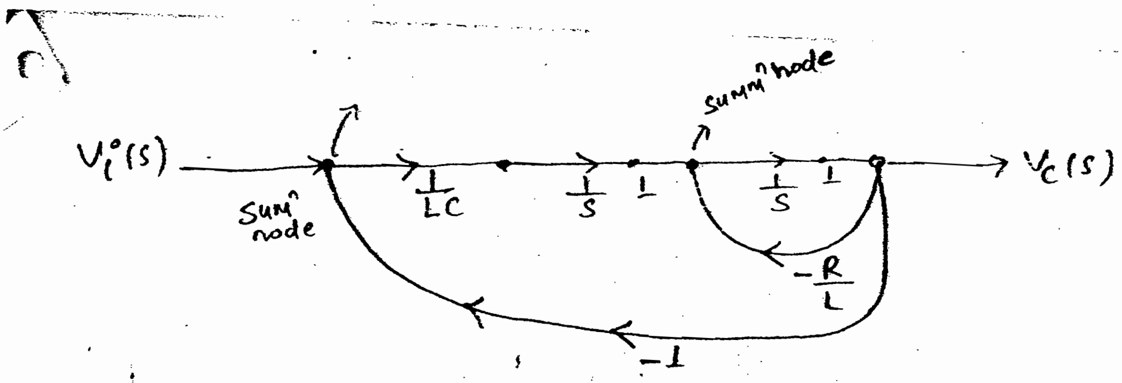


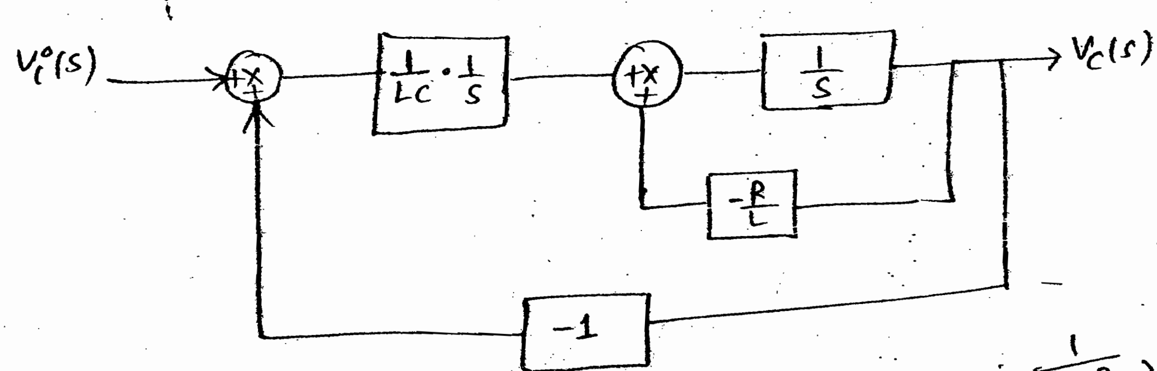
$$\frac{V_c(s)}{V_i(s)} = \frac{\frac{1}{CS}}{R + LS + \frac{1}{CS}}$$

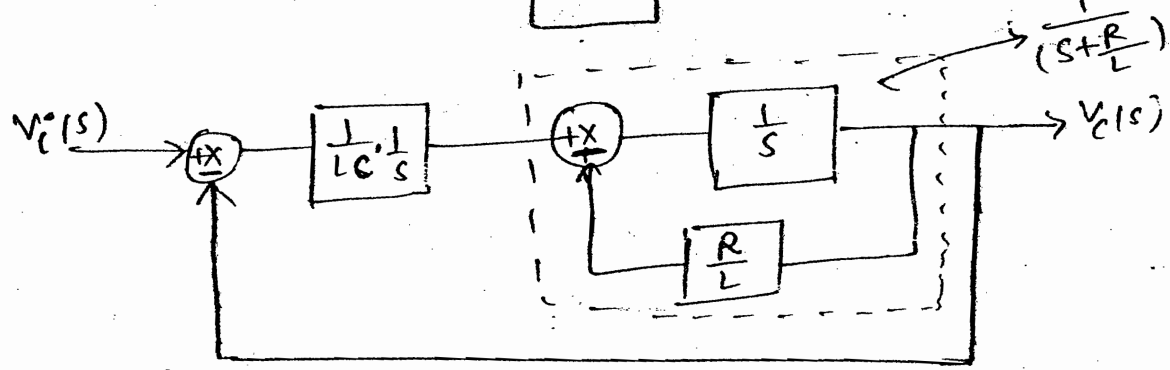
$$\frac{V_c(s)}{V_i(s)} = \frac{1/LC}{LCs^2 + RCs + 1} = \frac{(\frac{1}{LC})}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

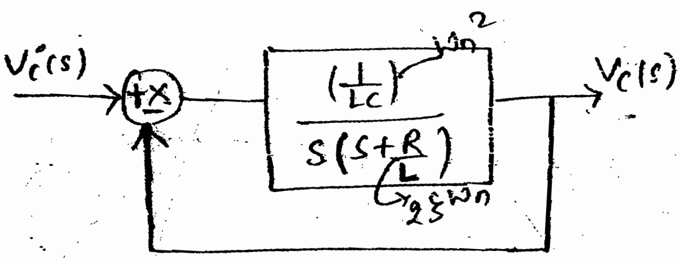
Highest powers से Num^r and Denomi^r से Divide for SFG/Mason Formula

$$\frac{V_c(s)}{V_i(s)} = \frac{(\frac{1}{LC}) \times \frac{1}{s^2}}{1 - \left[-\frac{(\frac{R}{L})}{s} - \frac{(\frac{1}{LC})}{s^2}\right]}$$



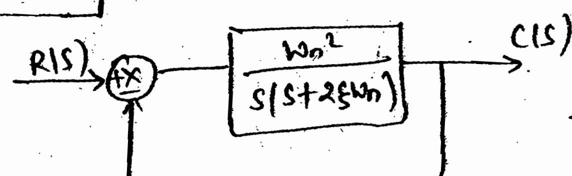






$$\frac{V_c(s)}{V_i(s)} = \frac{(\frac{1}{LC})}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

∵ In time response



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

on comparing $\omega_n^2 = \frac{1}{LC}$, $2\xi\omega_n = \frac{R}{L}$ ⇒ $\boxed{\omega_n = \frac{1}{\sqrt{LC}}}$ $\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$

open loop T/F (after remove summation)

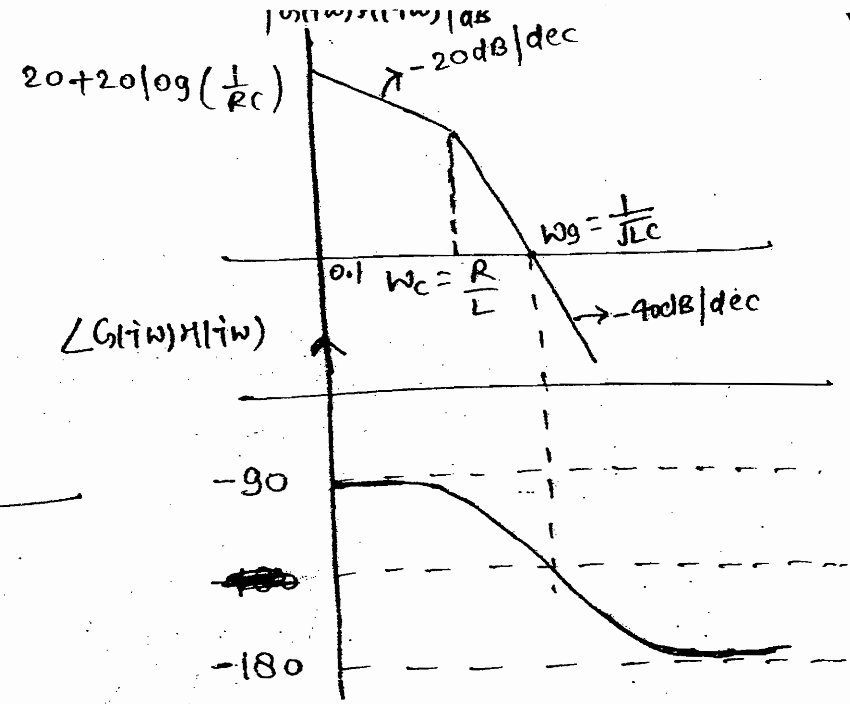
$$\boxed{G(s)H(s) = \frac{(\frac{1}{LC})\ K(\text{variable})}{s(s+\frac{R}{L})}}$$

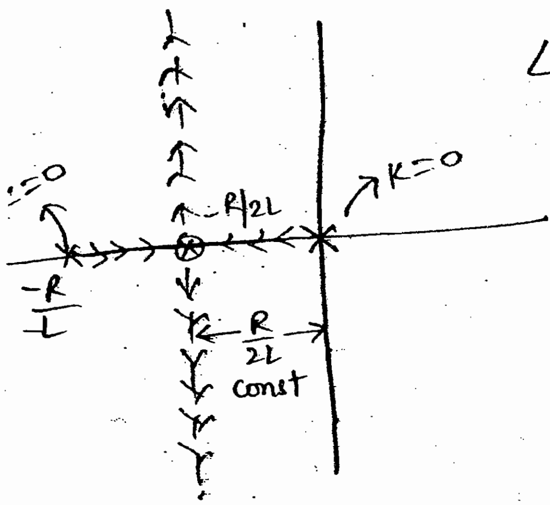
→ From this, ① Bode plot ② Nyquist plot ③ Root locus

const

K variable होगा 'C' को change करने से

Bodeplot

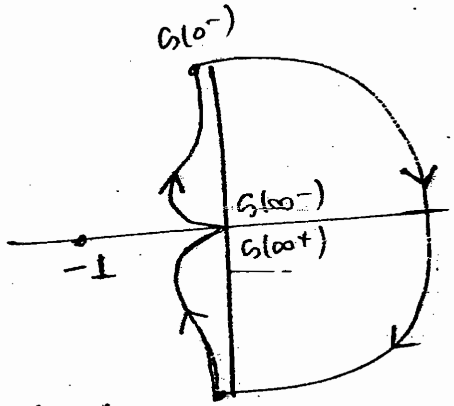




$P_+ = 0$

$N = 0$ (always)

$\boxed{Z_+ = 0}$ always



s/T Highly stable

& GM = +∞

If we take output accross Inductor

so we find T/F for that and farther same process.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & 1\\ 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}[u]$$

$$[y] = [1 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0][u]$$

A certain LTI s/T has state represenⁿ given above

(i) Find eigen value of s/T

(ii) If $u(t) = 1\delta(t)$ and $x_1(0) = x_2(0) = 0$ Calculate $x_1(t)$ & $x_2(t)$

(iii) If $u(t) = 0$ calculate $x_1(0)$ & $x_2(0)$ so that o/p $y(t) = Ae^{-2t}$

Solⁿ $[\lambda I - A] = \begin{bmatrix}\lambda+2 & -1\\ 0 & \lambda+3\end{bmatrix} \Rightarrow |\lambda I - A| = 0$

$\lambda = -2, -3$ (location of pole)

$\Rightarrow$ stable s/T

$$[X(s)] = [SI-A]^{-1}[X(0)] + [SI-A]^{-1}[B][U(s)] \quad \text{---- (1)}$$

$$[SI-A] = \begin{bmatrix} s+2 & -1 \\ 0 & s+3 \end{bmatrix} \Rightarrow [SI-A]^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+3 & 1 \\ 0 & s+2 \end{bmatrix}$$

$$\text{Cofactor} = \begin{bmatrix} (s+3) & 0 \\ +1 & (s+2) \end{bmatrix} \Rightarrow [SI-A]^{-1} = \frac{Adj}{|det|} = \frac{\begin{bmatrix} (s+3) & 1 \\ 0 & s+2 \end{bmatrix}}{(s+2)(s+3)}$$

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{\begin{bmatrix} (s+3) & 1 \\ 0 & (s+2) \end{bmatrix}}{(s+2)(s+3)} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} + \frac{\begin{bmatrix} (s+3) & 1 \\ 0 & (s+2) \end{bmatrix}}{(s+2)(s+3)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} U(s)$$

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{\begin{bmatrix} (s+3) & 1 \\ 0 & (s+2) \end{bmatrix}}{(s+2)(s+3)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \frac{\begin{bmatrix} (s+3) & 1 \\ 0 & s+2 \end{bmatrix}}{(s+2)(s+3)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1] = \frac{\begin{bmatrix} (s+3) \\ (s+2) \end{bmatrix} \cdot 1}{(s+2)(s+3)}$$

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{\begin{bmatrix} (s+3) \\ 0 \end{bmatrix} \cdot 1}{(s+2)(s+3)} = \begin{bmatrix} \frac{1}{(s+2)} \\ 0 \end{bmatrix} \Rightarrow x_1(s) = \frac{1}{s+2}, \; x_2(s) = 0$$

$$\boxed{x_1(t) = e^{-2t} \quad x_2(t) = 0}$$

iii)

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{\begin{bmatrix} (s+3) & 1 \\ 0 & (s+2) \end{bmatrix}}{(s+2)(s+3)} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} + \frac{\begin{bmatrix} (s+3) & 1 \\ 0 & (s+2) \end{bmatrix}}{(s+2)(s+3)} [0] \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{\begin{bmatrix} (s+3)x_1(0) + x_2(0) \\ (s+2)x_2(0) \end{bmatrix}}{(s+3)(s+2)} = \begin{bmatrix} \frac{x_2(0) + x_1(0)(s+3)}{(s+2)(s+3)} \\ \frac{x_2(0)(s+2)}{(s+3)(s+2)} \end{bmatrix}$$

$$x_1(s) = \frac{x_1(0)(s+3) + x_2(0)}{(s+2)(s+3)} \quad \& \quad x_2(s) = \frac{x_2(0)(s+2)}{(s+2)(s+3)}$$

$$y(s) = x_1(s) + x_2(s) \Rightarrow y(s) = \frac{(s+3)x_1(0) + x_2(0)}{(s+2)(s+3)} + \frac{(s+2)x_2(0)}{(s+2)(s+3)}$$

$$y(s) = \frac{(s+3)x_1(0) + (s+3)x_2(0)}{(s+2)(s+3)} = \frac{x_1(0) + x_2(0)}{(s+2)} = \frac{A}{s+2}$$

$$\boxed{x_1(0) + x_2(0) = A}$$

If $x_1(0) = 0$ Then $x_2(0) = A$

If $x_2(0) = 0$ Then $x_1(0) = A$

2 $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = z(t)$ For the given diff^n eq^n obtain its sol^n by using state space representation

$\dot{y}(0) = y(0) = 0$, $z(t) = e^{-1t}$

Initial cond^n given ⇒ इसी को state variable लेना है।

so calculate its o/p by both method state space & T/F method

$\because$ two state variable $y = x_1$ $x_1(0) = y(0) = 0$

$\boxed{\dot{x}_1 = x_2}$ —① $\Leftarrow \frac{dx_1}{dt} = \frac{dy}{dt} = x_2$ $x_2(0) = \dot{y}(0) = 0$

$$\frac{d}{dt}\left[\frac{dy}{dt}\right] + 1\left[\frac{dy}{dt}\right] - 2[y] = z(t)$$

$$\frac{d}{dt}[x_2] + 1[x_2] - 2[x_1] = z(t) \Rightarrow \boxed{\dot{x}_2 = 2x_1 - 1x_2 + 1z(t)}$$ ②

Now state matrix from state eq^n.

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 2 & -1\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}[z(t)] \rightarrow \text{state Matrix}$$

$$[y] = [1 \quad 0]\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + [0][z(t)] \rightarrow \text{output Matrix}$$

$$[SI - A] = \begin{bmatrix}(s) & -1 \\ -2 & (s+1)\end{bmatrix} \qquad \text{Co-factor} = \begin{bmatrix}(s+1) & 1 \\ 2 & s\end{bmatrix}$$

$$\text{Adj} = \begin{bmatrix}s+1 & 2 \\ 1 & s\end{bmatrix}$$

$$[SI - A]^{-1} = \frac{\begin{bmatrix}(s+1) & 2 \\ 1 & s\end{bmatrix}}{s^2+s-2} = \frac{\begin{bmatrix}(s+1) & 2 \\ 1 & s\end{bmatrix}}{(s+2)(s-1)} \rightarrow \text{unstable s/T}$$

$$[x(s)] = [SI - A]^{-1}[x(0)] + [SI - A]^{-1}[B][z(s)]$$

$$\begin{bmatrix}x_1(s) \\ x_2(s)\end{bmatrix} = \frac{\begin{bmatrix}(s+1) & 1 \\ 2 & s\end{bmatrix}}{(s+2)(s-1)}\begin{bmatrix}x_1(0) \\ x_2(0)\end{bmatrix} + \frac{\begin{bmatrix}(s+1) & 1 \\ 2 & s\end{bmatrix}}{(s+2)(s-1)}\begin{bmatrix}0 \\ 1\end{bmatrix}\frac{1}{(s+1)}$$

$$\begin{bmatrix}x_1(s) \\ x_2(s)\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix} + \frac{\begin{bmatrix}1 \\ s\end{bmatrix}}{(s+1)(s+2)(s-1)} = \begin{bmatrix}\frac{1}{(s+1)(s+2)(s-1)} \\ \frac{s}{(s+1)(s+2)(s-1)}\end{bmatrix}$$

$X_1(s) = \dfrac{1}{(s+2)(s+1)(s-1)}$ $\quad X_2(s) = \dfrac{s}{(s+2)(s+1)(s-1)} = s\{X_1(s)\}$

$x_1(s) = \dfrac{a_0}{s+1} + \dfrac{a_1}{s+2} + \dfrac{a_2}{s-1} = \dfrac{(-\frac{1}{2})}{s+1} + \dfrac{(1/3)}{s+2} + \dfrac{(1/6)}{s-1}$

$\boxed{x_1(t) = -\frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t} + \frac{1}{6}e^{+t}} \Rightarrow x_2(t) = \frac{d}{dt}x_1(t)$

$y(t)$

$\boxed{x_2(t) = \frac{1}{2}e^{-t} - \frac{2}{3}e^{-2t} + \frac{1}{6}e^{+t}}$ $\quad \Rightarrow \boxed{y(t) = x_1(t)}$ Ans

From T/F method because initial conditions are zero.

$\{s^2Y(s) - s\dot{y}(0) - y(0)\} + \{sY(s) - y(0)\} - 2Y(s) = Z(s)$

$s^2Y(s) + sY(s) - 2Y(s) = Z(s) \Rightarrow \dfrac{Y(s)}{Z(s)} = \left[\dfrac{1}{s^2+s-2}\right]$

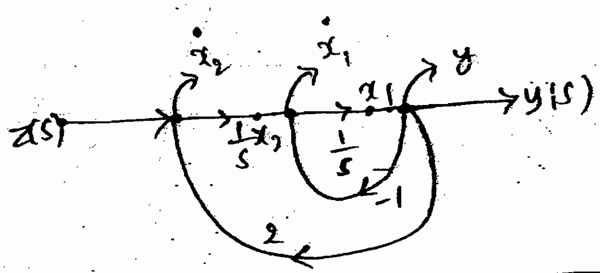
$Y(s) = \dfrac{1}{(s+1)(s+1)(s-2)} \quad \left\{Z(s) = \dfrac{1}{s+1}\right\} = \dfrac{a_0}{s+1} + \dfrac{a_1}{s+2} + \dfrac{a_2}{s-1}$

$Y(s) = \dfrac{-\frac{1}{2}}{s+1} + \dfrac{1/3}{s+2} + \dfrac{1/6}{s-1} \Rightarrow \boxed{y(t) = -\frac{1}{2}e^{-t} + \frac{1}{3}e^{-2t} + \frac{1}{6}e^{+t}}$

Now from T/F से solⁿ

$\dfrac{Y(s)}{Z(s)} = \dfrac{1}{s^2+s-2} = \dfrac{\frac{1}{s^2}}{1 - \left[-\frac{1}{s} + \frac{2}{s^2}\right]}$

Here gain = 1 so No require of additional Branch.



$\boxed{y = x_1}$ output eqⁿ.

$\dot{x}_1 = -1y + 1x_2 = -1x_1 + x_2$

$\boxed{\dot{x}_1 = -1x_1 + x_2}$

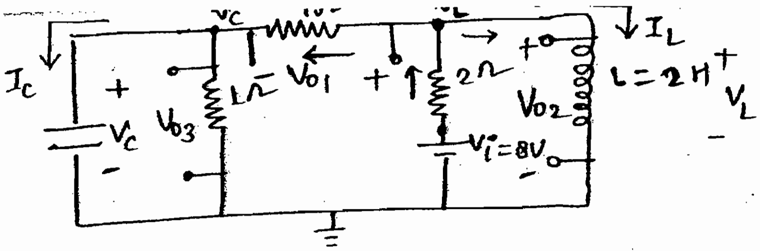
$\dot{x}_2 = 2y + Z \Rightarrow \boxed{\dot{x}_2 = 2x_1 + Z}$

State matrix

$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}[Z], \quad [y] = [1 \;\; 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0][Z]$

Now previous Matrix and this Matrix are different due to different state variable but solⁿ will be same $y(t)$ but $x_2(t)$ will be different

$\boxed{y(t) = -\frac{1}{2}e^{-1t} + \frac{1}{3}e^{-2t} + \frac{1}{6}e^{+1t} = x_1(t)}$ $\quad$ o/p → unique solⁿ



For the given electrical circuit obtain its state matrix as well as o/p Matrix

$$\frac{V_i - V_L}{2} = \frac{V_L - V_c}{1} + I_L \quad \left\{ \because V_L = L\frac{dI_L}{dt} = 2\frac{dI_L}{dt} \right\}$$

$$\frac{3}{2} V_L = V_c - I_L + \frac{V_i}{2} \Rightarrow \frac{3}{2} \times 2 \frac{dI_L}{dt} = V_c - I_L + \frac{V_i}{2}$$

$$\frac{dI_L}{dt} = \frac{1}{3} V_c - \frac{1}{3} I_L + \frac{1}{6} V_i \Rightarrow \boxed{\frac{dI_L}{dt} = \left(\frac{1}{3}\right)V_c + \left(-\frac{1}{3}\right) I_L + \left(\frac{1}{6}\right)V_i}$$

pply KCL

$$\frac{V_L - V_c}{1} = \frac{V_c}{1} + I_c \quad \left\{ \because I_c = C\frac{dV_c}{dt} = 1 \times \frac{dV_c}{dt} \right\}$$

$$I_c = -2V_c + V_L \Rightarrow \frac{dV_c}{dt} = -2V_c + 2\left\{\frac{dI_L}{dt}\right\}$$

$$\frac{dV_c}{dt} = -2V_c + 2\left\{\frac{1}{3}V_c - \frac{1}{3}I_L + \frac{1}{6}V_i\right\}$$

$$\boxed{\frac{dV_c}{dt} = \left(-\frac{4}{3}\right)V_c + \left(-\frac{2}{3}\right)I_L + \left(\frac{1}{3}\right)V_i}$$

contain state variable and input $\Rightarrow$ this is state eq^n.

$$\begin{bmatrix} \dot{I}_L \\ \dot{V}_c \end{bmatrix} = \underset{2\times 2}{\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{4}{3} \end{bmatrix}} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \underset{2\times 1}{\begin{bmatrix} \frac{1}{6} \\ \frac{1}{3} \end{bmatrix}} \begin{bmatrix} V_i \end{bmatrix} \quad \left\{ \begin{array}{l} [\dot{X}] = [A]_{n\times n}[X] + [B]_{n\times l}[U] \\ [Y] = [C]_{m\times n}[X] + [D]_{m\times l}[U] \end{array} \right\}$$

f o/p not defined then we cant write o/p equation.

$$V_{01} = V_L - V_c = \left\{\frac{2}{3}V_c - \frac{2}{3}I_L + \frac{1}{3}V_i\right\} - V_c \Rightarrow \boxed{V_{01} = -\frac{1}{3}V_c - \frac{2}{3}I_L + \frac{1}{3}V_i}$$

output arise accross 1Ω resistor.

$$\begin{bmatrix} V_{01} \\ V_{02} \\ V_{03} \end{bmatrix} = \underset{\substack{1\times 2 \\ 2\times 2 \\ 3\times 2}}{\begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} \\ -2/3 & 2/3 \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \underset{\substack{1\times 1 \\ 2\times 1 \\ 3\times 1}}{\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \end{bmatrix}} \begin{bmatrix} V_i \end{bmatrix}$$

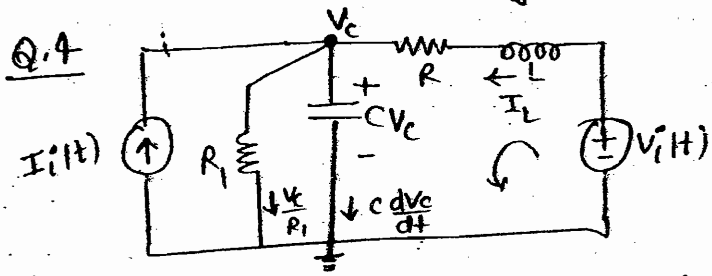
$$\begin{bmatrix} \frac{V_{01}(s)}{V_i(s)} \\ \frac{V_{02}(s)}{V_i(s)} \\ \frac{V_{03}(s)}{\ldots} \end{bmatrix} = \underset{\substack{1\times 2 \\ 2\times 2 \\ \ldots}}{[C]} \underset{\substack{2\times 2 \\ 2\times 2 \\ 2\times 2}}{[SI - A]^{-1}} \underset{\substack{2\times 1 \\ 2\times 1 \\ 2\times 1}}{[B]} + \underset{\substack{1\times 1 \\ 2\times 1 \\ 3\times 1}}{[D]}$$

$$\boxed{V_{o2} = V_L = \frac{2}{3}V_C - \frac{2}{3}I_L + \frac{1}{3}V_i}$$ second o/p accross inductor.

$$\boxed{V_{o3} = I V}$$

* We can calculate T/F simultaneously accross every or any component in a single Array. change only [C] & [D] Matrix

Q.4 obtain its state Matrix



Here 2 input given

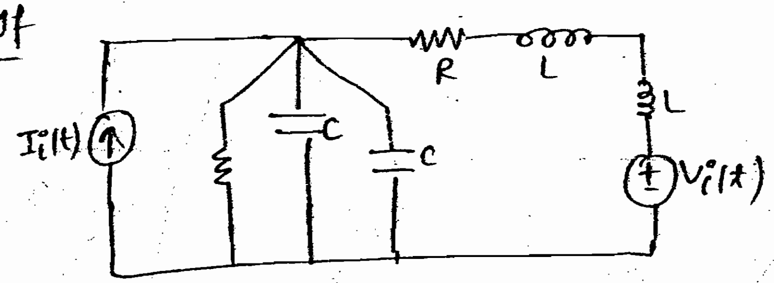
Sol^n

$$C\frac{dV_c}{dt} + \frac{V_c}{R_1} = I_i(t) + I_L \Rightarrow \boxed{\frac{dV_c}{dt} = \left(-\frac{1}{R_1 C}\right)\cdot V_c + \left(\frac{1}{C}\right)I_L + \left(\frac{1}{C}\right)I_i(t)}$$

$$V_i(t) - L\frac{dI_L}{dt} - RI_L - V_c = 0 \Rightarrow \boxed{\frac{dI_L}{dt} = \left(-\frac{1}{L}\right)\cdot V_c - \left(\frac{R}{L}\right)I_L + \frac{1}{L}V_i}$$

$$\begin{bmatrix} \dot{I}_L \\ \dot{V}_c \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{2\times 2} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & \frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_{2\times 2} \begin{bmatrix} I_i(t) \\ V_i(t) \end{bmatrix}$$ Ans → state Matrix

If

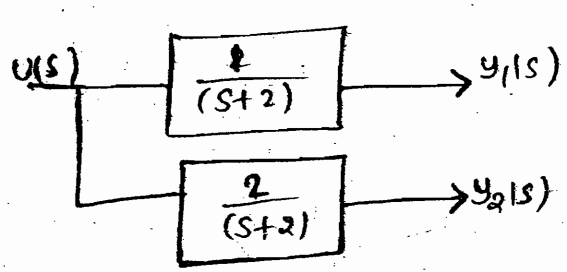


यहाँ भी 2 ही state variable होंगे

C & C in parallel where Voltage same

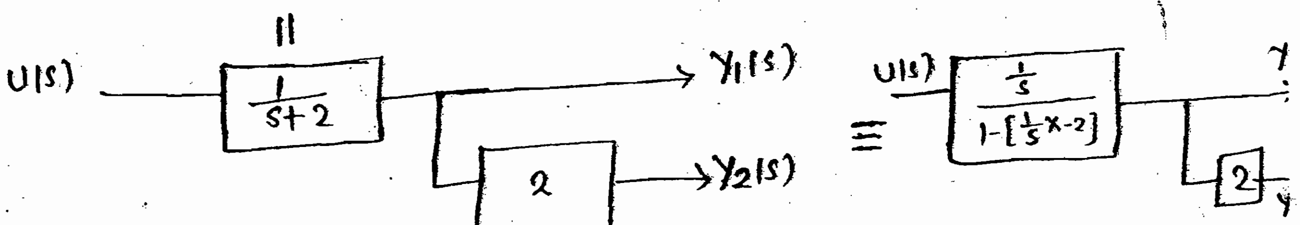
L & L in series where current same.

Q.5



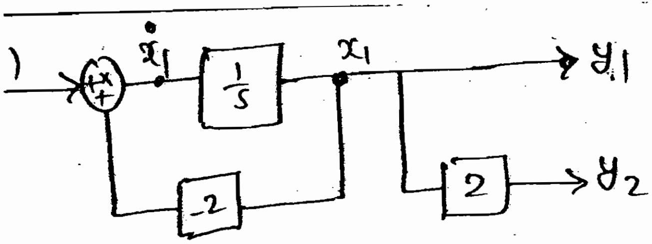
For the given block obtain its state Matrix

||



||

From this Block is.



Here 2 output
state variable = 1
input = 1

$\because y_1 = 1x_1$

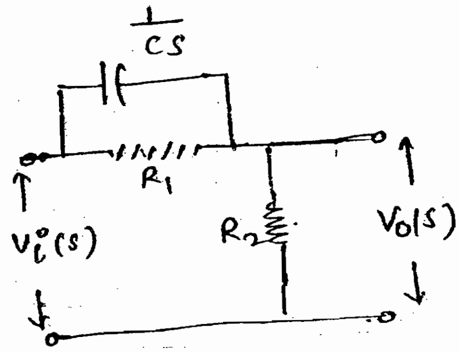
$y_2 = 2x_1$

$$[\dot{x}_1] = \underset{1\times1}{[-2]}[x_1] + \underset{1\times1}{[1]}[u]$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underset{2\times1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}[x_1] + \underset{2\times1}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}[u]$$ Ans

Compensator

Phase lead compensator



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + \frac{R_1 \times \frac{1}{CS}}{R_1 + \frac{1}{CS}}} = \frac{R_2}{R_2 + \frac{R_1}{R_1CS+1}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2[R_1CS+1]}{R_2R_1CS + R_2 + R_1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2[R_1CS+1]}{(R_2+R_1)\left[\frac{R_2R_1CS}{R_2+R_1}+1\right]}$$

$\because \boxed{\alpha = \frac{R_2}{R_1+R_2} < 1}$

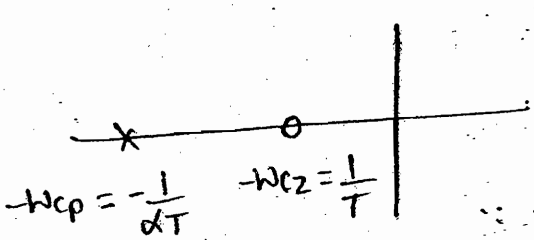
$\boxed{T = R_1C}$

$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha[TS+1]}{[\alpha ST+1]} = \frac{\alpha\left[\frac{S}{W_{cz}}+1\right]}{\left[\frac{S}{W_{cp}}+1\right]}$$

$\boxed{W_{cz} = \frac{1}{T}}$

$\boxed{W_{cp} = \frac{1}{\alpha T}}$

$\boxed{W_{cp} > W_{cz}}$



$\because$ zero closer to origin so phase lead compensator is zero dominant

$$\frac{V_o(jw)}{V_i(jw)} = \frac{\alpha[1+jwT]}{[1+jw\alpha T]}, \quad \left|\frac{V_o(jw)}{V_i(jw)}\right| = \frac{\alpha\sqrt{1+(wT)^2}}{\sqrt{1+(w\alpha T)^2}}$$

$$\left|\frac{V_o(jw)}{\ }\right| = \alpha\sqrt{1+(wT)^2}$$

$$\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right|_{dB} = 20\log\alpha + 20\log\sqrt{1+(\omega T)^2} - 20\log\sqrt{1+(\omega\alpha T)^2}$$

$\theta_1 > \theta_2 \Rightarrow$ +ve phase

$$\angle\frac{V_o(j\omega)}{V_i(j\omega)} = +\underbrace{\tan^{-1}\left(\frac{\omega T}{1}\right)}_{\theta_1} - \underbrace{\tan^{-1}\frac{\omega T\alpha}{1}}_{\theta_2}$$

$$\therefore \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{\alpha\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega\alpha T)^2}}$$

(L.F.R) At $\omega = 0$ $\frac{V_o}{V_i} = \alpha \Rightarrow V_o = \alpha V$

$V_o = \alpha V_i < V_i$ {Attenuati

At $\omega = \infty$ (H.F.R)

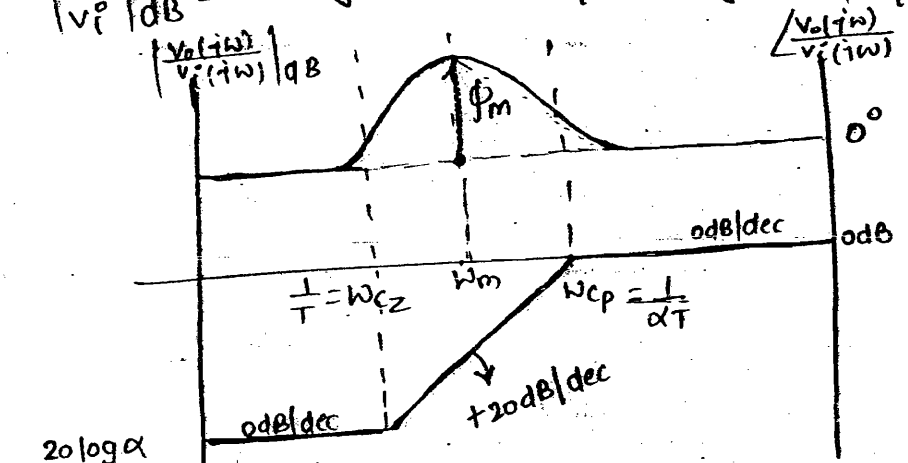
$$\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{\alpha\sqrt{(\omega T)^2}}{\sqrt{(\omega\alpha T)^2}} = \frac{\omega\alpha T}{\omega\alpha T} = 1 \Rightarrow \boxed{V_o = V_i}$$

$\omega = 0$

$$\left|\frac{V_o}{V_i}\right|_{dB} = 20\log\alpha \ (-ve) \ \{\because \alpha < 1\}$$

$\omega = \infty$

$$\left|\frac{V_o}{V_i}\right|_{dB} = 20\log 1 = 0\,dB$$



ω_m is A.Mean of ω_{cz} &

$$\log\omega_m = \frac{1}{2}\left[\log\omega_{cz} + \log k\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{1}{T}\right) + \log\left(\frac{1}{\alpha T}\right)\right]$$

$$= \frac{1}{2}\log\left(\frac{1}{\alpha T^2}\right)$$

$$= \log\left(\frac{1}{\alpha T^2}\right)^{\frac{1}{2}}$$

$$= \log\frac{1}{\sqrt{\alpha}T}$$

$$\omega_m = \frac{1}{\sqrt{\alpha}T} \quad \omega_m T = \frac{1}{\sqrt{\alpha}}$$

Now

$$\phi_m = \angle\frac{V_o(j\omega_m)}{V_i(j\omega_m)} = +\tan^{-1}\left[(\omega_m T) - \tan^{-1}(\omega_m T\alpha)\right]$$

$$\phi_m = \tan^{-1}\left(\frac{1}{\sqrt{\alpha}}\right) - \tan^{-1}(\sqrt{\alpha}) = \tan^{-1}\frac{(1-\alpha)}{2\sqrt{\alpha}}$$

$$\tan\phi_m = \frac{(1-\alpha)}{2\sqrt{\alpha}} = \frac{P}{B} \Rightarrow P = (1-\alpha),\ B = 2\sqrt{\alpha}$$

$$h = \sqrt{P^2+b^2} = \sqrt{(1-\alpha)^2 + 4\alpha}$$

$$h = 1+\alpha$$

$$\sin\phi_m = \frac{(1-\alpha)}{(1+\alpha)}, \quad \cos\phi_m = \frac{2\sqrt{\alpha}}{(1+\alpha)}$$

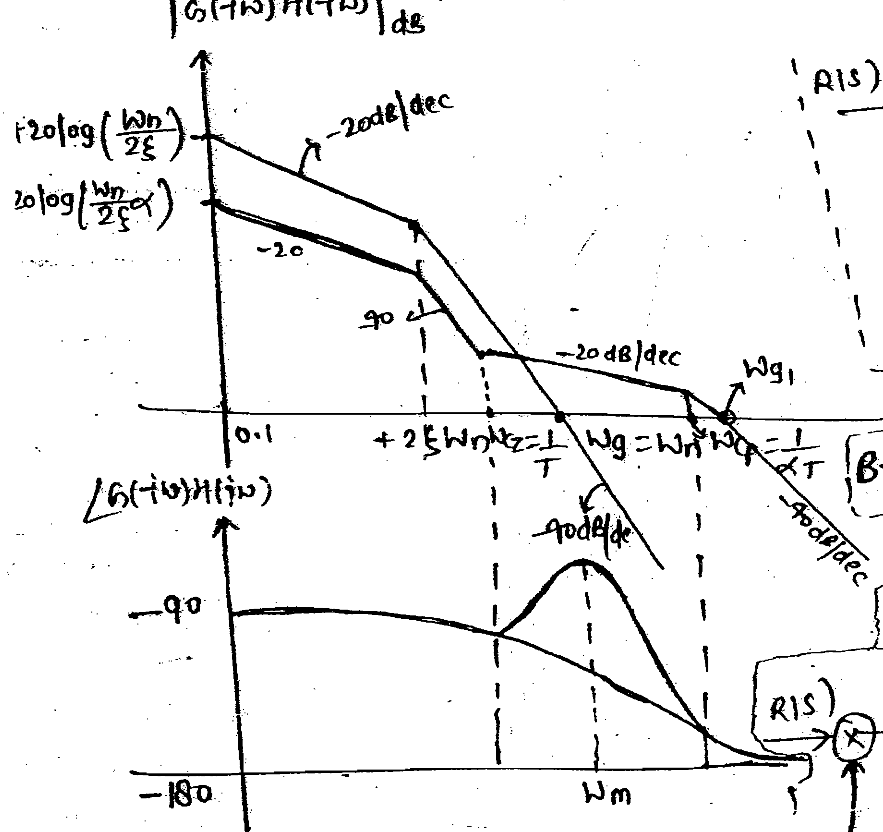
* Phase lead compensator is zero dominant because zero is closer to the origin
* Phase lead compensator act as High pass filter
* P.L. Compensator shift Gain Cross-over freq^n to Higher

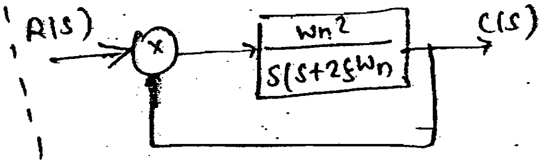
bandwidth of S/T will ↑ %age overshoot will also ↑
settling time will ↓

P.L.C will not affect steady state error.

As P.L.C. comp. is zero dominant It increase the bandwidth of system.

Consider $G(s)H(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)} \Rightarrow G(s)H(s) = \frac{\frac{\omega_n}{2\xi}}{s\left[\frac{s}{2\xi\omega_n}+1\right]}$



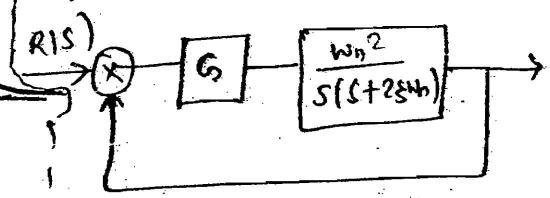


$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$B.W = \omega_c = \omega_n\sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}}$

$\omega_n \to \omega g$

Now connect P.L.C.



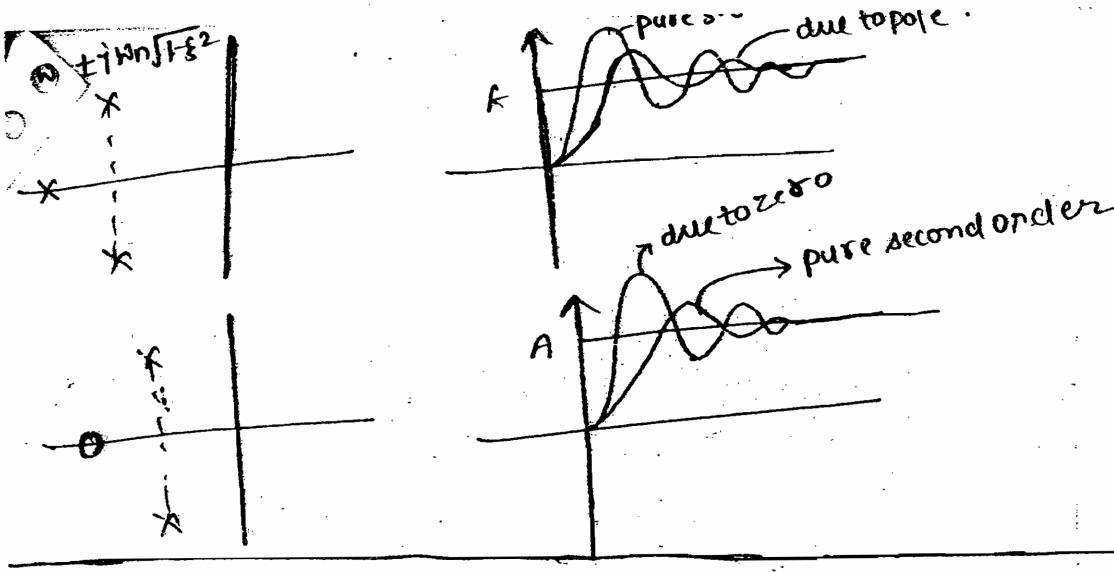
$$G(s)H(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)} \cdot \frac{\alpha\left[\frac{s}{\omega_{c_2}}+1\right]}{\left[\frac{s}{\omega_{cp}}+1\right]}$$

$$= \frac{\left(\frac{\omega_n}{2\xi}\alpha\right)\left(\frac{s}{\omega_{c_2}}+1\right)}{s\left[\frac{s}{2\xi\omega_n}+1\right]\left[\frac{s}{\omega_{cp}}+1\right]}$$

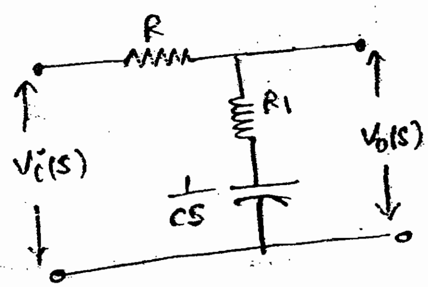
In case of second order system P.L.C Increase the phase Margin of the System but It will not affect the gain Margin

Thus whenever faster response is required we will use phase lead compensator.

∵ B.W depends on $\omega_n = \omega g$ ⇒ when we use P.L.C. ωg ↑ Hence B.W ↑



Phase-Lag compensator



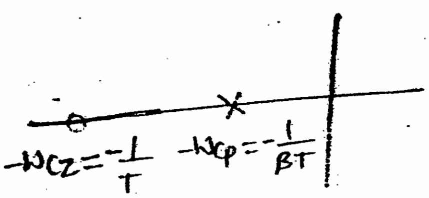
$$\frac{V_0(s)}{V_i(s)} = \frac{R_1 + \frac{1}{CS}}{R_2 + R_1 + \frac{1}{CS}} = \frac{(R_1C_1s+1)}{\left[\frac{(R_1+R_2)}{R_1}R_1Cs+1\right.}$$

$$\boxed{\because \beta = \frac{R_1+R_2}{R_1} > 1} \quad \boxed{\beta > 1}$$

$$\boxed{T = R_1C}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{(Ts+1)}{(\beta Ts+1)} = \frac{1\left[\frac{s}{\omega_{cz}}+1\right]}{\left[\frac{s}{\omega_{cp}}+1\right]}$$

$$\boxed{\omega_{cz} = \frac{1}{T}} \quad \boxed{\omega_{cp} = \frac{1}{\beta T}} \quad \boxed{\omega_{cp} > \omega}$$



$$\frac{V_0(j\omega)}{V_i(j\omega)} = \frac{1+j\omega T}{1+j\omega\beta T}$$

$$\left|\frac{V_0(j\omega)}{V_i(j\omega)}\right| = \frac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega\beta T)^2}}$$

* p. Lag compensator is pole dominant.

$$\angle\frac{V_0(j\omega)}{V_i(j\omega)} = +\underbrace{\tan^{-1}(\omega T)}_{\phi_1} - \underbrace{\tan^{-1}(\omega\beta T)}_{\phi_2} \quad \because \phi_1 < \phi_2 \Rightarrow \text{result } (-ve) \text{ so } \phi_m \ -ve$$

$$\left|\frac{V_0(j\omega)}{V_i(j\omega)}\right|_{dB} = 20\log 1 + 20\log\sqrt{1+(\omega T)^2} - 20\log\sqrt{1+(\omega T\beta)^2}$$

$$\because \left|\frac{V_0(j\omega)}{V_i(j\omega)}\right| = \frac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega\beta T)^2}}$$

At L.F.R. $\Rightarrow \omega = 0$

$$\frac{V_0}{V_i} = 1 \Rightarrow \boxed{V_0 = V_i}$$

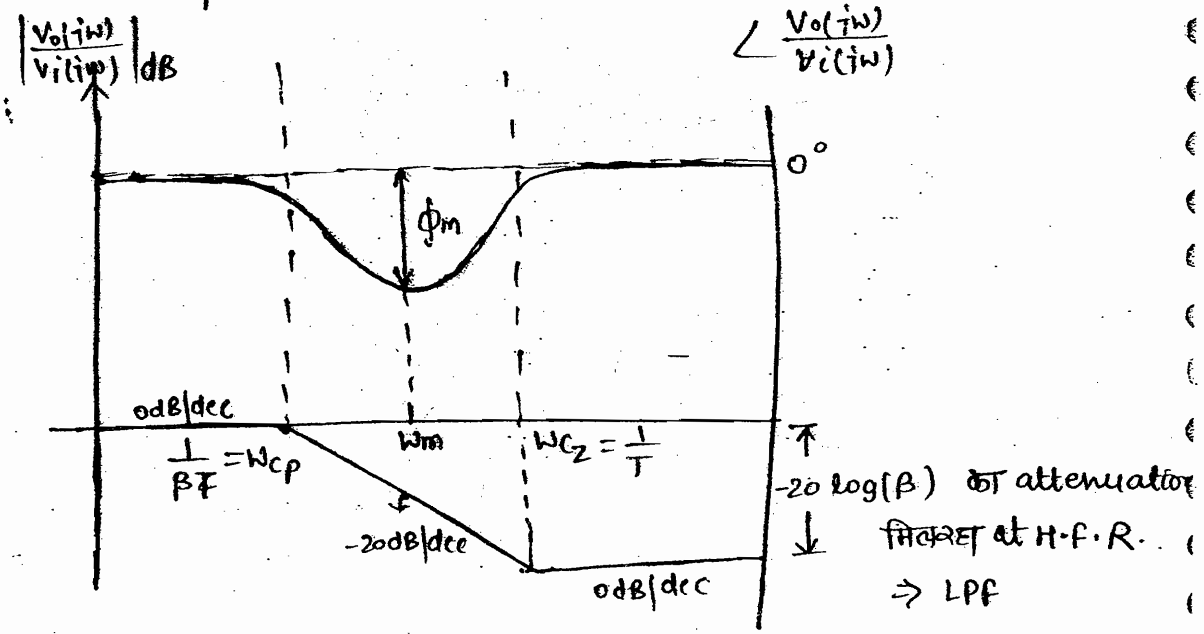
$$\left|\frac{V_0}{V_i}\right|_{dB} = 20\log 1 = 0\,dB$$

$\therefore R \Rightarrow \omega = \infty$

$$\therefore = \frac{\sqrt{(\omega T)^2}}{\sqrt{(\omega \beta T)^2}} = \frac{1}{\beta} \Rightarrow \boxed{V_o = \frac{1}{\beta} V_i}$$ Attenuation

o/p β times decrease..

$\frac{V_o}{V_i}\Big|_{dB} = -20 \log \beta$



$$\log \omega_m = \frac{1}{2}\left[\log \frac{1}{T} + \log \frac{1}{\beta T}\right] = \frac{1}{2} \log \frac{1}{\beta T^2} = \log \left(\frac{1}{\beta T^2}\right)^{\frac{1}{2}}$$

$$\log \omega_m = \log \frac{1}{T\sqrt{\beta}} \Rightarrow \omega_m = \frac{1}{T\sqrt{\beta}}, \quad \omega_m T = \frac{1}{\sqrt{\beta}}$$

$$\phi_m = \angle \frac{V_o(j\omega_m)}{V_i(j\omega_m)} = +\tan^{-1}(\omega_m T) - \tan^{-1}(\omega_m T \beta)$$

$$\phi_m = \tan^{-1}\left(\frac{1}{\sqrt{\beta}}\right) - \tan^{-1}(\sqrt{\beta}) = \tan^{-1} \frac{1-\beta}{2\sqrt{\beta}}$$

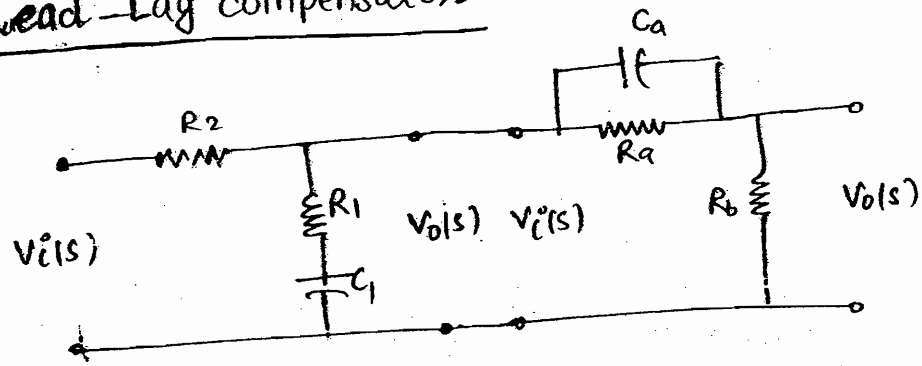
$$\tan \phi_m = \frac{1-\beta}{2\sqrt{\beta}}, \quad \sin \phi_m = \frac{1-\beta}{1+\beta}, \quad \cos \phi_m = \frac{2\sqrt{\beta}}{1+\beta}$$

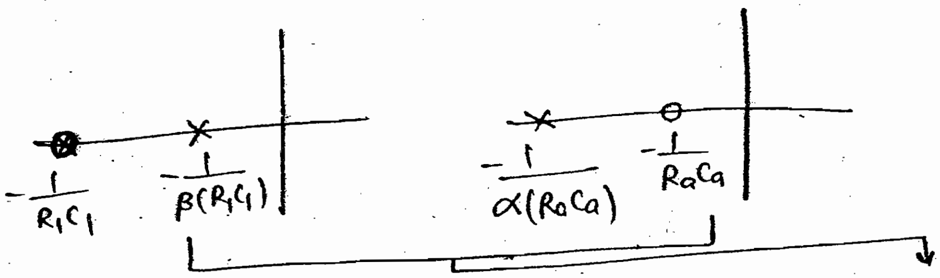
phase lag compensator is pole dominant

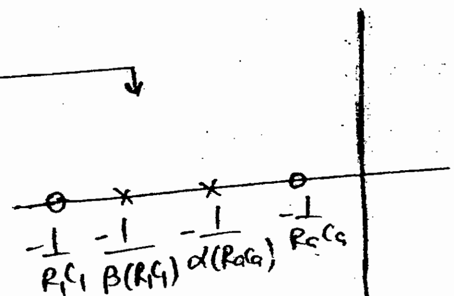
phase lag compensator act as low pass filter

- phase lag compensator shift Gain crossover freq^n to lower value so B.W of system will decrease
- p.lag.c. decrease steady state error. Thus if we want to improve steady state error in that case we will use phase-lag compensator.

Lead-Lag compensator



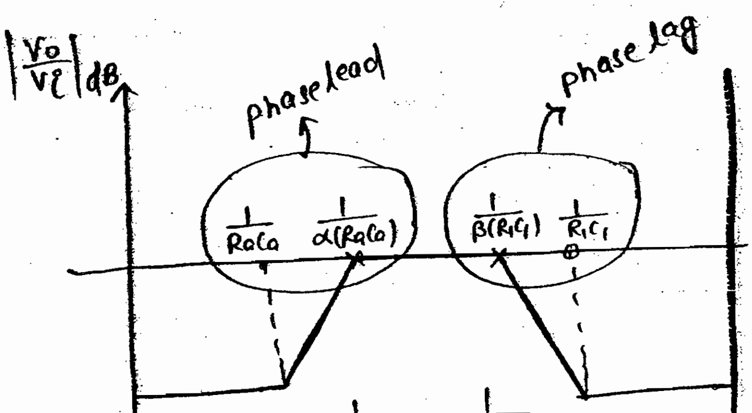




To achieve this condⁿ

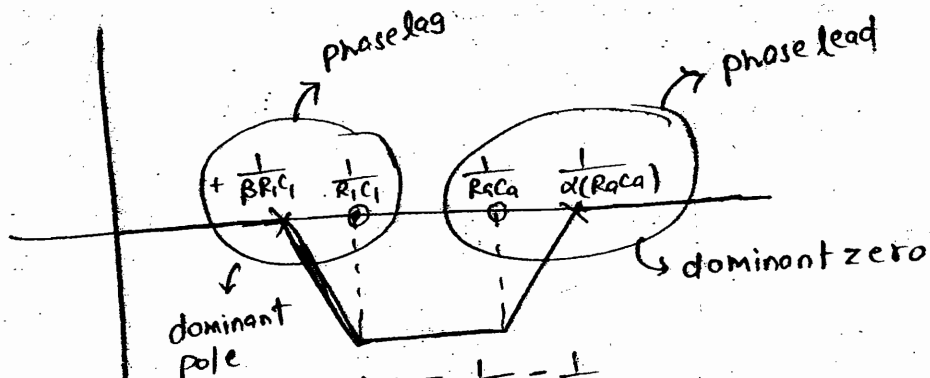
$$\frac{1}{R_aC_a} < \frac{1}{R_1C_1}$$

$$\boxed{R_1C_1 < R_aC_a}$$



$$\Delta W = \frac{1}{\beta(R_1C_1)} - \frac{1}{\alpha(R_aC_a)}$$

→ This is Band pass filter

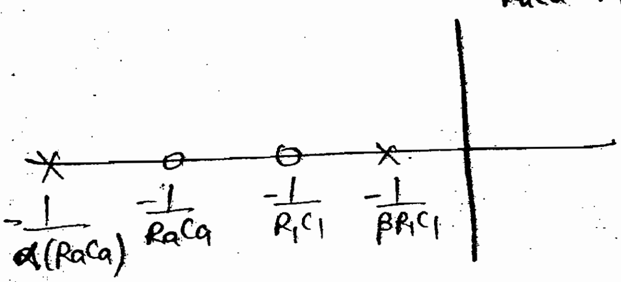


$$\Delta W = \frac{1}{R_aC_a} - \frac{1}{R_1C_1}$$

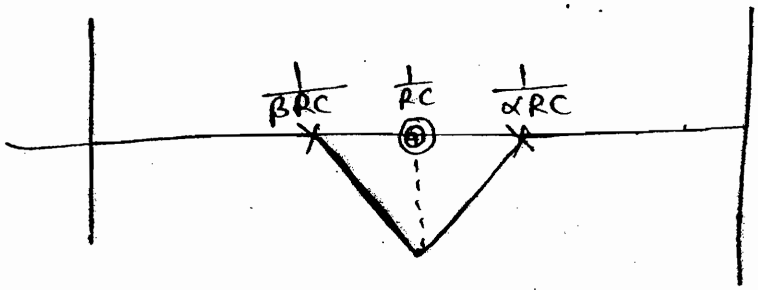
To achieve this requirement

$$\frac{1}{R_aC_a} > \frac{1}{R_1C_1}$$

$$R_1C_1 > R_aC_a$$



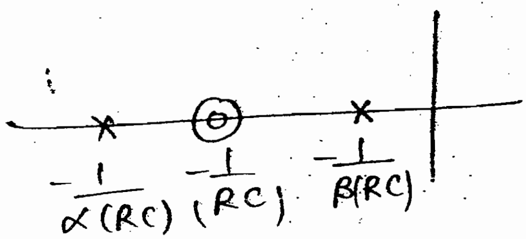
This is Band elimination filter



$R_1C_1 = R_2C_2 = RC$

$\omega = \frac{1}{RC}$ पर heavy attenuation देता

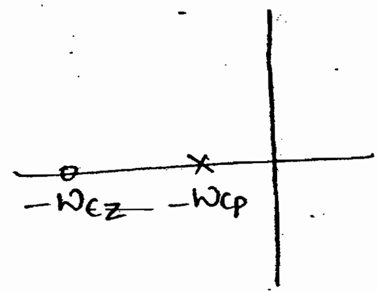
→ Notch filter



inimum phase and Non-minimum phase system

inimum phase system

$$T(s) = G(s)\cdot H(s) = \frac{K\left[\frac{s}{W_{cz}}+1\right]}{\left[\frac{s}{W_{cp}}+1\right]}$$



$W_{cp} < W_{cz} \Rightarrow \frac{W_{cp}}{W_{cz}} < 1$

$$G(j\omega)H(j\omega) = \frac{K\left[1+\frac{j\omega}{W_{cz}}\right]}{\left[1+\frac{j\omega}{W_{cp}}\right]}$$

$$\left|G(j\omega)H(j\omega)\right| = \frac{K\sqrt{1+\left(\frac{\omega}{W_{cz}}\right)^2}}{\sqrt{1+\left(\frac{\omega}{W_{cp}}\right)^2}}$$

$$\left|G(j\omega)H(j\omega)\right|_{dB} = 20\log K + 20\log\sqrt{1+\left(\frac{\omega}{W_{cz}}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{W_{cp}}\right)^2}$$

$$\angle G(j\omega)H(j\omega) = \tan^{-1}\frac{\omega}{W_{cz}} - \tan^{-1}\frac{\omega}{W_{cp}}$$

Mathematical Modelling

In case of Mechanical system there is a fixed node known as Ground node and (or) datum node and second node is known as variable node on application of force variable node will move in that direction to achieve equilibrium condn.

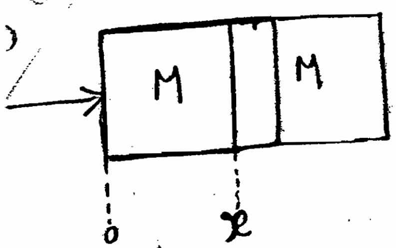
Mechanical system containing mass, spring and damper and force is applied on the mass in this case each mass will have its own variable node and force, spring and damper will use the variable node of this Masses

Mechanical system containing Mass, spring, damper and force is not applied on the mass in this case each mass will have its own variable node, force will also have its own variable node while spring and damper will use the variable node of either force or Mass.

- Mechanical system containing spring and damper in series in this case both spring and damper will have its own variable node
- Mass is a second order system, damper is first order system while spring is zero order s/T Mass and damper are responsible for transient behaviour of the s/T while spring is responsible for steady state behaviour.

* If any mechanical s/T contain N no of mass then overall order of that s/T will be '2N'

Mass

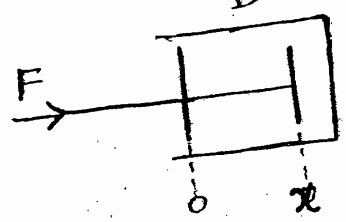


F_M = Mass x Acceleration

$$\boxed{F_M = M\left(\frac{d^2x}{dt^2}\right)}$$ If x = Const

$F_M = M \cdot \frac{d}{dt}(V)$ ↳ If V constant Then $F_M = 0$

(2) Damper



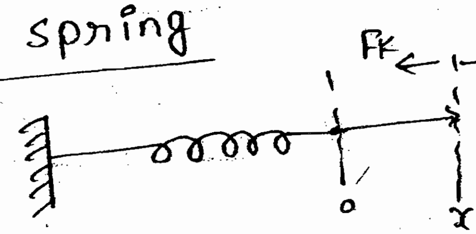
$$\boxed{F_D = D\left(\frac{dx}{dt}\right)}$$

If x = const Then $F_D = 0$

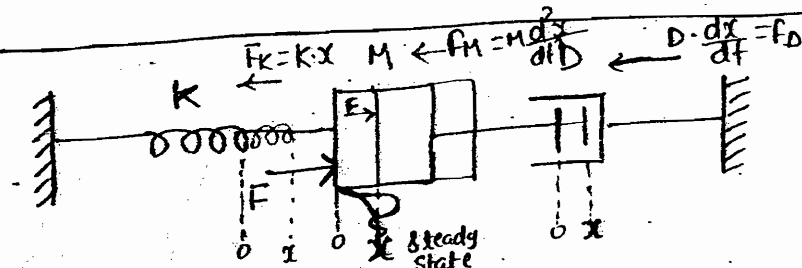
Friction coefficient

* Steady state condn means x becomes constant and $F_M = 0$ & $F_D = 0$
⇒ After removing Force x = Cons

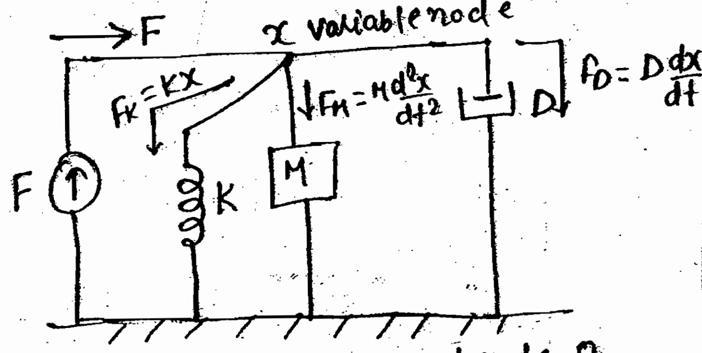
(3) Spring



$F_K = K \cdot x$ ↳ even x constant then F_K not $= 0$



When F = 0 Then All elements on Position 0
at t = 0 पर apply Force
Then all move by a distance x



F, M, K and D goes away from Ground node to a distance of x. When apply Force.

$$F = F_M + F_D + F_K \Rightarrow F = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx$$

Apply Laplace transform

$$F(s) = Ms^2X(s) + DsX(s) + KX(s)$$

$$F(s) = (Ms^2 + Ds + K)X(s)$$

ransfer function $\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Ds + K}$

$$\frac{X(s)}{F(s)} = \frac{\left(\frac{1}{M}\right)}{s^2 + \left(\frac{D}{M}\right)s + \frac{K}{M}} \Rightarrow \text{II}^{nd} \text{ order system.}$$

$$s^2 + \left(\frac{D}{M}\right)s + \frac{K}{M} = s^2 + 2\xi w_n s + w_n = 0$$

on comparing $\boxed{w_n = \sqrt{\frac{K}{M}}}$, $2\xi w_n = \frac{D}{M} \Rightarrow$

$$\boxed{\xi = \frac{D}{2\sqrt{KM}}}$$

se 1 underdamped condition.

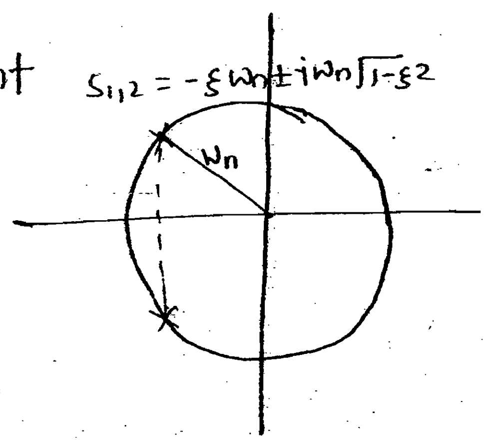
For underdamped condⁿ $\boxed{0 < \xi < 1}$

: w_n should be constant

for this K, M should be = constant

or different ξ we can vary D

$$\xi = \frac{D}{2\sqrt{KM}} < 1 \Rightarrow \boxed{D < 2\sqrt{KM}}$$



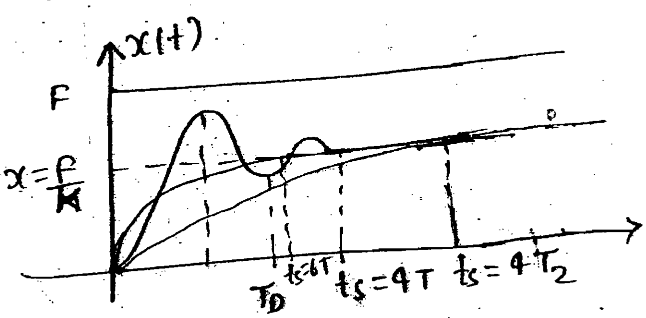
$$S_{1,2} = -\xi w_n \pm j w_n \sqrt{1-\xi^2}$$

$$S_{1,2} = -\frac{D}{2M} \pm i \sqrt{\frac{K}{M}} \sqrt{1 - \frac{D^2}{4KM}}$$

time constant

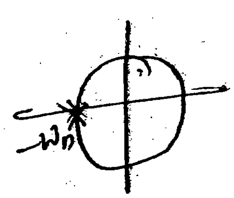
$$T = \frac{1}{\xi w_n} = \frac{2M}{D}$$ for 2% error band $t_s = 4T = \frac{8M}{D}$

$$T_D = \frac{2\pi}{w_d} = \frac{2\pi}{\sqrt{\frac{K}{M}} \cdot \sqrt{1 - \frac{D^2}{4KM}}}$$



ase 2 Critically damped

$\boxed{\xi = 1}$ $\frac{D}{2\sqrt{KM}} = 1 \Rightarrow \boxed{D = 2\sqrt{KM}}$



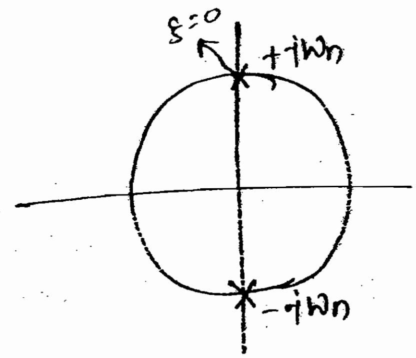
$s_{1,2} = -\omega_n = -\sqrt{\frac{K}{M}} \Rightarrow T = \frac{1}{\omega_n} = \sqrt{\frac{M}{K}}, \quad t_s = 4T = 4\sqrt{\frac{M}{K}}$

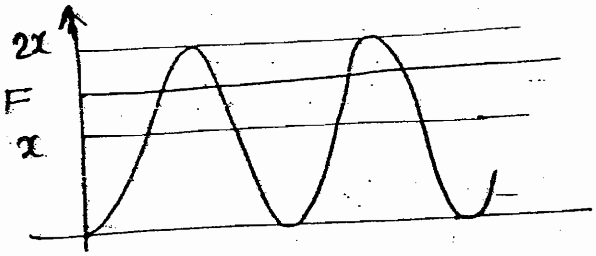
Case-III undamped cond^n

$\boxed{\xi = 0}$

$D = 0 \Rightarrow s_{1,2} = \pm j\omega_n = \pm j\sqrt{\frac{K}{M}}$

$T = \infty, \quad t_s = \infty$



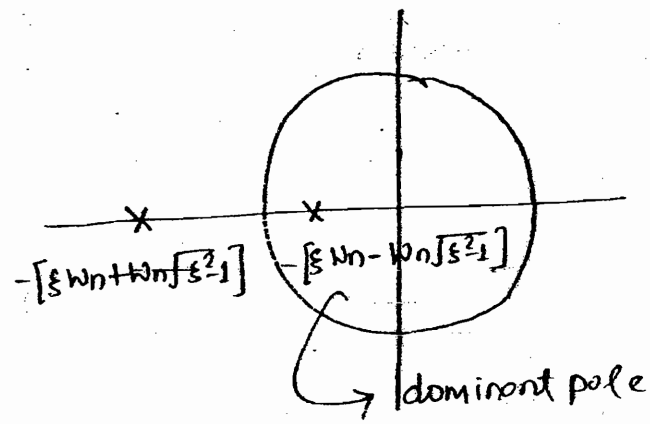


Marginally stable never settle this

Case IV overdamped cond^n

$\boxed{\xi > 1} \Rightarrow \frac{D}{2\sqrt{KM}} > 1$

$\boxed{D > 2\sqrt{KM}}$

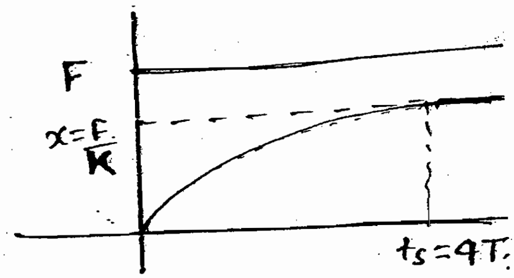


$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$s_{1,2} = -\frac{D}{2M} \pm \sqrt{\frac{K}{M}} \cdot \sqrt{\frac{D^2}{4KM} - 1}$

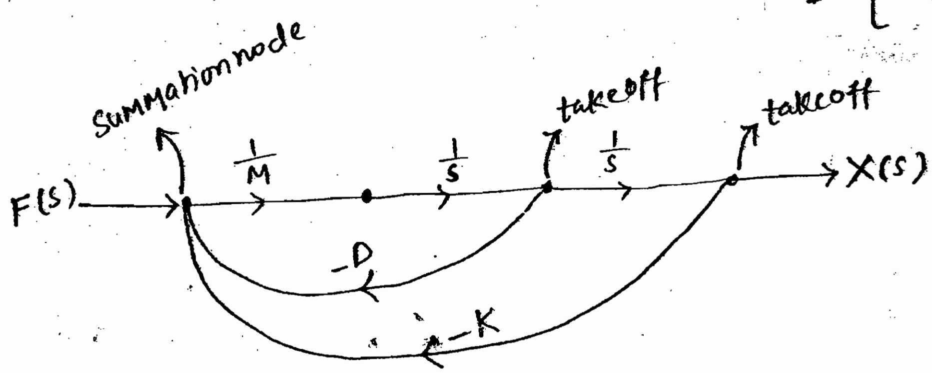
time constant $T_2 = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}} =$

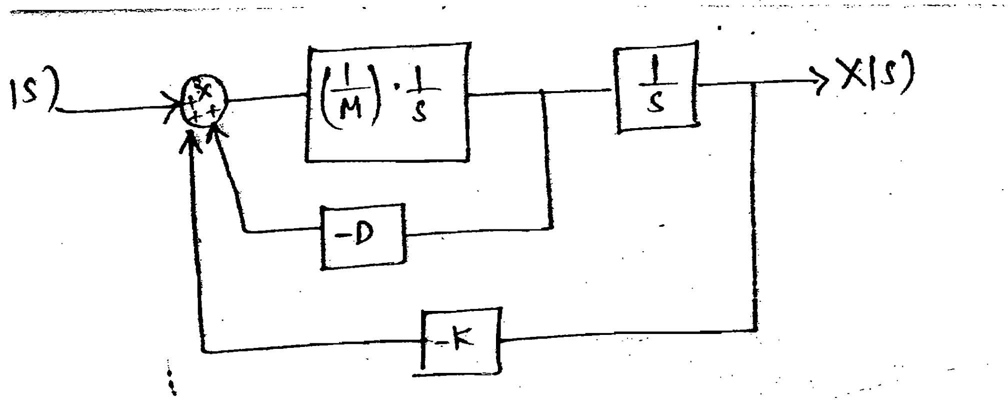
$T_2 = \frac{1}{\frac{D}{2M} - \sqrt{\frac{K}{M}} \cdot \sqrt{\frac{D^2}{4KM} - 1}}, \quad t_s = 4T_2$

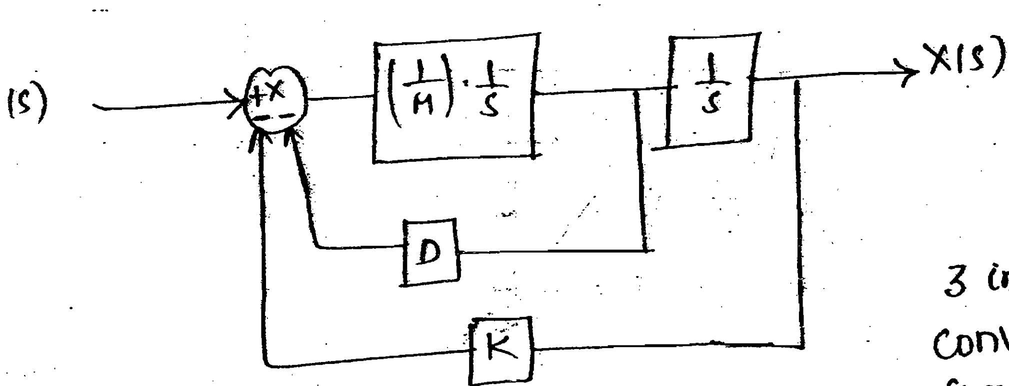


Now

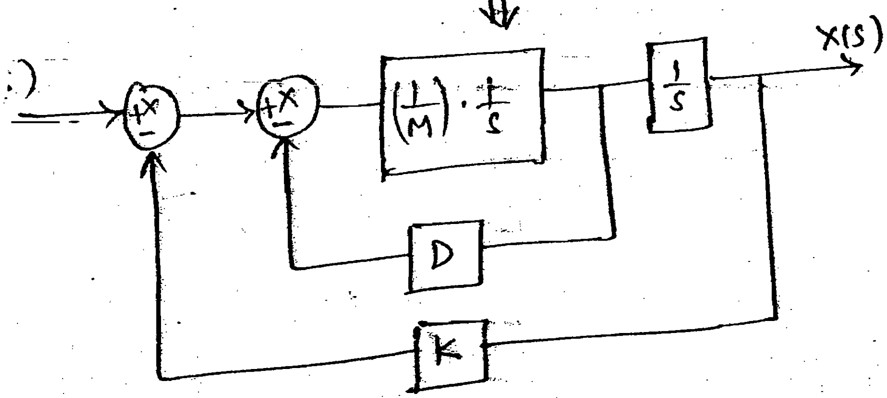
T|F $\frac{X(s)}{F(s)} = \frac{\left(\frac{1}{M}\right)}{s^2 + \left(\frac{D}{M}\right)s + \left(\frac{K}{M}\right)} = \frac{\left(\frac{1}{M}\right)\frac{1}{s^2}}{1 - \left[-\frac{\left(\frac{D}{M}\right)}{s} - \frac{\left(\frac{K}{M}\right)}{s^2}\right]}$

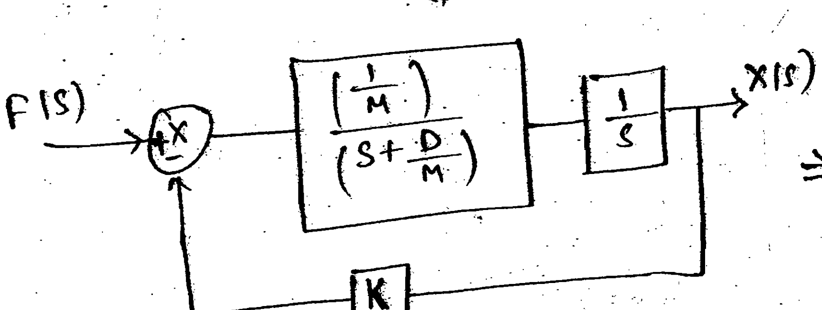




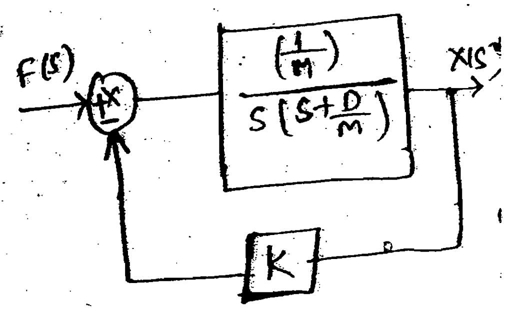


3 input summation convert into 2,2 input summⁿ side by side





⇒



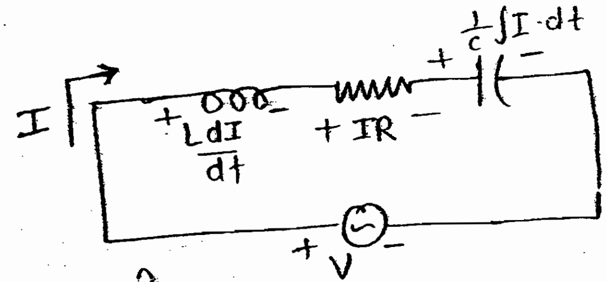
$$\boxed{\frac{X(s)}{F(s)} = \frac{\frac{1}{M}}{s^2+\left(\frac{D}{M}\right)s+\frac{K}{M}}}$$

* It is closed loop non unity Feedback s/T
to make unity feedback s/T
Then we have to put K = 1 (spring constant)
because o/p settle at

Force Voltage and Force current Analogy

Force voltage analogy

$V = L\frac{dI}{dt} + I \cdot R + \frac{1}{C}\int I \cdot dt$



$$\boxed{V = M\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q}$$ $\left\{\because I = \frac{dQ}{dt}\right\}$

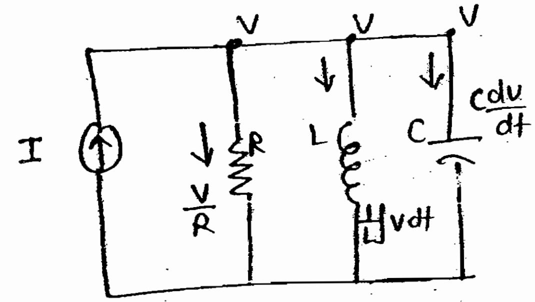
on comparing with Force eq^n

$F \leftrightarrow V$, $M \leftrightarrow L$, $D \leftrightarrow R$, $K \leftrightarrow \frac{1}{C}$, $x \leftrightarrow Q$, $v \leftrightarrow$ (Velocity)

Mechanical in series ⇒ electrical N/W will be in parallel

Force current analogy

$I = C\frac{dV}{dt} + \frac{V}{R} + \frac{1}{L}\int V \cdot dt$



$\because V = \frac{d\varphi}{dt}$ so

$$\boxed{I = C\frac{d^2\varphi}{dt^2} + \frac{1}{R}\frac{d\varphi}{dt} + \frac{\varphi}{L}}$$

on compairing with Force eq^n.

$F \leftrightarrow I$, $M \leftrightarrow C$, $D \leftrightarrow \frac{1}{R}$, $K \leftrightarrow \frac{1}{L}$, $x \leftrightarrow \varphi$, $v \leftrightarrow V$

* In Force-Voltage analogy if Mechanical system is connected in parallel then its analogous electrical S/T will connected in series. and vice-versa.

* In Force-current analogy if Mechanical system is connected in parallel then its analogous electrical S/T will also connected in parallel and if mech. S/T is connected in series and its analogous electrical S/T will also connected in series.